Term 1

Numicon 6 Student Book

Collaborative problem-solving activities for Year 8

numicon

START

$\frac{3}{4} =$

$\frac{1}{3} = \frac{4}{12}$

$\frac{1}{2} = \frac{6}{12}$

OXFORD

Numicon 6 Student Book

Information for Year 8 teachers

Planning for mathematical progression

This book is designed to be used alongside the Numicon 6 Teaching Handbooks, which can be found on the Teaching Handbooks tab of your *Numicon Online* subscription. The Teaching Resource Handbooks include a suggested order for teaching the activity groups through the year, which the Student Book follows. There are four Student Book pages for each activity group.

Where to find the main teaching

The main teaching to cover before children work on a page from the Student Book is shown at the top of the page. For example, before children work on page 2 of this book, teach Activity 1 from Numbers and the Number System 1.

Using the Student Book

The pages in the Student Book provide further practice and challenging questions to follow up the main teaching activity and deepen the learning. You can use the questions in the Student Book selectively, to meet the needs of children in your class.

Using apparatus alongside the Student Book

The Student Book questions aim to stimulate rich mathematical discussion. Children are encouraged to make use of structured apparatus and imagery in responding to the questions and communicating their ideas.

Where to find answers

You can find complete answers to the questions in this book in *Numicon Student Book 6 Answers* on *Numicon Online*. The answer book also contains an introduction to the Student Books by Dr Tony Wing and a chart to support your planning.

Assessing understanding with the Explorer Progress Books

After completing work on an activity group, you can use the Explorer Progress Book to assess how well children have understood the key learning.

At the end of each 4 page section in the Student Book, you will find a reference to the Explorer Progress Book. For example, page 5 of the Student Book links with pages 4–5 of *Numicon Explorer Progress Book 6a*. You can find the Explorer Progress Books by going to the Online Index on *Numicon Online*.

Tracking children's progress with Numicon Online

You can use assessment evidence from the Explorer Progress Books, and from children's work throughout the activity group as a whole, to record progress on the Numicon 6 Milestone Tracking spreadsheet on *Numicon Online* (on the Oxford Owl website).

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Term 1

Numicon 6 Student Book

Collaborative problem-solving activities for Year 8

Written by Jayne Campling, Andrew Jeffrey,
Adella Osborne and Dr Tony Wing

OXFORD

Term 1 Contents

Buy a Bike

$\frac{1}{3}$ off

$100

TOP BIKE

30% off

$100

How to use this book

Welcome to the Numicon 6 Student Book.

I'm Tia.

My name is Ben.

I'm Molly.

I'm Ravi.

Here are some of the things you will find inside this book.

Coloured sections
You will notice that the sections of this book are coloured red, yellow, blue, green or purple.

Each colour shows a different maths theme:

Pattern and Algebra

Numbers and the Number System

Calculating

Geometry

Measurement

In this book you can try out new calculations…

… and new methods for finding answers.

There are opportunities to look for patterns…

…and to think about how different maths ideas are connected.

Practice

These questions help you to practise and explore the new maths ideas you have learned.

Going deeper

These questions give you extra challenge and make you think deeply.

You will need to work with a partner on questions that have this symbol.

When you see this grey symbol you can do these activities in the Explorer Progress Book pages online.

Exploring large numbers

Norway population:
5 223 256

Switzerland
population: 8 341 600

More people live in Switzerland than in Norway.

Molly

Ben
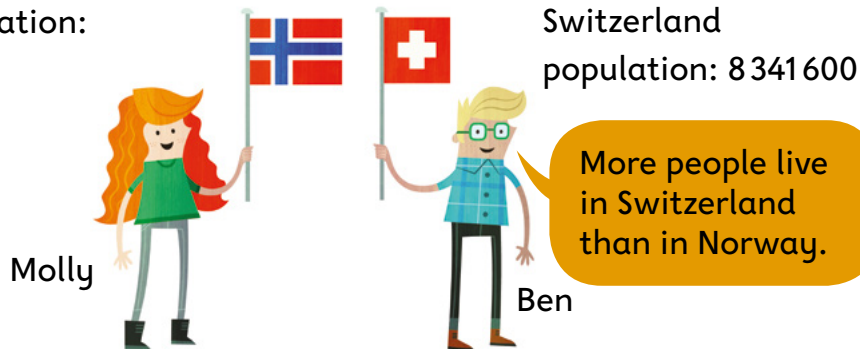
Practice

1 **a** Can you say both Molly's and Ben's numbers to your partner?

   b Is Ben correct? Explain to a partner how you know.

   c What is the value of the '3' in each of the numbers?

2 **a** Which of these countries' populations is the greatest?

Seychelles 91 437	Hong Kong (China) 7 234 400	Luxembourg 576 300
Sierra Leone 7 085 631	Hungary 9 823 070	Serbia 8 778 826

   b Can you order the populations from the greatest to the least? Compare your answers with a partner's.

3 What is the value of each '7' in the populations above? Can you explain how you work out these values?

Going deeper

1 Can you write the number that is 10 000 more than 91 400?

2 Can you write down the numbers between 9 531 712 and 9 611 712, leaving 10 000 between each number?

Looking at the value of digits in large numbers

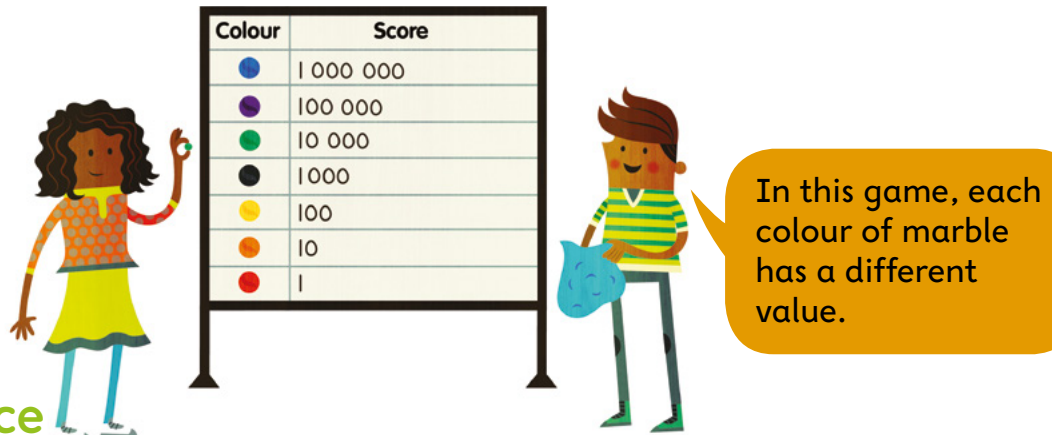| Colour	Score
🔵	1 000 000
🟣	100 000
🟢	10 000
⚫	1000
🟡	100
🟠	10
🔴	1

In this game, each colour of marble has a different value.

Practice

1 Ravi pulls out the marbles below. What number has he scored?

🔵 🔵 🟣 ⚫ ⚫ ⚫ ⚫ ⚫ 🟡 🟠 🟠 🔴

2 Tia pulls out the marbles below and says her number is 113 142, but she is not correct. Can you explain why not and give the correct answer?

🔵 🟣 🟢 🟢 🟢 ⚫ 🟠 🟠 🟠 🟠 🔴 🔴

3 Ravi and Tia play again and this time they each take 20 marbles out.

   a How many of each colour of marble does Ravi need to pull out to make the number 3 045 602?

   b Can you find six different combinations of 20 marbles that would beat Ravi's score and use more than one colour of each marble?

Going deeper

1 If there are ten of each kind of marble in the bag:

   a what is the biggest multiple of 5 you can make using ten marbles

   b what is the smallest multiple of 3 you can make using ten marbles?

2 Can you make a number that has no thousands or hundreds using ten marbles?

3 Tia pulls out ten marbles. She pulls out twice as many yellow marbles as green marbles and twice as many green marbles as orange marbles. She pulls out three blue marbles. What number must she have?

