BEAM research papers

Learning about numbers with patterns

using structured visual imagery (Numicon) to teach arithmetic

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Summary of a research project carried out at an infant school in England

Contents

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Purpose

The focus of this teacher-led action-based research was to construct a programme of multi-sensory teaching activities, thereby developing mental arithmetic capability in children from Nursery through Key Stage 1. The basis of the teaching approach involved the use of structured imagery and apparatus within an image/symbolic-rich classroom environment. Key Stage 1 National Test scores of children following the activity programme were compared with national scores.

Context

Highhill Infant School[1] has a two-form entry with 180 pupils on its roll plus a 25-full-time-equivalent nursery class; one third of its pupils are on the special needs register and just over 20% of children qualify for free school meals. Baseline assessment results show many children enter the school with poor language skills.

Teachers in the school – in collaboration with the University of Brighton School of Education – carried out a Teacher Training Agency (TTA) Teacher Research Project during 1996-7, to begin to construct in Reception and Year 1 an effective programme of multi-sensory arithmetic activities. An extension of the project in 1997-8 enabled the activity programme to be extended into Nursery and Year 2. Early indicators of the potential success of the programme were obtained in relation to children's performance on the Key Stage 1 National Tests in 1998.

Since the study further research, carried out by Brighton EAZ, Bristol Learning Support Service and Downs Educational Trust, has investigated the effectiveness of the teaching activities for children with various special needs including Down Syndrome. This paper reports on the work with mainstream children.

Key findings

1. Children whose arithmetic had been supported using structured imagery during the research project showed a dramatic improvement in attainment in Key Stage 1 National Tests 1998 in comparison with the results of the previous cohort (1997), whose learning had not been supported by structured imagery. Since the study, similar levels of attainment have been sustained year on year at the school.

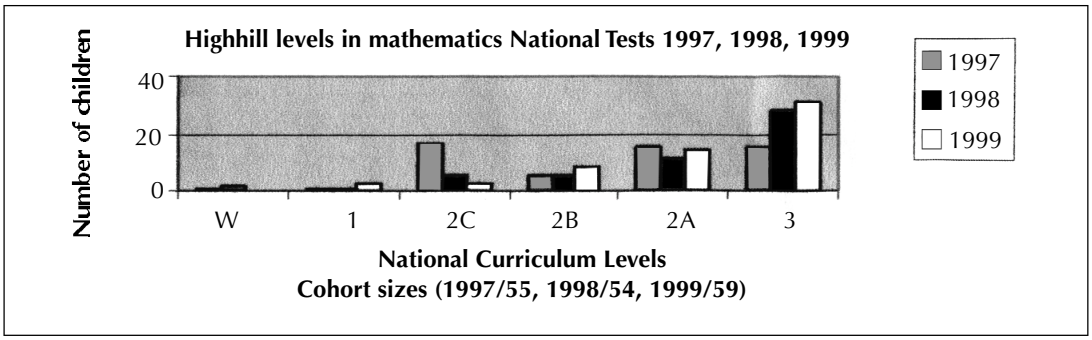


Figure 2: Key Stage 1 National Test results at Highhill

1. The school's name has been changed for the purpose of this paper.

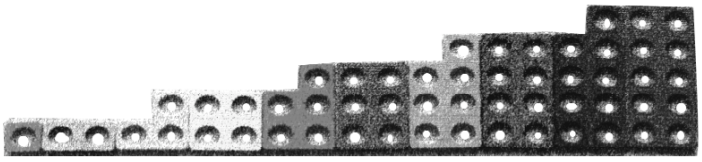
2. The programme of teaching activities helps to meet the early learning goals of the Foundation Stage and the Key Stage 1 objectives of the National Numeracy Framework.
3. Children were drawn to the apparatus and used its structured patterns to show their understanding of number and arithmetic.
4. Teachers found the programme of activities easy to follow and found it easy to assess children's understanding by observing and listening to children's explanations as they used the structured apparatus.

Background

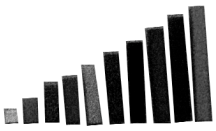
In the period before 1997, and as a result of teaching successive classes, we became aware that many children had not substantially mastered arithmetic. We tried out various published mathematics schemes, all of which relied on counting as the basis for arithmetic and moved children very quickly from counting towards formal symbols in mathematics. Some children were able to arrive at correct answers without necessarily knowing why. We felt schemes put an artificially low ceiling on teacher expectations for some children.

The serendipitous discovery of a copy of Catherine Stern's book, *Children Discover Arithmetic*[2], led us to challenge our pedagogy and explore the use of the visual images originally used by Stern within a structured programme of teaching activities that we developed during the project. The activities used shapes and rods and encouraged children to develop a systematic mental imagery of number, to develop mathematical language and to apply their arithmetic to real-life problems.

