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Supporting more able students in the Numicon Approach

Managing Student's Learning when they are above

Being clever at maths includes understanding all the strands, not just number.

Being able to work with patterns, relationships and confidently converse in mathematical language.

Making connections in maths and solving problems by applying rules and principles is necessary for all children to demonstrate mastery of a concept and connected maths ideas.

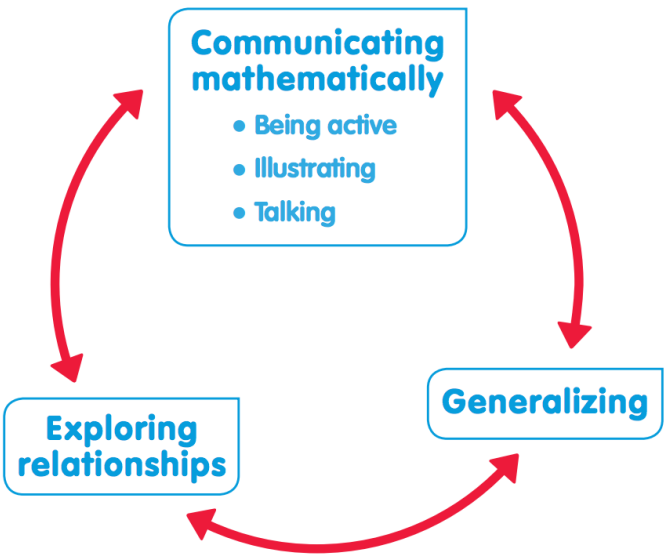
For this reason, Numicon supports children to broaden their understanding, go deeper and think mathematically from Year 0.

The Curriculum clearly asks for children to investigate, explain and justify in every strand in Maths.

The Numicon Approach draws all children to work together to explore the relationships and patterns in maths. Maths is a language. This aspect is also included in the Curriculum.

Numicon makes this a key aspect of the teaching and the learning. Words and terms are provided for the teacher along with opportunities through each week to incorporate this use of these words and terms in the conversations and activities.

Generalising in maths is the Going Deeper. Just answering the question is not beneficial to the long term understanding for the children. We want them to really know the how maths happens not just the what.



If children are consistently demonstrating all these qualities and behaviours, use the suggestions below to develop a rich learning experience for them.

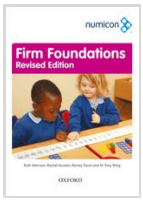
Children who are gifted in mathematics will need these adaptations and if necessary, promotion to the learning in the next year's curriculum level.

- Refer to the Numicon Mapping Chart, downloadable from Numicon Online NZ to familiarise yourself, understand and appreciate the context of the entire curriculum in all strand knowledge required in the years before and after.
- In considering the future years note how they develop from the present year that you are teaching. Teach from the year expectations rather than the year ahead.
- Go deeper with these concepts by providing many opportunities for the students to explain their thinking and reasoning and justifying the findings. When children have gained mastery, they can use the knowledge flexibly. Only when these adaptations are exhausted, move onto the next year.