Rounding large numbers

Norway population:
5 223 256
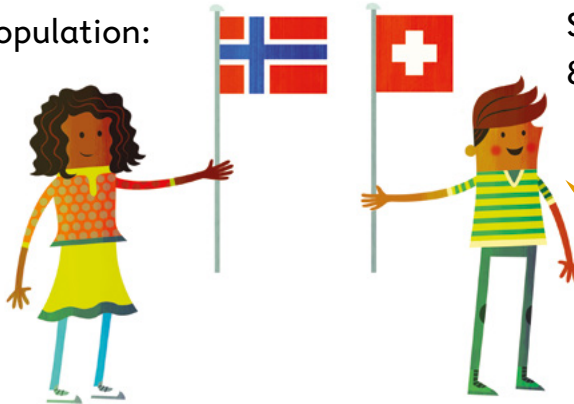
Switzerland population:
8 341 600

The population in Switzerland rounded to the nearest 100 000 is 8 300 000.

Practice

1 What is the population of Norway rounded to the nearest 100 000? Explain to a partner how you know.

2 Can you copy this number line and mark approximately where the number 5 223 256 will appear?

5 200 000   5 300 000

3 Can you round the population of Norway to the nearest:

   a 10 000   **b** 1000

   c 100   **d** 10?

4 Sierra Leone has a population of approximately 7 085 631. What is this rounded to the nearest:

   a 1 000 000   **b** 100 000   **c** 10 000

   d 1000   **e** 100   **f** 10?

Going deeper

1 Can you list five numbers that will round up to 3 340 000 to the nearest 10 000?

2 Can you write a number that would round to 1 100 000 to the nearest 100 000 and 1 062 000 to the nearest 1000?

3 Hayley is thinking of a number. When it is rounded to the nearest ten thousand it is 1 280 000. Can you work out the range of numbers her number must fall into? Discuss with a partner.

Exploring the values of digits in decimal numbers

Practice

1 **a** Can you explain why this statement is true?
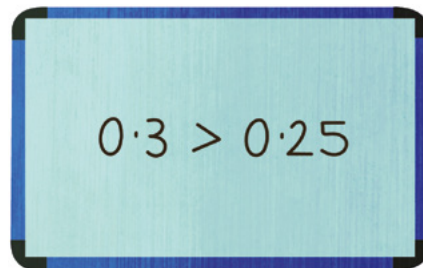
   b What is the value of the 3, the 2 and the 5?

$$0·3 > 0·25$$

2 Can you write each pair of numbers below with either a > or < sign between the numbers to make the statements correct?

   a 1·8   1·65   **b** 0·503   0·65   **c** 2·067   2·12

3 Can you write a number with three decimal places that is bigger than 0·5 but less than 0·75?

Going deeper

1 Can you list three numbers that lie between 1·45 and 1·46?

2 Ella is thinking of a number with three decimal places. Its hundredths digit is twice its tenths digit and the thousandths digit is three times the tenths digit.
The number is less than 0·25.
Which **two** possible numbers could Ella be thinking of?

3 Can you create some similar 'thinking of a number' questions to give to a partner to solve?

4 Try this game with a partner.

   • Take it in turns to write a 7-digit number down, keeping it hidden.

   • Give your partner clues, including two rounding clues.

   • Keep giving clues until your partner guesses correctly.

Using negative numbers

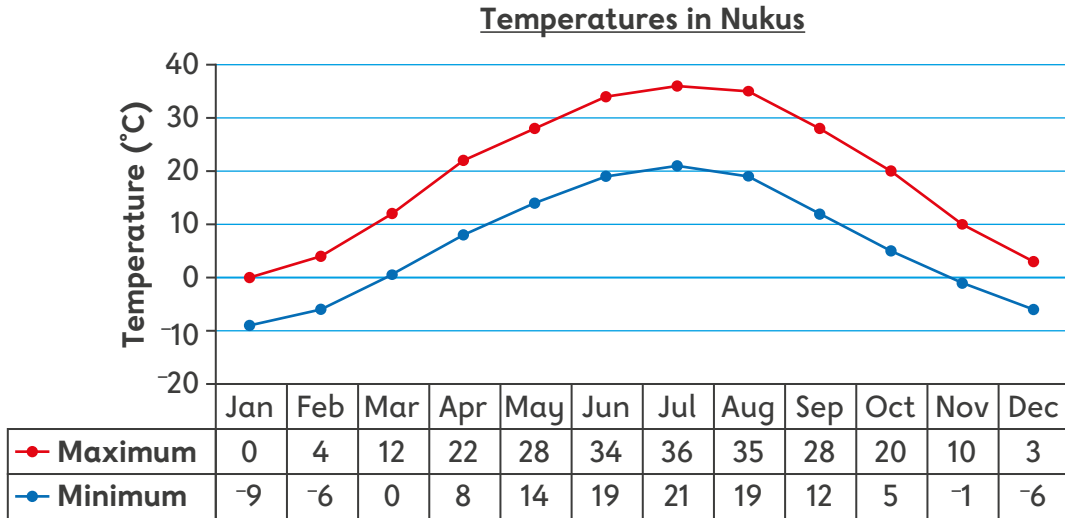**Temperatures in Nukus**

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maximum	0	4	12	22	28	34	36	35	28	20	10	3
Minimum	⁻9	⁻6	0	8	14	19	21	19	12	5	⁻1	⁻6

Practice

1 Jack has found a graph showing the monthly maximum and minimum temperatures in Nukus, Uzbekistan.

   a In which months does the temperature **sometimes** fall below freezing?

   b Which is the coldest month?

   c Which months are generally hotter than the month of May?

2 Which months have the same minimum temperature?

Going deeper

1 Can you copy and complete the sentences below with 'colder' or 'warmer'?

   a 10°C is _____ than ⁻5°C   **b** 15°C is _____ than ⁻20°C

   c ⁻12°C is _____ than ⁻18°C

2 Can you give a whole number that is between:

   a ⁻9 and ⁻6   **b** ⁻6 and 4?

3 Can you suggest a temperature for each missing box to keep the temperatures in order from coldest to warmest?

   ⬜°C   ⁻50°C   ⬜°C   ⁻15°C   ⬜°C   1°C   18°C

Finding differences

−12°C −10°C −8°C −6°C −4°C −2°C 0°C 2°C 4°C 6°C 8°C 10°C 12°C
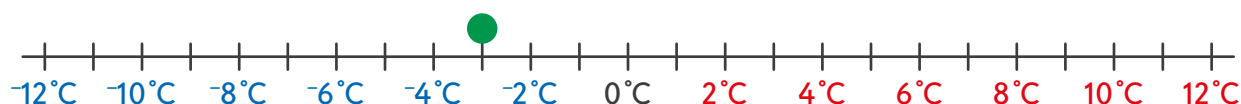
Hana places her counter on −3°C. She turns over a card and has to move her counter to follow the instruction.

Practice

1 What number will Hana's counter be on after she has followed the instruction on the card to the right?

| 2°C fall |

2 Next, Hana turns over these three cards:

| 9°C rise | | 15°C fall | and | 4°C rise |

Where will her counter be after she has completed all three moves?

3 Can you work out the difference between:

a 0°C and −9°C   **b** 4°C and −6°C   **c** −1°C and 10°C?

Going deeper

1 a Hazel recorded the temperature at night as a whole number. It was colder than 0°C, but warmer than −4°C. Can you list the temperatures it could have been?

b Can you think of a temperature and then give clues to help your partner work out what it is?

2 Can you give three pairs of temperatures (one positive and one negative) that have a difference of 12°C?

3 Can you explain how we can use a number line to show infinite pairs of temperatures with the same difference?

7

Profit and loss

Bill runs a local store. He sells ice blocks there. It costs Bill $1 to buy each ice block. He sells each ice block for $1·80.

Ice block
$1·80

The running costs for the store are $50 a day. Bill records the number of ice blocks sold each day in this table.

Day	Ice block sales	Total money taken
Monday	30	$54
Tuesday	10	$18
Wednesday	50	$90
Thursday	70	$126
Friday	60	$108
Saturday	120	$216
Sunday	140	$252

Practice

1 On which days does Bill's store make a loss?

2 Can you calculate the profit or loss for each day? Give your answers for each day in negative or positive amounts.

3 An ice cream company made $25 000 one month but its costs were $40 000. How much of a loss did it make that month?

Going deeper

1 Look at the table above. The next week, sales at the store were halved due to bad weather. Can you work out the profit or loss for each day? Give each answer as a negative or positive amount.

