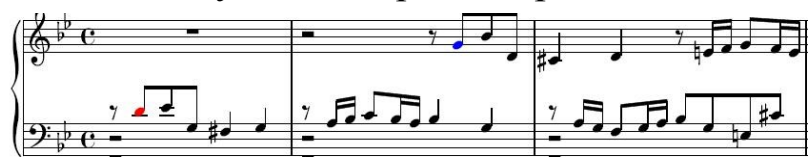# Mathematics and Music

Down the ages people have been struck by the connections and parallels between maths and music. Pythagoras is said to have noticed that the different musical notes emitted from a blacksmiths workshop depended on the lengths of the metal objects being struck. Important results were established by the mathematicians of ancient India through their study of the rhythms of music.

The basic musical interval (frequency ratio between notes) is 2:1 and is called an *octave*. The next most important intervals are the *fourth* and the *fifth*. These intervals are found in music of all cultures. The frequency ratio is 4:3 for the former and 3:2 for the latter.
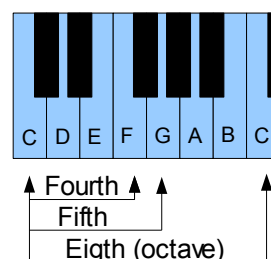
Western music divides the octave into 12 intervals called semi-tones. Up until the 17th century musical instruments were tuned purely by what sounded right. This meant that the frequency ratio between successive semi-tones (adjacent notes on the keyboard) varied by a small amount. This means that if musicians tried to play a sequence of music slightly higher or lower it didn't sound the same. The solution to this problem was to tune musical instruments on mathematical principles so that the frequency ratio between semitones is exactly $2^{1/12} = 1.0594631$. Musical instrument tuners developed various techniques for achieving this exact division of notes which is called the "equal temperament scale". All modern western instruments are tuned to this scale.

J.S. Bach (1685-1750) wrote his *48 Preludes and Fugues* to demonstrate the versatility of the equal temperament scale. He composed preludes and fugues in all 12 major keys and all twelve minor keys (2x2x12 = 48).

The opening 2-bar phrase of this fugue is transposed to a different pitch. Even if you don't read music you can see the repeated pattern of notes.

In addition to the mathematics of musical notes there are many mathematical features of music itself. Thus in a fugue the same pattern is *translated* (*transposed)* to different pitches. Often the same patterns are *stretched* or *compressed* or *reflected*. These are all ways in which a mathematical function can be modified. There are many other direct analogies between mathematics and music so it is no wonder that so many mathematicians and musicians have been interested in each others' work.