Phase 3

Number

	During year 7 Informed by prior learning, teach students to:	During year 8 Informed by prior learning, teach students to:	Teaching considerations
Number structure	identify, read, write, compare, and order whole numbers using powers of 10 (e.g., 10,000 = 10^4) **NPC5** NNS 1.5, 1.6 **NPC5** P&A 1.3 **NPC5** Calc 10.4, 10.5	identify, read, write, compare, and order whole numbers and decimals using powers of 10 (e.g., 0.01 = $\frac{1}{100} = 10^{-2}$) **NPC6** NNS 1, 2 **NPC6** Calc 5.1	**Represent** and order numbers using place-value (PV) expanders or charts and number lines.
	find the highest common factor (HCF) of two numbers under 100, and find the least common multiple (LCM) of two numbers under 10 **NPC5** P&A 3	use prime factorisation to represent a number and to find the HCF of two numbers **NPC6** P&A 1	**Represent** factors using factor trees, or systematic lists. **Connect** HCFs to simplifying fractions, and LCMs when renaming fractions. **Generalise** conjectures about prime or composite numbers by investigating factors.
	use exponents to represent repeated multiplication, and identify square roots of square numbers up to at least 100 **NPC5** P&A 4.4, 4.5, 4.6, 4.7, 5.7 **NPC6** Inv 1	identify and describe the properties of prime and composite numbers up to at least 100 and cube numbers up to at least 125 **NPC5** P&A 3.4, 3.6, 3.7, 4.7 **NPC6** P&A 1.2, 4.5	**Investigate** and **generalise** divisibility tests for composite and prime numbers, and **connect** the results to square and cube numbers and square roots. **Investigate** and **explain** patterns in repeated multiplication and **represent** them using exponent notation. **Connect** prime and composite numbers with factors, and **represent** a number as a product of its prime factors (prime factorisation).
Operations	use rounding and estimation to predict results and to check the reasonableness of calculations **NPC5** NNS 4.8 **NPC5** Calc 3.2, 3.3, 5.1, 5.3, 5.4	use rounding, estimation, and benchmarks to predict results and to check the reasonableness of calculations **NPC6** NNS 1.3 **NPC6** Calc 3	**Explain** efficient methods for supporting estimation (e.g., when adding a long list of numbers, look for numbers that can be grouped and summed to roughly 10, 100, 1,000). **Connect** operations to benchmarks to make estimates (e.g., 73% is roughly $\frac{3}{4}$). **Explain** and **justify** findings, by **connecting** to estimates and other checking methods such as using the inverse operation.
	round whole numbers to any specified power of 10, and round decimals to the nearest tenth, hundredth, or whole number **NPC5** NNS 4 **NPC6** Calc 3	round whole numbers to any specified power of 10, and round decimals to the nearest tenth, hundredth, thousandth, or whole number **NPC5** NNS 4 **NPC6** Calc 3	
	recall multiplication facts to at least 10 × 10 and identify and describe the divisibility rules for 2, 3, 5, 9, and 10 **NPC4** Calc 5, 6 **NPC5** Calc 4.1 **NPC5** P&A 4.1, 6.2 **NPC6** P&A 4.5	identify and describe the divisibility rules for 2–11 **NPC6** P&A 4.5 **(covers divisibility by 2, 3, 5, 9, 10)** **Activity to follow**	**Investigate** patterns in multiples in 100s boards and multiplication charts to **generalise** divisibility rules.



Years 0 and 1 follow the learning progressions and related activities in Firm Foundations.

Surrounding the number knowledge concepts are related measurement, geometry, algebra, statistics and probability activities to build a connection and richness in maths. Children make generalisations about numbers and patterns and their relationships and equally develop their conversational confidence in using mathematical language.

All these knowledge and surrounding activities have the potential for even greater depth. Language and conversation is a core focus of the learning activities.

Introducing Numicon Shapes

Activity Card **1**

As you welcome children into the classroom, encourage them to explore their environment and resources, playing alongside them. Watching what they do, listening to what they say and how they count, will establish their starting points and counting range.

As you help children settle by establishing daily routines, children will see Numicon Shapes being used every day to solve mathematical problems, e.g. How many children are here today?

This card's activities focus on colour, size and comparison of various objects, including Numicon structured manipulatives. Listen for children using comparative language and note how they refer to Numicon Shapes. Some may simply describe them as 'big' and 'small'; others may use colour names, use number names spontaneously or count holes. Decide together on suitable colour names, e.g. (in order, 1–10) orange, light blue, yellow, light green, red, turquoise, pink, green, purple, blue, and use these consistently. When children refer to Numicon Shapes by number names, acknowledge correct use and amend any mistakes so they do not practise incorrect names.

Children freely explore the size, pattern and orientation of Numicon Shapes independently. Look for children trying out how to turn and flip the Shapes to fit them together, noting those who can do so without many attempts.

Arrange a parent meeting to explain how maths will be taught and how they can be involved in their child's learning (see the Agenda for parents' meeting and the Home learning overview available in *Numicon at the Seaside*).

Watchpoints

At this stage, avoid telling children number names for the Numicon Shapes as it is important that they make this connection themselves. Telling them hinders your opportunity to assess their number understanding.