2 Can you make up four calculations that will give you an answer of ⁻$650?

3 Work with your partner to turn your calculations from **question 2** into word problems.

8

Adding and subtracting large numbers

The picture shows how much luggage each aeroplane can hold and how much the luggage weighs on a flight.

a

Capacity of 18 000 kg

Luggage weight 15 762 kg

b

Capacity of 13 500 kg


Luggage weight 9050 kg

Practice

1 Can you calculate how much extra luggage each plane could still hold? Discuss and agree your method. Can you think of other methods?

2 Can you calculate 12 564 – 8395? What do you think is the best method for this calculation? Can you explain why?

3 Can you solve 18 635 + ▓ = 254 729? What do you think is the best method for this calculation? Can you explain why?

Going deeper

1 Without calculating the answer to each subtraction, can you solve these problems?

   a 18 000 – 15 762 = 16 500 – ▓   **b** 13 500 – 9050 = ▓ – 10 000

   c Can you explain your thinking?

2 Copy and complete this table.

3 Can you create your own table puzzle for a partner to solve?

+	16 472	?
?	56 423	?
46 284	?	64 732

Multiplying using factors

25 seats in a row
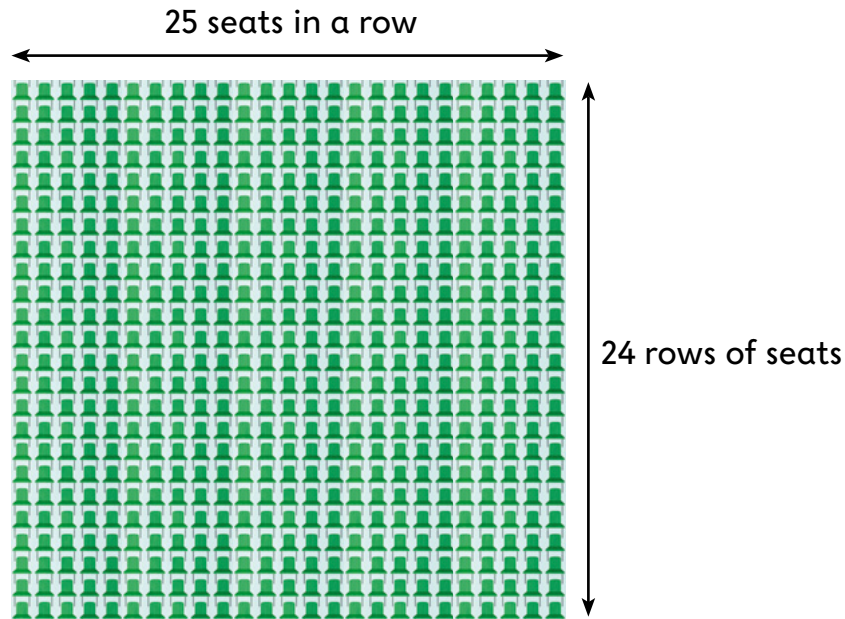
24 rows of seats

A football stadium has 24 rows of 25 seats.

Practice

1 How many seats does the football stadium have altogether? Can you use factors of 24 or 25 to simplify the calculation 24 × 25? Share and compare your strategy with your partner.

2 Using factors can you calculate:

a 15 × 36   b 18 × 25   c 120 × 42?

Going deeper

1 The 6-key is broken on a calculator. How could you use this calculator to work out 46 × 16? Make a record of which keys you would press on the calculator.

2 If you know the answer to 24 × 25, how could you work out 12 × 50? Can you explain your thinking?

3 Using factors, what do you think is the best way to calculate 175 × 28?

Multiplying using partitioning
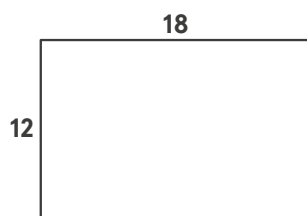
Neema is calculating 18 × 12 and is using this diagram to help her.

Practice

1 How might you split up the rectangle to simplify Neema's calculation? Now solve it and use a drawing to show your thinking.

2 Using the same approach, can you solve these multiplying calculations by partitioning?

a 15 × 32 **b** 52 × 45 **c** 38 × 19

3 Compare your methods with a partner for working out the answers to **question 2**. Can you split the calculations up in any other way?

Going deeper

1 How could you simplify and solve 180 × 12?

2 Using a rectangle 25 × 64, can you find five different ways of splitting the rectangle to calculate its area?

3 Take it in turns to turn over two numeral cards from a 1–99 pack. Write the multiplying sentence and then try to beat your partner to the answer by using a method that simplifies the calculation.

Partitioning and dividing

64 ÷ 4 is the same as 32 ÷ 2.

150 ÷ 6 is the same as 50 ÷ 2.

Practice

1 Can you explain why both Tia and Ben are correct?

2 Can you solve these dividing calculations by making them simpler first?

   a 300 ÷ 6   b 848 ÷ 16   c 540 ÷ 9

3 Can you find two different ways of calculating 160 ÷ 32?

Going deeper

1 Can you find another way to work out 150 ÷ 6 using common factors? Explain your strategy to a partner.

2 Can you simplify $\frac{192}{12}$ and write four different dividing calculations that give the same quotient?

3 How might you solve 342 ÷ 18? Discuss your reasons with your partner.

Multiplying and dividing by 10, 100 and 1000

INPUT		OUTPUT
50	→	0·5
1·5	→	0·015
10	→	0·1
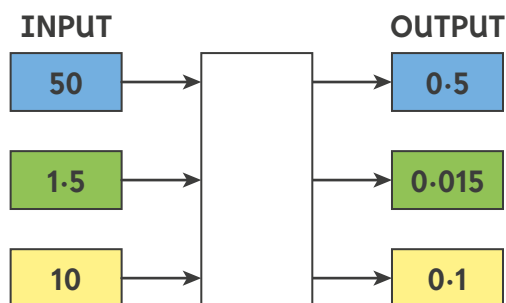
Christopher puts some numbers into a function machine.

Practice

1 What function must be inside the machine? Can you explain how you worked this out?

2 Can you put three more numbers into the function machine and then give the new numbers that would come out?

3 Can you multiply these numbers by 1000?

   a 6·4   **b** 10·75   **c** 140·8

4 How many times larger is 12 500 than 12·5?

5 How many times smaller is 1·05 than 105?

Going deeper

1 Play this game with a partner:

- write down a number
- your partner then secretly multiplies or divides the number by 10, 100 or 1000 and passes the answer to you
- work out what their calculation must have been and write it down.

2 Can you give a number that, when divided by 1000, lies between 0·1 and 0·2?

Introducing the mean (average)

What's our average score?

When we talk about the average, we are usually referring to the mean.

76% 46% 52% 62%

Tia   Molly   Ravi   Ben
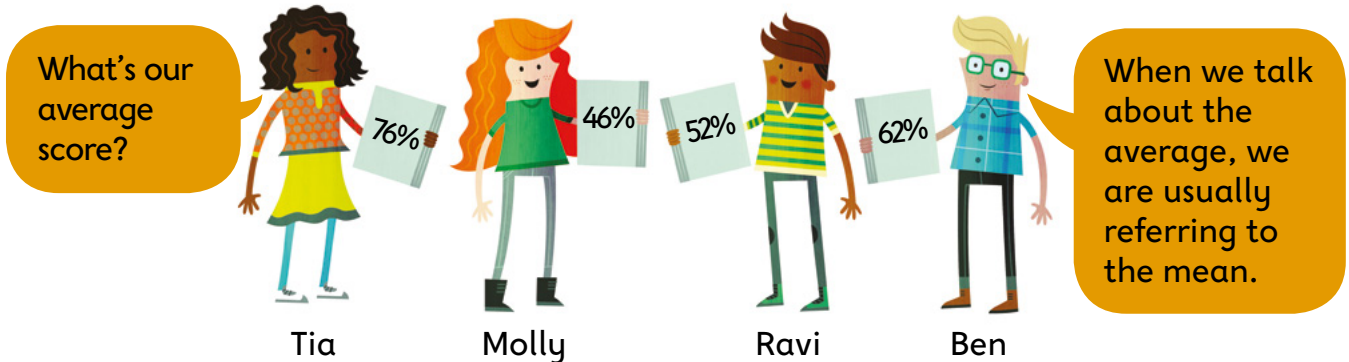
Practice

1 The children have just had their test papers marked and returned to them. Can you explain how to find their mean score?