The structured images which we used are shown below:



shapes



rods

We wanted children to develop an understanding of number that relates numbers to each other (relational understanding). For example, fitting together the 3 and 7 shapes provides a visual model of why the answer is 10 and how 3 + 7 relates to 2 + 8 and 4 + 6. They could then draw on this understanding when it came to solving new problems – instead of just applying learned procedures and falling back on counting when they met something unfamiliar.

2. Stern, C. *Children Discover Arithmetic* New York: Harper & Row 1949 [1971]

Our research coincided with the publication of the National Numeracy Strategy's *Framework*[3], which emphasized mental arithmetic strategies and set high expectations of children's achievement. Paradoxically, the *Framework* did not put specific emphasis on the direct teaching of relational understanding and it avoided the use of structured apparatus – although it expected children to employ relational understanding in mental arithmetic.

The project

Throughout the project, the research was led by the maths coordinator and the headteacher, in collaboration with Tony Wing from Brighton University. In the first phase of the research the children were taught by the maths coordinator, who was teaching in Year 1, and one Reception teacher. In the extension phase of the research we wanted to see whether the programme of activities could be taught successfully throughout the school by non-specialist teachers, so all seven class teachers used the programme of activities as the basis for their arithmetic teaching.

In each class a sample of six children was chosen. To give a balance of age and gender, in each group of six there were three boys and three girls who were autumn, spring and summer born; otherwise, the children were randomly chosen, resulting in a spread of ability. At the start of each phase of the research we established the starting-point for each child on a progress chart that we devised after looking at several published baseline assessments. During the research period we extended this chart to cover the whole Key Stage 1 curriculum for number and arithmetic.

The progress chart was matched to the programme of teaching activities. The children's progress was recorded throughout the research period. The nursery staff made regular observations of children as they worked independently with the structured apparatus. At the end of the research period we were able to measure the progress that had been made by each of the chosen children. We compared the results of the whole Year 2 cohort – who did Key Stage 1 National Tests in May 1998 and who had used structured apparatus throughout Years 1 and 2 – with the results of the previous cohort, who took their Key Stage 1 Tests in May 1997 and who had been taught largely without the visual images provided by the structured apparatus.

We also looked carefully at the strategies used by both cohorts of children to answer the number questions in their Key Stage 1 National Tests.

Teaching and learning

All classrooms and the Nursery created a visually rich mathematics environment, giving number a high profile so that children could see number being used. For example, drawers were numbered as well as labelled; storage pots were marked to show how many pencils they should contain; and teachers were encouraged to take opportunities to use clocks and calendars and to show

3. Department for Education and Skills *Framework for teaching mathematics from Reception to Year 6* DfES 1999

numbers in daily use in data handling situations like 'How many children are having a school dinner today? And how many are having a packed lunch?'

Displayed prominently in every classroom was a large display number line. This was a line with numbers from 0-20 marked at regular intervals, each interval marked by the numeral, the corresponding structured image and the number word (see figure 2). There was also a smaller number line, also for display, showing numerals from 0-100.

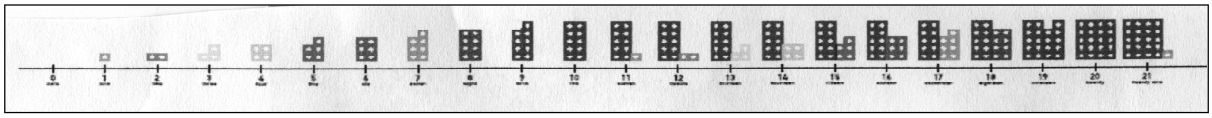
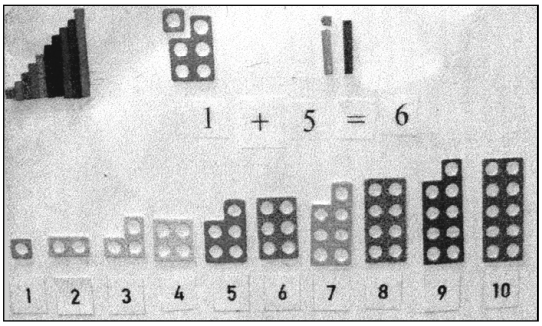


Figure 2: large display number line

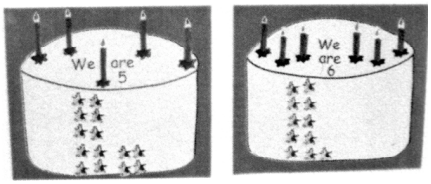
Each classroom also had a mathematics area. This contained an interactive display offering a variety of independent activities including counting, numeral recognition games, pattern making, problem solving in terms of puzzles, and construction apparatus. Here, children could practise what they had learned and make new discoveries.