Key mathematical ideas (Teaching Guide, page 8)

The main focus is on:
Contrasting, comparing and combining, counting

Children will also encounter:
Equivalence, order, sequences and direction, one-to-one correspondence, the successor relationship, part-whole relationships, pattern and generalizing, shape and space reasoning and logic

Communicating

Model these words and terms, and listen for children using them in the same ways as you use them:

- **Words for counting**, e.g. number names (one, two, three, ...), number, count, how many?
- **Words for comparing size and quantity**, e.g. bigger, smaller, largest, smallest, same size, too big, too small, all, some, part, whole, more, fewer, less, same, different
- **Words for position**, e.g. before, after, first, last, next, in between, upside down
- **Words for action**, e.g. match, order, put in order, cover, flip, turn over, turn around, point to
- **Colour names for Numicon Shapes**

Learning opportunities

- To chant the count sequence in order.
- To count objects using one number name for each object counted.
- To become familiar with the Numicon Shapes and their colour names.
- To cover the whole Numicon Baseboard with Numicon Shapes by matching and manipulating them to fill the available spaces.
- To notice the size order of Numicon Shapes.
- To use the language of size and make simple size comparisons, e.g. big and small.

Assessing

Look and listen for children who:

- Chant number names in order consistently (note children's counting range).
- When counting objects, say one number name for each object counted (note children's counting range).
- Use colour or number names for Numicon Shapes.
- Match a Numicon Shape to its Large Foam Shape.
- Match Numicon Shapes to their pictures.
- Rotate and flip Shapes to cover the Baseboard.
- Use sizes of objects as a criterion for making comparisons.

As children explore the Shapes, they will start to use them to cover the Baseboard. Encourage them to match Shapes to spaces. Talk about colour, size, pattern, position.

> Can you cover the whole Baseboard?
>
> Can you find a big Shape to fill this space?

Increased challenge: try without the extra 1- and 2-shapes; without using 10-shapes; using only four colours of Shape.

> Can you cover the Baseboard without using any orange Shapes?
>
> Which Shape will fit this space?

Increased challenge: children sort the objects into a group of small and a group of large, count the number in each group and compare the sizes of the two groups.

Maths games and puzzles

- Put out Numicon Shapes, Baseboards and Overlays for children to continue to explore with their friends.
- Put out puzzles involving manipulation of shapes, e.g. one-piece and multiple-piece inset puzzles.
- Children play colour-matching games, e.g. colour dominoes.
- Children play counting games, e.g. board games, collecting counters.

Role-play, small world and construction

- Set up role-play equipment to provide children with opportunities for exploring big and small, e.g. 'The Three Bears'.

- Invite children to show 'big' and 'small' using poses or movements.

> Can you make a big shape with your body?
>
> What is the smallest shape you can make with your body?

Understanding the world

Look around the immediate environment and talk about the similarities, differences and patterns in size and colour that you can see.

Years 2 to 8

The teaching progressions provide a spiral approach to learning, enabling the concepts to be frequently visited so children are constantly reminded of previous learning and make connections through all the strands.

Strand and Activity Group Number		Activity Group Title
Getting Started		Getting started with apparatus and imagery
Numbers and the Number System	1	Working with numbers up to a million
Numbers and the Number System	2	Exploring equivalence with fractions
Numbers and the Number System	3	Understanding decimals
Geometry	1	Measuring angles
Calculating	1	Developing fluency with adding and subtracting calculations and understanding inverse relationships
Calculating	2	Strategies for bridging when adding and subtracting mentally
		NPC Milestone 1

The weekly Overview or summary provides the connecting points for the teacher to build a rich learning experience. It begins with the Key Mathematical Ideas. The teacher is provided with the explanation for these in the Implementation Guide link within the online lessons or in the accompanying printed book. It is from this Implementation Guide that the teacher can read how to make the learning deeper for the more able children. Eg, developing the skills of generalising.