2 Two of the children realize that the mean score of their two tests is 49%. Can you work out which two?

3 One child wants to take the test again and their score is taken out of the mean. The mean is now 58%. Can you work out which child decided to take the test again?

Going deeper

1 To qualify for the final of a 100 m race, Jonah needs to do three runs with a mean time of less than 11 seconds. His first two runs take 10·4 and 11·2 seconds respectively. How fast must Jonah run his third run in order to qualify?

Can you explain how you worked this out?

2 Manaia is counting how many times he can skip in one minute. After his first five attempts his mean number of skips is 62. After one more attempt this goes up slightly to 64. How many skips did he manage on his sixth attempt?

Using the mean

493·5g

502·8g

498·7g

501·3g

496·6g

500·1g

503·3g

504·4g

507·1g

498·6g

Practice

1 Samara is checking the weight of these ten bags of potatoes to ensure that they meet the regulations.

   a Samara has been told to reject any bags that are more than 1% lighter than the mean mass. Which bags she should reject?

   b Do you think it would be fair to label the bags 'Half Kilo' bags? Can you explain your thinking?

Going deeper

1 Samara is asked to check the volume of liquid in some drinks cartons that claim to hold 330 ml. From a sample of four cartons, she is pleased to find that the mean average is exactly 330 ml. The first three cartons she tests contain 331·1 ml, 327·2 ml and 329·4 ml. Can you find the volume of liquid in the fourth carton?

2 Tim, aged 40, joins a band and becomes its fifth member. He raises the mean average age of the band members to 36. Can you work out what the average age was before Tim joined?

Average speeds and units of speed

Track length: 200 m
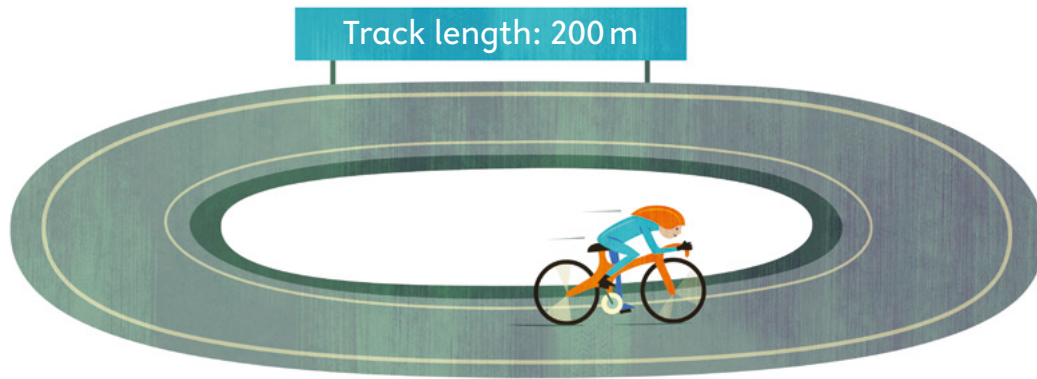
Practice

1 George is cycling around this track at a velodrome. After 10 minutes he has completed exactly 30 laps. What is his average speed in km/h?

2 Draw axes with time up to 1 hour on the *x*-axis, and distance up to 40 km on the *y*-axis. Using your answer from question 1, can you draw a line to show George's average speed?

3 Can you use the graph to find out how far George could cycle in 20 minutes?

4 Now try using your graph to find out how long George would take to cycle 75 laps.

Going deeper

1 Anahera drives on the motorway for half an hour, at an average speed of 90 km/h. She then drives through some roadworks and has to slow down to an average speed of 60 km/h. If she spends a quarter of an hour driving through the roadworks, how long does Anahera's journey take altogether and how far has she driven?

2 Zane is visiting his friend Nadia in the USA. They run a 10-mile race. Nadia says that 1 mile is roughly 1.6 km. Nadia runs at an average speed of 5 mph, and Zane runs at an average speed of 9 km/h. Who wins, and by approximately how much time? Talk to a partner about how you worked this out.

Constructing and interpreting pie charts

Practice

The children are playing a game. They have to throw as many balls into their bucket as they can.

Ravi got 2 balls in.

Ben got 8 balls in.

Molly got 8 balls in.

Tia got 18 balls in.
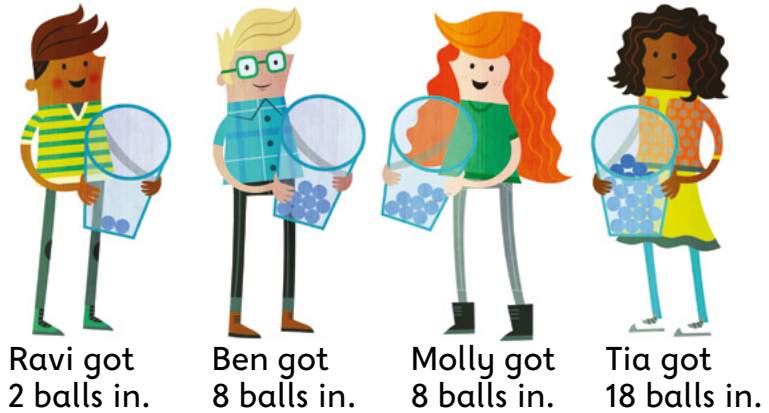
1 a Draw a bar chart and a pie chart to show how many balls are in each bucket.

b Which chart do you think is more useful? Can you explain why?

2 Which chart is most useful for answering each question below?

a What fraction of the balls is in Tia's bucket?

b What is the difference between the number of balls in Ravi's bucket and the number of balls in Tia's bucket?

Going deeper

1 a A designer asks people what colour dresses they prefer. Use the information to complete the table.

Twice as many people said 'red' as 'blue'. Half the people said either 'white' or 'red'. The number of people who said 'blue' was double the number who said 'yellow'.

| Black	White	Blue	Yellow	Green	Red
25%	?	?	?	?	20%

b Now choose a diagram to draw which will show this data in a useful way.

c If the designer makes 200 dresses, how many should be green?

Exploring factors and multiples
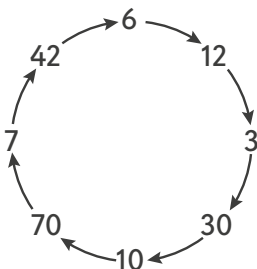
Practice

1 Can you describe how this circular number chain has been made?

2 Can you make a similar circular chain for yourself, starting with any whole number in the range 1–10, without repeating any numbers?

3 What is the longest circular number chain like these that you can make? How do your choices affect the length of the chain?

4 How many different factor trees, like those below, can you make beginning with 80?

$24 = 2 \times 2 \times 2 \times 3$ $24 = 3 \times 2 \times 2 \times 2$ $24 = 2 \times 2 \times 2 \times 3$
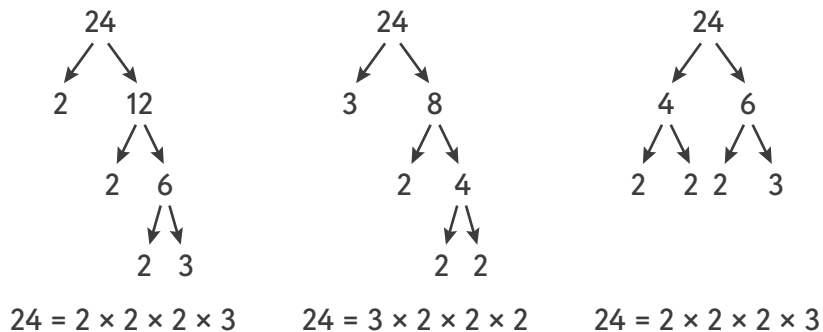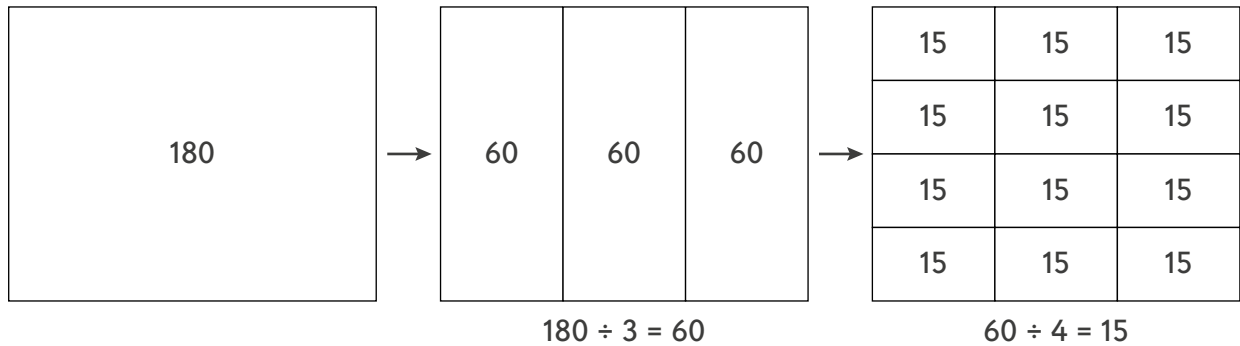
Going deeper