The programme of teaching activities using structured apparatus was taught throughout the school in the daily mathematics lesson. The activities were predominantly practical and multi-sensory, because they involved the children seeing and feeling the structured images, whilst they were hearing and saying connected mathematical language. The tendency to move too quickly into formal symbols was resisted. The early activities in the programme focused on teaching children the patterns and how each related to other patterns before the numerals were named. Children were not required to record their arithmetic on paper until they had shown understanding in a practical context.

In whole class or focus teaching the structured apparatus was used either on a table top or on a magnetic white board to illustrate teaching points, and children used it independently either individually or in group work.



Teachers were encouraged to make connections between classroom teaching activity using the structured apparatus and the 'real world'. For example, if the children were working on addition, they would be invited to make up their own addition story to apply the number bonds they had learnt. When learning about '1 less' in subtraction, links might be made with number songs which involve decrease like 'Five little firemen'. When handling data (for example, how many children were having school meals and



how many were having a packed lunch) counters would be arranged into the Stern patterns to show how many in each data set.

The programme of activities was carefully designed to scaffold children's learning so they were not expected to take on too many new ideas at a time. Each activity built on and extended previous learning. Sometimes familiar activities were revisited with a deeper teaching focus. Inbuilt was the opportunity for children to practise and rehearse what they had learnt. The activities were simple to set up but challenged the children to think about the patterns. For example, to practise addition children might throw a number die and then feel in a 'feely bag' for two shapes that made that number. Alternatively they might build a model with the apparatus to represent number bonds of ten. Using rods children might be asked to arrange 1-10 rods in a 'staircase' pattern and then fill spaces with more rods to show all combinations of two rods which make 10.

Findings

1. The Year 2 children's scores in the National Tests at the end of Key Stage 2 showed dramatic improvement over previous cohorts' scores (see figure 1). They had developed a range of strategies to solve arithmetic problems, appeared to see numbers as related 'wholes' and generally did not solve arithmetic problems by counting. Many were able to apply their arithmetic to solve problems in context.
2. With few exceptions children had developed confident and positive attitudes to maths. Children found the images attractive and enjoyed working with them.
3. Children benefited from frequent opportunities to count ever larger sets of objects. This gave them experience of higher numbers and understanding of the structure of the number system. Teachers found this a useful assessment tool.
4. It was important to remind children to use their mental imagery – phrases such as 'let your fingers be your eyes' if they were feeling for shapes in a feely bag and 'try to see the shapes in your mind's eye' when they were doing mental arithmetic, were found to be helpful.
5. Children were not asked to record arithmetic until they had understood arithmetic symbols and knew addition and subtraction facts to 10 in practical activities. However, when they started to record arithmetic they were able to do this quickly and accurately. It was found that it was also important that they had developed pencil control so the writing process was not laborious for them.
6. Teachers found the activities were straightforward once they were familiar with the approach. They benefited from time to read and understand the rationale behind the approach. They found the programme of activities and assessment sheet useful in planning and assessment and that they supported appropriate grouping of children for focus teaching and independent work.
7. Teachers found that the imagery also provoked ideas for the pupil on position, action, pattern, colour, shape, odd and even, more and less as well as the value of numbers. In this supportive mathematical context children were helped to understand much mathematical language.
8. Parents recognized their children's success and began to see evidence of their children's understanding of arithmetic at home.

Subsequent development

In 1998 at the end of our research project we wrote "We expect it is highly likely that it is the strategies children use when answering arithmetic questions at the end of Key Stage 1 SATs which are the significant performance indicator for potential success at Key Stage Two SATs, and not the overall level. The arithmetic questions at Key Stage One SATs are simple and many can be solved purely by counting; children can achieve level 2C having answered very few number questions. However at the end of Key Stage Two children will not be able answer the arithmetic questions if counting is their only strategy; relational understanding and strategic arithmetic are essential for success. Our children's impressive success us in the end of Key Stage One SATs raised the question of how high our expectations should be of their achievement at the end of Key Stage Two".

Four years later our first research cohort of children (who had taken Key Stage 1 National Tests in 1998) took their Key Stage 2 National Tests in 2002. Their results show a marked improvement over the results of the previous cohort. The Key Stage 2 results for 2003 show that the improvement has been sustained.