Geometry 1: Measuring angles

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Key mathematical ideas Angles, Rotation, Equivalence, Invariance, Ordering, Being logical

Educational context

In the *Geometry, Measurement and Statistics 5 Teaching Resource Handbook*, Geometry 3, children encountered both the 'static' and the 'dynamic' aspects of angles – that is, angles as fixed properties of shapes and angles as amounts of turn or change in direction. *Geometry, Measurement and Statistics 5* builds on this experience, providing children with an opportunity to refine and develop their understanding of this learning through practical exploration and discussion.
In this activity group children are introduced to the measurement of angles in degrees, and have the opportunity to practise classifying, estimating, measuring and drawing angles accurately, using a protractor, in a variety of meaningful contexts. They also begin to consider relationships between angles, in particular angles which meet at a point or on a straight line, and to use this understanding to work out missing angles. This work lays the foundations for Geometry 3, in which children will explore more general ideas relating to angles in 2D shapes, and encounter geometric proof.
Throughout the activity group, support children to investigate for themselves and in practical ways. Provide resources and time to allow them to explore, in particular, the relationship between different illustrations of angles, and between angles as turns and describing directions as 'fixed' quantities.

Learning opportunities

- To understand that an angle is an amount of turn and is measured in degrees (°).
- To understand that a full turn is 360°.
- To identify acute, obtuse and reflex angles and estimate their size, in degrees.
- To measure and draw angles, in degrees, using a protractor.
- To find missing angles at a point or at a point on a straight line.

Terms for children to use

angle, turn, rotation, rotate, clockwise, anticlockwise, vertex, degrees, acute angle, right angle, obtuse angle, straight angle, reflex angle, quarter turn, half turn, three-quarter turn, full turn, circle, semicircle, direction, distance, points of the compass, estimate, measure, protractor, accurate, precise, rounding

Assessment opportunities

Look and listen for children who:
- Use the terms for children to use effectively.
- Explain that an angle is an amount of turn or a measure of change in direction and is measured in degrees.
- Know that a full turn or full circle is 360°, and know or can work out the size of other fractions of a full turn or circle, e.g. that a quarter turn or right angle is 90°.
- Classify an angle as acute, right, obtuse, straight or reflex, and use this understanding to give a reasonable estimate of its size.
- Use a protractor to measure and draw angles.
- Calculate missing angles at a point and at a point on a straight line
- Understand the types of shapes that belong in each part of a sorting diagram, e.g. in the intersection of sets on a Venn diagram.

GMS Milestone 1

- Estimate and classify angles between 0° and 360° (GMS 5:1a)
- Measure angles using a protractor, correct to the nearest degree (GMS 5:1b)
- Find missing angles at a point and in one full turn, or at a point on a straight line and in a half turn, using knowledge that the total angle is 360° or 180°, respectively (GMS 5:1c)

Explorer Progress Book 5, pp. 2–3

After completing work on this activity group, give small focus groups of children their Explorer Progress Books and ask them to work through the challenges on the pages. As children complete the pages, assess what progress they are making with the central ideas from the activity group. Refer to the assessment opportunities for assistance.

Explore More Copymaster 1: Open Book

After completing work on Activity 2, give children Explore More Copymaster 1: Open Book to take home.

Pupil Book 5, pp. 14–17

These pages in the Pupil Book provide further practice and challenging questions. You can use them to follow up the activities and deepen the learning.

Focus activities

1. Introducing degrees
2. Measuring angles with a protractor
3. Measuring angles and planning a route
4. Drawing angles and plotting a route
5. Finding missing angles at a point and on a straight line

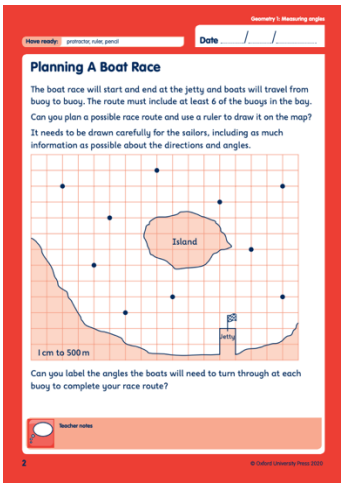
> Being logical in doing mathematics usually involves using what is called deductive logic; and deducing something involves moving from a general statement to a particular one. For example, knowing that 'the exterior angles of any polygon add up to 360°' allows us to deduce that the exterior angles of a triangle will add up to 360° because a triangle is a polygon. By using generalizations in various logical ways in mathematics, we can be sure that our reasoning is reliable.