1 Can you make a circular number chain using the numbers 5, 6, 9, 30, 45 and 54?

2 Can you explain why circular number chains make circles?

3 Can you explain a good strategy for creating factor trees? Illustrate your strategy by making a factor tree for the number 224.

18

Using factors when multiplying and dividing

This diagram shows 180 ÷ 12 = 180 ÷ 3 ÷ 4.

180 ÷ 3 = 60

60 ÷ 4 = 15

Practice

1 Can you draw a diagram like the one above to show that 225 ÷ 15 = 225 ÷ 3 ÷ 5?

2 Rewrite the calculations below in as many different ways as you can. Can you choose which versions you think would be the easiest to use to calculate the answer?

a 16 × 24   **b** 504 ÷ 24   **c** 198 ÷ 18   **d** 25 × 15

3 Can you rewrite the calculations below as fractions, and then simplify them?

a 300 ÷ 18   **b** 252 ÷ 21

Going deeper

1 How many different ways can you find to calculate each of the following?

a 23 × 15   **b** 16 × 36   **c** 28 × 13   **d** 19 × 17

2 Which calculation in **question 1** did you find the hardest and why?

3 How might knowing the prime factors of 720 help you to calculate 720 ÷ 16? Can you explain why?

Using the lowest common multiple (LCM)

Disco light instructions:
- Press start.
- The red light will flash every 10 seconds.
- The blue light will flash every 14 seconds.
- The green light will flash every 25 seconds.

Practice

1 Can you work out when all three lights will first flash together? Give your answer in seconds.

2 Choose two whole numbers between 2 and 12, and find all their common multiples in the range 50–150. Can you find their LCM?

3 Which numbers in this list are common multiples of smaller numbers?

| 24 | 39 | 60 | 84 | 74 | 48 | 54 | 30 |

Can you say which smaller numbers these are common multiples of?

Going deeper

1 Using their prime factors, can you draw a Venn diagram to help you work out the LCM of 8 and 30?

2 This Venn diagram shows the prime factors of two numbers. Can you say what the two numbers are, and name their LCM?

3 How could you have used a Venn diagram to help you solve the disco lights problem above?
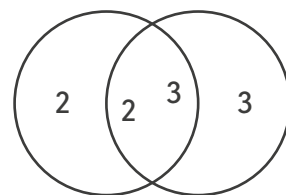
Exploring highest common factors (HCF)
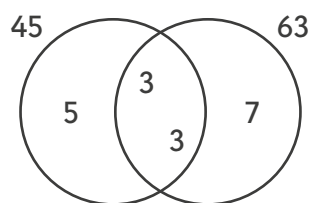
The HCF of 45 and 63 is 3 × 3 = 9.

Practice

1 Can you use a Venn diagram to calculate the HCF of these numbers?

   a 56 and 96   b 72 and 90   c 81 and 27

2 Without using the number '7' itself, can you find five pairs of numbers that have an HCF of 7?

3 How many different pairs of numbers can you find between 50 and 100 that have an HCF of 6?

Going deeper

1 Can you work out the largest possible HCF of a pair of whole numbers up to and including 100?

2 Can you work out the answer to question 1 again, but this time the smaller number of the pair cannot be a factor of the larger number?

3 Can you list all the number pairs within the range 50–100 whose only common factor is 7? How do you know you have found them all?

Exploring fractions

All these fractions are greater than $\frac{1}{2}$.

$$\frac{3}{4} \quad \frac{3}{8} \quad \frac{7}{10} \quad \frac{4}{5} \quad \frac{9}{16}$$

Practice

1 Can you explain to a partner whether Ben is correct or not?

2 Can you order the fractions above from smallest to largest? You can use apparatus to support you.

3 Can you suggest three more fractions that are greater than $\frac{1}{2}$?

4 Can you order the fractions in each group below from smallest to largest?

a $\frac{2}{3}$ $\frac{7}{9}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{4}$ b $\frac{5}{12}$ $\frac{1}{3}$ $\frac{5}{6}$ $\frac{1}{8}$ $\frac{5}{24}$ c $\frac{3}{7}$ $\frac{1}{4}$ $\frac{3}{14}$ $\frac{5}{7}$ $\frac{1}{2}$

Going deeper

1 Which of Ben's fractions above are bigger than $\frac{3}{4}$? Can you explain how you know?

2 Can you write five proper fractions in order from smallest to largest using numbers from the lists below? You can only use each number once. Compare your fractions with a partner. Can you agree a definition for a proper fraction?
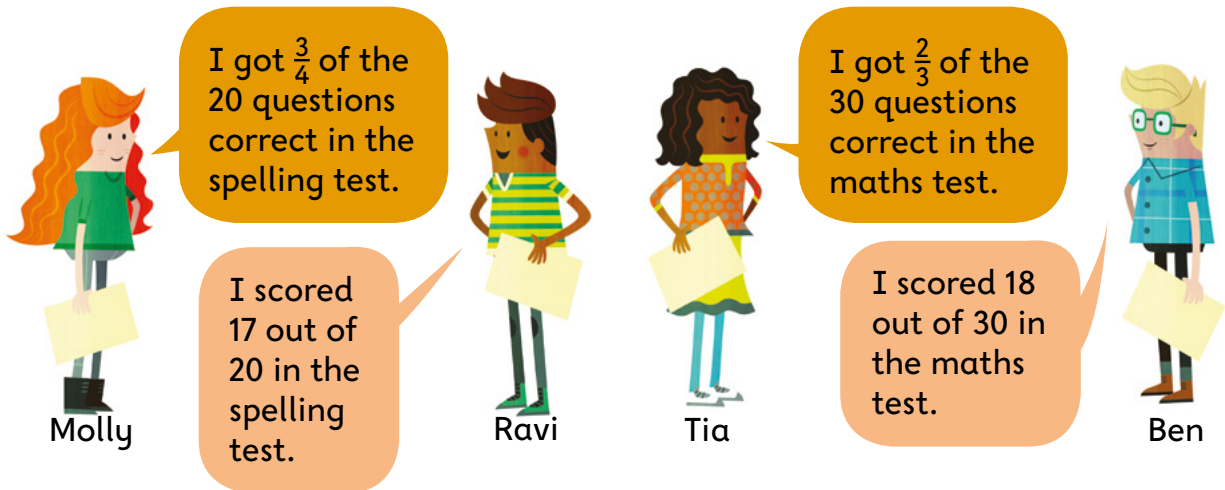
Numerators
1 5 9 3 6

Denominators
9 15 12 8 6

3 Take turns to give your partner a proper fraction, for example $\frac{2}{3}$. Your partner must give you a smaller proper fraction that has a different denominator, for example $\frac{1}{6}$. Record your answers in this way: $\frac{2}{3} > \frac{1}{6}$.

Converting and comparing fractions

I got $\frac{3}{4}$ of the 20 questions correct in the spelling test.

I scored 17 out of 20 in the spelling test.

Molly

Ravi

Tia

I got $\frac{2}{3}$ of the 30 questions correct in the maths test.

I scored 18 out of 30 in the maths test.