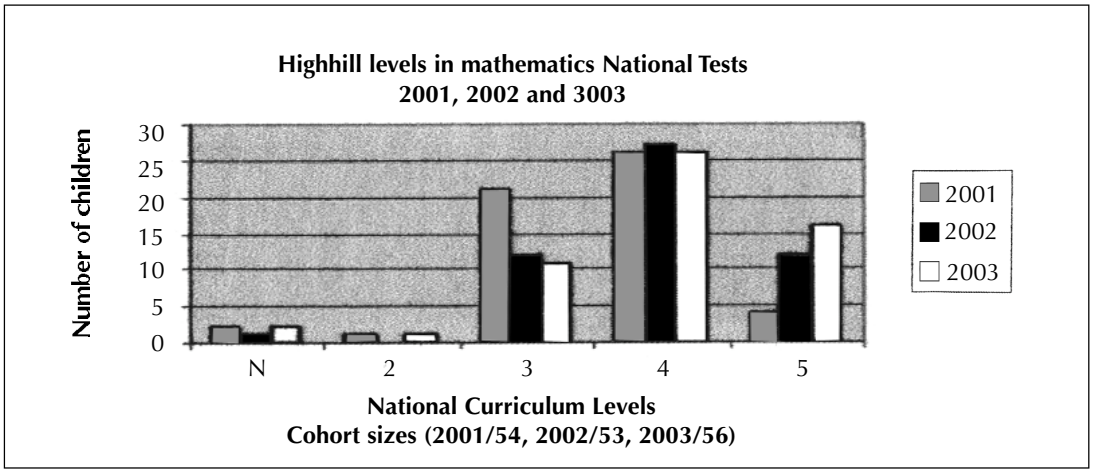


Figure 3: Key Stage 1 National Test results at Highhill

Since our original study further research has been carried out by Wiltshire County Council Psychological Service in conjunction with the Downs Educational Trust. Their findings suggest that the approach is particularly relevant to children with Down Syndrome who tend to be good visual learners, and are good with patterns but tend to have poor auditory memory. The Wiltshire Project was a pilot for a much larger scale study which is currently underway led by the Down Syndrome Educational Trust in Portsmouth.

Interest in our work has been shown by the Dyslexia Institute which suggests the present extent of research indicates that support for pupils with some specific difficulties in mathematics should be structured, cumulative and multi-sensory.

Further development work in Key Stage 2 has been carried out through a DfES Best Practice Research Scholarship at a school on the South Coast, supported by Brighton EAZ and the University of Brighton, Brighton.

The programme of activities developed through the research project has now been published and is available to teachers as Numicon.

Ways forward

Our findings show striking evidence of children (from all ability groups) responding positively to visual structured images and the related programme of activities. Colleagues in other schools who have adopted the approach since dissemination of the findings report similar responses, as do ITT students using the approach. We feel this justifies much larger-scale development work to further test the findings, especially since the approach enables achievement of the National Numeracy Strategy's objectives in the *Framework*.

Other research into arithmetic teaching suggests that current approaches in many schools, and in many published mathematics materials, continue both to underplay confident, relational mental arithmetic and to allow children to build their arithmetic laboriously upon limiting counting procedures. Unfortunately the models and images for number recently published by the National Numeracy Strategy do not include any references to structural apparatus. We find this disappointing in the light of the many children's successes with Catherine Stern's images and approaches that we have seen, but perhaps classroom teachers will continue to choose for themselves approaches and equipment that they know work.

Further reading

Atkinson, Ruth; Tacon, Romey; Wing, Dr. A, *Teachers' Books for Foundation Stage Year One, and Year 2*, Numicon Ltd. 1999-2003

Bird, Gillian, *Number skills development for infants with Down syndrome (0-5 years)*, The Down Syndrome Educational Trust 2001

Bird, Gillian and Buckley, Sue, *Number skills for individuals with Down syndrome – an overview*, The Down Syndrome Educational Trust 2001

Dyslexia Institute, *Dyslexia and Mathematics*, The Dyslexia Institute, 133 Gresham Road, Staines, TW18 2AJ

DfEE, *The National Numeracy Strategy*, March 1999

Haylock, Derek, *Mathematics Explained for Primary Teachers*, Paul Chapman Publishing, l995

Henderson, Anne, 'Working with Dyscalculia', *Learning Works* 2003

Hughes, Martin, 'Children and Number', *Difficulties in Learning Mathematics*, Blackwell, l986

Skemp, Richard R., *The Psychology of Learning Mathematics*, Penguin Books l971

Stern, C., *Children Discover Arithmetic*, Harper & Row, New York l949

Wing, Tony, 'Serendipity, and a special need', *Mathematics Teaching* 174, Association of Teachers of Mathematics, March 2001

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