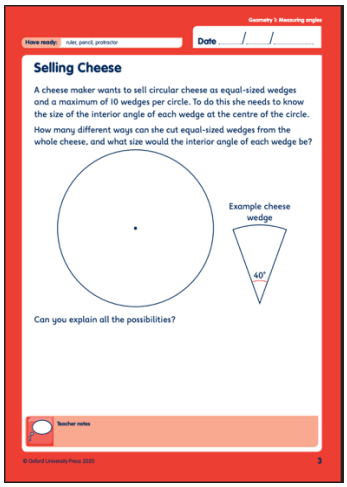
Educational Context - An invitation to consider the previous learning to build on for this week with information on the learning for this week and insight in developing mathematical thinking. It's at this point that a teacher can provide further activities to enrich learning

> Throughout the activity group, support children to investigate for themselves and in practical ways. Provide resources and time to allow them to explore, in particular, the relationship between different illustrations of angles, and between angles as turns and describing directions as 'fixed' quantities.

Learning Opportunities, Words and terms for use in conversation, Explore More activities to use in the classroom and play at home further enrich the learning and potential for deeper enrichment.

The assessment resources provide opportunities for children to demonstrate not only what they know but how they apply their learning. The second page of each Explorer Progress set of pages provides the students to show their deep reasoning and making connections.





The weekly Focus Activities are designed for all children to participate in learning the key mathematical ideas. The opening activities/lessons serve as a review from previous learning and a foundational introduction to the learning throughout the week. The activities increase in impact and complexity through the week with the final activities written for the more able students. These activities are strong in their generalising and application to other settings to discover the algebraic implications. There are many opportunities through the week for challenge and investigations. The follow-up Student Book activities provide practice and investigations to go deeper in the understanding of the concepts.

Focus activities

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2. Measuring angles with a protractor
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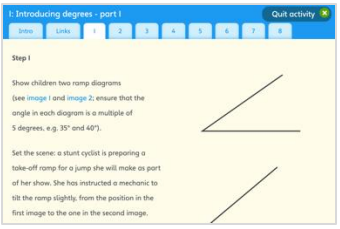
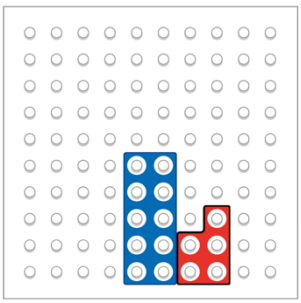
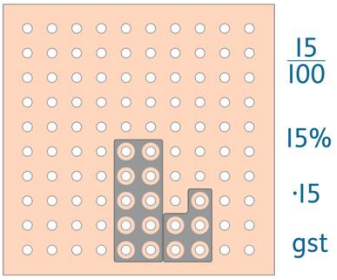


Consolidation is built throughout the programme so that children can grow in their learning at whatever stage of learning they are on. This schema builds ideas and connections, drawing from their past learning. It's like a jigsaw puzzle of seeing how maths ideas connect with prior learning.
It's important to not rush this process.
The practice of using the same manipulatives to illustrate many concepts help with the building of the schema for each child. Eg. Using the same materials for fractions, decimal, percentages and GST and links the concepts together. This enables them to go deeper into more complex reasoning.





Once the activities have been introduced children will then work in their groups. It is at this point that the extension activities are introduced to develop investigations, problem solving and creative maths throughout the week. The grouping builds a community of learning but also an opportunity to see possibilities when working alongside all learners, especially in discussions.

Using manipulatives allow children to explore *how* they know rather just *what* they know.