Ben

Practice

1 a Who scored the higher mark in the spelling test? How do you know?

b Who scored the higher mark in the maths test? How do you know?

2 Another child gained a spelling test mark that was between Molly's and Ravi's. What mark did they score, out of 20?

3 Another child got $\frac{5}{6}$ of the maths test correct. What do you think their score was?

4 What is the common denominator for each group of fractions below? List the fractions with their new denominator.

a $\frac{3}{8}$ $\frac{1}{6}$ $\frac{5}{12}$ $\frac{1}{3}$ 　　　**b** $\frac{7}{10}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{3}{5}$ 　　　**c** $\frac{7}{9}$ $\frac{5}{6}$ $\frac{2}{3}$ $\frac{7}{12}$

Going deeper

1 Can you order these fractions, starting with the smallest?　$\frac{3}{4}$　$\frac{17}{20}$　$\frac{2}{3}$　$\frac{18}{30}$

2 Can you name two fractions which lie between:

a $\frac{1}{6}$ and $\frac{2}{5}$ 　　　**b** $\frac{1}{3}$ and $\frac{3}{7}$?

3 Can you name five proper fractions that are greater than $\frac{4}{5}$? Can you explain your strategy for finding these fractions?

Simplifying fractions

Tia and Ben are interested in which scores occur most often when they throw two dice and record the total. They throw the two dice 80 times.

The score 7 came up most frequently. 18 times out of 80, or $\frac{18}{80}$.

Score	Frequency
2	2
3	4
4	6
5	8
6	11
7	18
8	12
9	10
10	5
11	3
12	1

Practice

1 Can you simplify the fraction $\frac{18}{80}$? How could you explain what 'simplifying' a fraction means to your partner?

2 Can you write and simplify the fractions for the other scores in the table? Are there some that cannot be simplified? Can you explain why not?

3 Can you simplify these fractions?

a $\frac{5}{40}$ b $\frac{3}{27}$ c $\frac{16}{56}$ d $\frac{48}{72}$

4 Can you name three fractions that would all simplify to $\frac{2}{5}$? What is a good method for finding these?

Going deeper

1 Write five fractions that can be simplified for your partner to simplify. What makes some fractions more difficult to simplify than others?

2 "To simplify fractions, halve the numerator and denominator until you can't do it any more." Is this statement sometimes, always or never true? Can you explain your thinking?

Ordering mixed numbers

Four children are completing a sponsored relay by doing laps of their school field.

Name	Laps completed
Jamie	$4\frac{3}{4}$
Raj	$3\frac{1}{2}$
Isla	$3\frac{2}{3}$
Kiana	$4\frac{5}{6}$

Practice

1 Who ran the furthest? Can you explain your thinking?

2 Can you order all the mixed numbers above, showing the smallest number of laps first?

3 Can you order the mixed numbers below from smallest to largest?

a $1\frac{3}{8}$ $1\frac{2}{5}$ $2\frac{3}{10}$ $2\frac{1}{4}$ **b** $1\frac{2}{9}$ $1\frac{1}{6}$ $2\frac{1}{4}$ $2\frac{5}{12}$

4 Can you name a mixed number that would lie between $1\frac{1}{4}$ and $1\frac{1}{3}$? How did you choose this, and how do you know you are correct?

Going deeper

1 Can you write one mixed number that would lie between each pair of results in the ordered set of relay results from **question 2**?

2 Can you write three mixed numbers that would lie between $2\frac{1}{2}$ and $2\frac{3}{4}$?

3 Take it in turns to think of a mixed number and then give clues to help your partner guess what it is.

Estimating and rounding

Country	Car production numbers 2015
Germany	6 033 164
Japan	9 278 238
UK	1 682 156
France	1 970 000
Spain	2 733 210
Italy	1 014 223
India	4 125 744
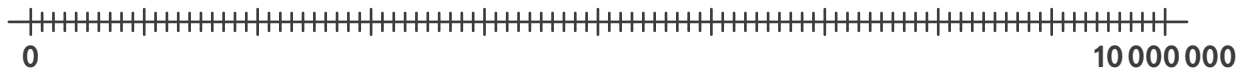
0   10 000 000

Practice

1 a Can you round each of the car production numbers to the nearest 10 000?

  b Draw a number line like the one above. Can you place the countries in order along your line?

2 Can you round each of the car production numbers to:

  a the nearest 100 000   b the nearest 1000   c the nearest 100?

Going deeper

1 Which country do you think makes roughly twice as many cars as France?

2 If Italy's car production were to increase by 10%, how many extra cars would this be, to the nearest 1000?

3 If 1% of the cars produced in Japan were sold to the UK, how many cars would this be, to the nearest 100?

Estimating quantities and costs

> I use about 133 litres of water each day for everything, including drinking, cooking and washing.

Practice

1 As there are 365 days in a year, Ravi estimates that he will therefore use roughly 40 000 litres of water each year. He does this by rounding 133 and 365 to the nearest hundred, giving 100 × 400 = 40 000.
Can you make a closer estimate than Ravi's?

2 Assume that everyone uses the same amount of water as Ravi and that there are on average 2·3 people in every household. Molly estimates that one household uses 2 × 50 000 = 100 000 litres of water each year.
Can you explain how she estimated?
Can you make a closer estimate than Molly's?

3 1000 litres (or 1 cubic metre) of water costs $1·948. Molly estimates that water will cost an average household roughly $200 each year.
Do you think Molly's estimate will be higher or lower than the actual cost? Can you explain why?

Going deeper

1 Assume that the average amount of water that one person uses annually is 133 × 365 = 48 545 litres. Rounding each figure to the nearest 100 gives 40 000 litres, but rounding each figure to the nearest 10 gives 130 × 370 = 48 100 litres. Can you explain why rounding both figures to the nearest 10 is so much more accurate?

2 Edinburgh, in Scotland, had 230 831 households in 2015. Can you estimate how many cubic metres of water Edinburgh used in one day in 2015? Explain how you made your estimate. Do you think your estimate will be higher or lower than the actual amount used?

Rounding

Practice

1 a Flour comes in bags of 1·4 kg. Can you work out how many bags of flour will be needed to make 300 cheese straws?

Recipe for 15 cheese straws

350 g flour
225 g butter
150 g cheddar cheese
2 eggs
$\frac{1}{4}$ teaspoon salt

b The cheese straws are sold in packs of 8 and the bakers are planning to make 100 packs in a day. How many bags of flour are needed for one day's cheese straw production?

c At the end of a day's production, the bakers want to use up all the flour they bought. How many extra packs of cheese straws can they make?
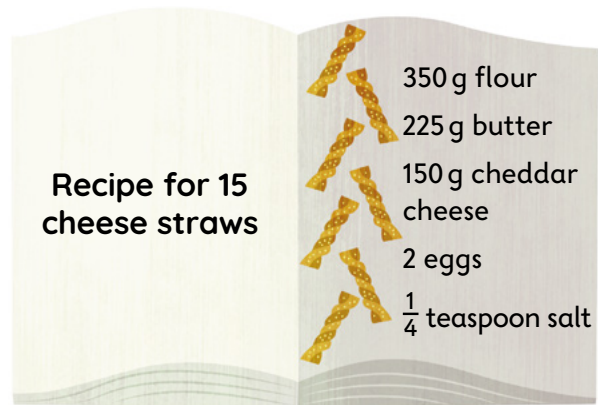
Going deeper

1 Can you write a 7-digit number that gives the same number when rounded to either the nearest 10 000 or to the nearest 100 000?

2 The children each have one of these cards.

| 425 129 | 420 356 | 399 874 | 399 996 |

Can you work out who has which card?

My number has exactly 3998 hundreds in it.

My number is 420 000 to the nearest 10 000.
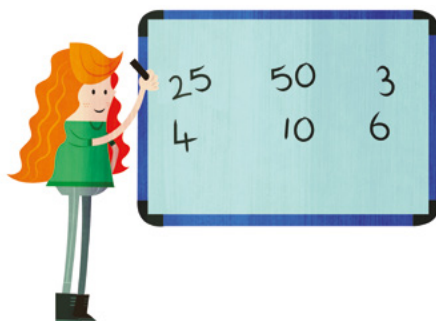
My number is 400 000 to the nearest 100 000.

My number is 400 000 to the nearest 10.

Molly   Ravi   Ben   Tia

Estimating answers to calculations

Practice

1 Using any of Molly's numbers once only, and any of the operations +, −, × or ÷, how close can you get to the answers below?

> **a** 274 **b** 256 **c** 732 **d** 598 **e** 412

2 Explain how you would estimate answers to each of these calculations. Do you think each estimate will be higher or lower than the actual answer?

a 659 + 328   **b** 123 × 4·8   **c** 1472 − 748   **d** 270 ÷ 8·7

e 324 + 573   **f** 8·9 × 21·5   **g** 3347 − 419   **h** 62·7 ÷ 6·8

Going deeper

1 Think about the calculation 9·3 × 138·8. If you can only round **one** of the decimal numbers to the nearest whole number, which one would you round to get closest to the actual answer? Can you explain why?

2 A country made 5·1 million cars in 2014 (to the nearest hundred thousand), compared with 5 million cars (to the nearest million) in 2013. Sanjay thinks that they made more cars in 2014 than in 2013. Is he correct, do you think? Can you explain why?

3 Using **all** of the numbers 75, 25, 2, 7, 4 and 8 **once** each, and any of the operations +, −, × or ÷, how close can you get to each of these answers:

> **a** 348 **b** 504 **c** 769 **d** 230?

Exploring column methods for adding

A 0·158 kg
B 0·067 kg
C 0·193 kg

Practice

1 a Tia wants to post parcel A and parcel B. Can you estimate the combined mass of the parcels? Now calculate the answer exactly using the written column method. How close were you?

b Can you estimate and then calculate how much parcel B and parcel C weigh altogether? How close were you this time?

c Estimate and then calculate how much parcel A and parcel C weigh altogether. How close were you? Are you getting better at estimating?

2 If Tia ties a length of string around each parcel, adding 0·007 kg to each weight above, how much will each parcel weigh now?

Going deeper

1 Can you add the masses below by estimating first and then calculating using the written column method? Discuss your strategies with a partner.

a 0·28 kg and 0·817 kg   **b** 0·704 kg and 0·097 kg   **c** 0·345 kg and 0·886 kg

2 Tia wraps parcel D which weighs 0·275 kg. Which parcels A–D would give a combined mass of no more than $\frac{1}{2}$ kg? List the masses of the different combinations in order of size.

3 Choose one of your combination calculations from above and check your adding by doing some subtracting.

Working out change by subtracting

$3·75 per kg   86c per kg $2·69 per kg $2·24 per kg

Practice

Ben's Money $8·45

Ravi's Money $10·24

1 **a** Ben buys a kilogram of rice and a kilogram of flour. How much money will he have left?

b Ravi buys a kilogram of sugar and a kilogram of oats. How much money will he have left?

2 Can you use a written column method to calculate:

a 9·67 – 2·38   **b** 10·05 – 7·14

c 8·1 – 5·65   **d** 6·11 – 3·74?

Now use the inverse to check that your answers are correct.

Going deeper

1 Look at the money Ben and Ravi start with above. Do they each have enough money to buy a kilogram of each item? How much money is left over or how much more do they need?

2 Looking at the calculations below, can you work out what has gone wrong? Discuss what you think has happened with a partner. Complete the correct calculation for each.

a

```
    8 · 0 6 8
  - 3 · 4 7 4
  ───────────
    5 · 1 4
```

b

```
    6 · 4 5
  - 2 · 5 7 4
  ───────────
    3 · 9 8 4
```

c

```
    5 · 0 0 7
  - 3 · 1 2 9
  ───────────
    1 · 9 8 8
```

3 Can you find two different subtracting calculations that will give the answer 2·547?

31

Subtracting using equal adding

Lily is working out 456 – 168. This is the method she is using.

$$
\begin{array}{ccc}
 & 4 & {}^{1}5 & {}^{1}6 \\
- & {}^{2}\!\not{1} & {}^{7}\!\not{6} & 8 \\
\hline
 & 2 & 8 & 8 \\
\end{array}
$$

Practice

1 Can you explain Lily's method to your partner?

2 Using the same method as Lily, can you work out:

 a 864 – 238 **b** 542 – 185

 c 15·65 – 9·73 **d** 12·04 – 7·46?

3 Can you solve each of these empty box problems, without doing the subtracting calculation on the left?

 a 673 – 134 = 675 – ▇ **b** 1002 – 875 = ▇ – 873

 c 4·6 – 1·25 = ▇ – 1·5 **d** 12·5 – 7·7 = 15 – ▇

 Compare your answers with your partner and explain your thinking.

Going deeper

1 Can you write a similar equal adding calculation to Lily's above that requires:

 a adding 10 to each number

 b adding 10 to each number and then 100 to each number?

2 Can you each create four balancing calculations like the ones in question 3 for your partner to solve? Discuss and agree your answers by explaining your methods to each other.

Missing number problems

		5	·	6	⭐		1	⭐	·	4	8		1	4	·	⭐	6		1	⭐	·	3	⭐
+		8	·	⭐	5	+		5	·	8	4	−		6	·	1	⭐	−	1	2	·	8	4
	1	4	·	1	2		1	8	·	⭐	2			8	·	0	8			5	·	⭐	5

Ali spilled his drink and smudged some of the numbers in his book.

Practice

1 Can you work out what missing digits are hidden by the splodges for each calculation? Check by completing the calculations.

2 a Choose one number from each list to create an adding calculation. Do this three times with different number pairs.

> A: 56·8   32·43   16·126   11·73   0·562   125·67

> B: 8·92   24·57   13·06   35·75   18·3   5·649

Rub out some of the digits and ask your partner to work them out.

b Now choose one number from each list and come up with a subtracting calculation. Repeat three times and rub out some of the digits for your partner to work out.

Going deeper

1 a Using the key, can you work out the missing numbers in this calculation?

⭐ = multiple of 4

▲ = multiple of 3

🟩 = multiple of 5

$$\begin{array}{r} ▲\ 🟩\ 7\ ▲ \\ +\ 🟩\ ▲\ 🟩\ ▲ \\ \hline ⭐\ ▲\ ▲\ 🟩 \end{array}$$

b Can you write your own calculation for a partner using the same key?

Exploring percentages, fractions and decimals

Practice

1 Ravi's calculator is very simple, and doesn't have keys for fractions or percentages. Can you explain what calculations he can do to compare these two offers?

2 Decide which is greater in each of the pairs below. Can you explain how you know for each pair?

   a $\frac{2}{3}$ or 0·66   **b** 28% or $\frac{13}{50}$ **c** 0·75 or 80%

3 Which of the comparisons that you made in **question 2** was easiest to decide? Can you explain why?

Going deeper

1 Olivia sees a mountain bike in a second-hand bike shop sale. After a 30% discount, it would cost her $140. Then she sees a second-hand bike online that would cost her $160 after a 50% discount. Can you explain why the bike offered with the biggest discount actually costs more?

2 When or where have you noticed fractions being used outside school?

3 When or where have you noticed decimals being used outside school?

4 When or where have you noticed percentages being used outside school?

Comparing percentages

Three people's daily spend on food

	Koa	Lily	Bo
Vegetables	$2	$2	$5
Fruit	$4	$2	$6
Dairy	$11	$5	$8
Other	?	?	?
Total money spent	$20	$10	$20

Practice

1 a Look at how much money each person spent in total and how much they spent on fruit. Who spent the highest percentage of their total money on fruit?

b Who spent the highest percentage of their total money on dairy?

c Who spent the lowest percentage of their total money on vegetables?

2 Who spent the lowest percentage of their total money on other food?

3 Altogether, what percentage of their total money did each person spend on dairy?

Going deeper

1 Taken altogether, did Koa, Lily and Bo spend more or less than 10% of their total money on vegetables? Can you explain how you know?

2 At Dylan's school during one week, Rimu Class (35 children) had 7 absences, and Kauri Class (30 children) had 3 absences. Which class had the better attendance record for that week? Can you explain why?