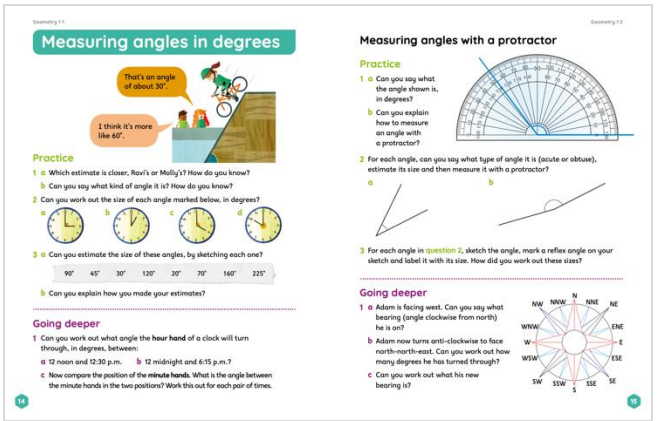
Using the manipulatives makes the harder maths more accessible for everyone. This is inclusive teaching practice. The lessons, through all the levels, include many opportunities to extend the thinking of the more able children.

Encourage children to identify any other factors of 360 that they may not have found using this method, e.g. 15 × 24, 8 × 45.

Encourage children to consider situations where angles might need to be described very precisely, e.g. angling a telescope to look at a particular star, altering a ship's course, adjusting the rear wing of a racing car.

Challenge children to create their own map and navigation instructions, for others to plot the route according to the instructions, then check whether they are successful by overlaying them on the map. Children could choose their own theme for the map, e.g. a maze, a treasure trail.

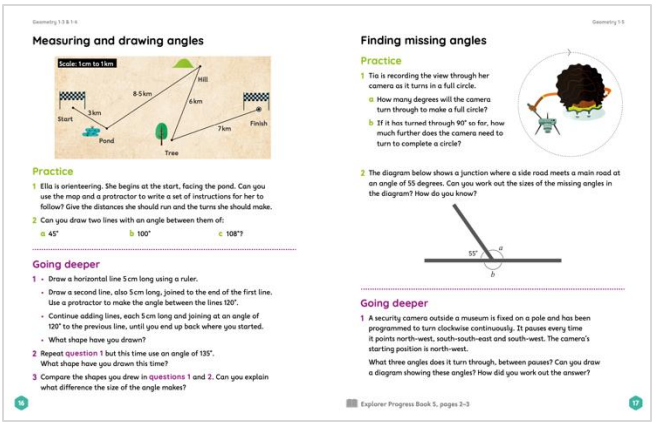
Encourage children to share and check their work with each other and, if their findings are very different, to try to work out why.



The first part of the activities demonstrate the consolidation to show what they really know.

The more able will go onto the Going Deeper section with understanding and enthusiasm. These questions prompt to really think about the why and extend the learning.

Again, the children work in collaboration to build the language and confidence to articulate their learning.



- At the conclusion of the lesson, during the Connecting and Reflecting Time, Numicon Whole Class Discussions serve the students to also think deeper about their learning, how it might be applied to everyday life and other connections in maths. They also challenge children to think deeper in maths problems.
The Implementation Guide also suggests questions as an encouragement for them to think deeper. These questions and problems help them become consciously aware of what they know and begin to monitor their own learning. eg. 'Is there something you would like to do again?'

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Practice and discussion: Whole-class

- Discuss with children how and when the mathematics they have been learning could help them in solving problems.
- Call out an angle between 0 and 180 degrees. Children should bring both arms out in front of them, hold one still and move the other arm to make the angle.
- Show children a range of acute, obtuse and reflex angles and ask children to identify each one. Ask children to order the angles from smallest to largest.
- Invite children to solve missing angle problems using adding and subtracting facts, e.g. show two angles on a straight line, one labelled and the other unlabelled, or show a circle divided into two sectors with one labelled internal angle and the other unlabelled for children to find the missing angle.

- Numicon has written a specific programme that can be purchased for these students - **'Investigations with Numicon'.** This resource of ten challenges provides guidance for teachers and suggested teaching approaches. The students will make conjectures and realise the importance of working systematically as they search for patterns. They will need to think logically as they test their conjectures while working towards making generalisations.

Further...

- Use the resources on Tahurangi - The 'Figure It Out' books
- Use your assessment data to inform the starting point of your teaching to create the learning journey for your students in this transition year. Plan this as a team to ensure the coverage of the TM curriculum throughout the year.
- Contact the Gifted Children's website for more information. Use the resources from Mindplus.
- For Year 8 students, contact your local HOD at the secondary school(s) to create a learning journey.