Percentage increases and decreases
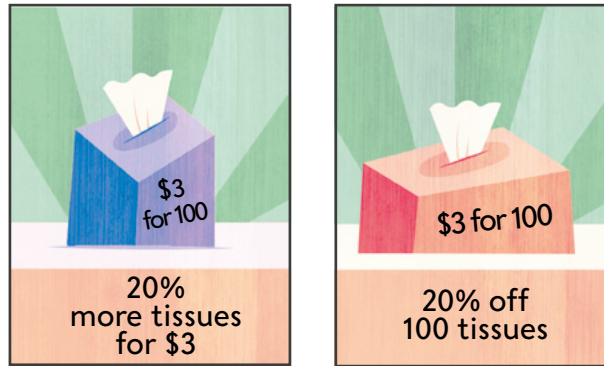
Practice

1 Which of the two offers above is better value? Can you explain why?

2 Choose a percentage increase from list B below, apply it to an amount from list A, and write down the increased amount. Can you do this for five different percentages and amounts in the lists?

> **A:** 22 km, 2 kg, 3820 ml, 7·8 L, 840 cm², 6 km, 358 seconds
>
> **B:** increases of: 1%, 5%, 7·5%, 10%, 15%, 70%, 95%

3 Choose a percentage discount from list D below, apply it to an amount from list C and then write down the reduced amount. Can you do this for five different percentages and amounts in the lists?

> **C:** $12, $48, $300, $245, $13·50, 80c
>
> **D:** discounts of: 5%, 7·5%, 25%, 33%, 60%, 62·5%

Going deeper

1 What is your best method for calculating 17·5% of $50? Can you explain your method to your partner?

2 If you add 20% on to an amount, and then take 10% off the total, will that give you the same result as taking 10% off the amount first and then adding 20% on? Can you explain why, or why not?

Using percentages with data

NZ population at 2023 census ≈ 5 million

NZ population by Island at 2023 census

Island	Approximate percentage of total population
North Island	75%
South Island	25%

NZ population by age at 2023 census

| Age group	Approximate percentage of total population
0–14 years	19%
15–64 years	65%
65 years and over	16%

Practice

1 a Roughly how many 0–14 year olds were living in NZ in 2023, do you think? Can you round your answer to the nearest 100 000?

   b Can you say roughly how many people lived in the South Island in 2023?

   c Roughly how many people in NZ lived in the North Island in 2023?

2 In a bag of marbles, 20% are red. There are 15 red marbles. How many marbles are in the bag? Explain how you know.

Going deeper

1 Clara wants to increase this recipe by 25%. Can you work out what the new amounts will be for each ingredient, and how many rolls this new recipe will make?

Recipe for 8 sourdough rolls

240 g flour
30 g butter
50 ml water
30 g sugar
120 g sourdough starter
1 beaten egg (50 g)
1 pinch of salt

2 In the 30 years leading up to 2014, the number of sparrows in Europe fell by 147 million. This was a fall of roughly 60%. What do you think the number of sparrows in Europe was in 1984, to the nearest million? Can you explain your answer?
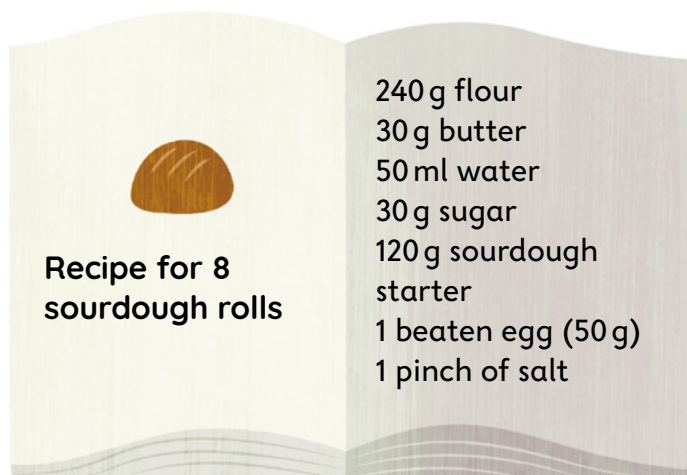
Exploring 2D shapes and angles

Tia is exploring all the different triangles she can make with a perimeter of 16 cm. She has found one so far.
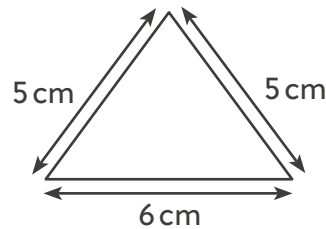
Practice

1 Draw Tia's triangle using a ruler and compass and then measure the angles with a protractor to the nearest degree. Can you label each angle and name the kind of triangle you have drawn?

2 Can you find any other triangles with a perimeter of 16 cm and sides that are a whole number of centimetres? Use a compass and ruler to draw, label and name each type of triangle.

3 Now can you draw some triangles with whole-number side lengths and a perimeter of 20 cm?

4 Using a ruler and a compass, can you construct a scalene triangle that is:

   a acute-angled   b obtuse-angled   c right-angled?

Going deeper

1 Look at your triangles from question 2. What generalizations can you make about the side lengths of a triangle with a perimeter of 16 cm? What side lengths can it not have? Explain why.

2 Two sides of a triangle are 8 cm and 5 cm, and the angle between these sides is 55°.
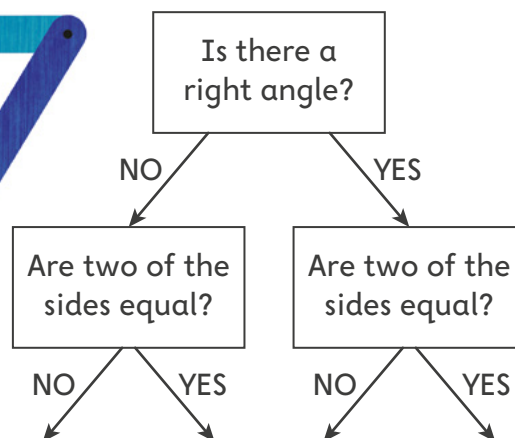
   a Can you draw the triangle using a compass, ruler and protractor? What is the other side length and the perimeter of the triangle?

   b What other triangles can you construct with the same perimeter (with whole-number side lengths)? Label the side lengths and angles. Can you make all 'types' of triangle? If not, why not?

   c Sort your triangles into categories.

Exploring quadrilaterals

Ryan is making quadrilaterals and using a branching diagram to sort them.

Is there a right angle?

NO · YES

Are two of the sides equal?

NO · YES

Are two of the sides equal?

NO · YES

Practice

1 Draw or make a variety of quadrilaterals using isometric paper or geo strips. Can you create four different quadrilaterals that have two sets of equal sides?

2 Can you create a branching diagram and use it to classify the quadrilaterals you found in question 1?

Going deeper

1 Can you explore and explain if it is possible to create a quadrilateral that has:

   a three obtuse angles   b three acute angles   c three right angles?

2 Which of these statements are sometimes true, always true or never true? Can you explain your reasoning?

A square is a rhombus.

A trapezium is a rectangle.

A quadrilateral has two obtuse angles and two acute angles.

A parallelogram has one pair of obtuse angles.

Exploring angles in regular polygons

| Shape	Number of sides or angles	Sum of interior angles	Size of each angle
Equilateral triangle	3	180°	60°
Square	4	360°	90°
Regular pentagon	5	540°	108°
Regular hexagon	6	720°	120°

Mindy has been exploring the sizes of interior angles in regular polygons.

Practice

1 **a** Can you explain why the external angles of any polygon will always total 360°?

  b Can you explain why the sum of the exterior and interior angles at any vertex make 180°?

2 What will the sum of the interior angles in an octagon be? Can you explain why?

3 Can you work out what size the interior and exterior angles will be in a regular dodecagon (12-sided polygon)? Explain your reasoning to your partner.
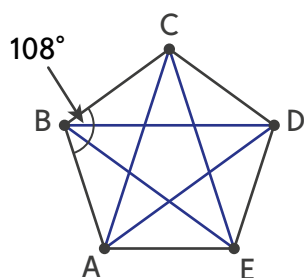
Going deeper

1 Can you work out the sum of the internal angles in a regular heptagon?

2 Can you work out a formula for finding the size of the internal angles for any regular polygon with n sides?

heptagon

3 Investigate the number of triangles that can be drawn inside any polygon. Is there a generalization you can make?
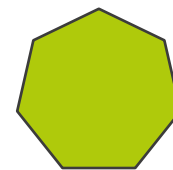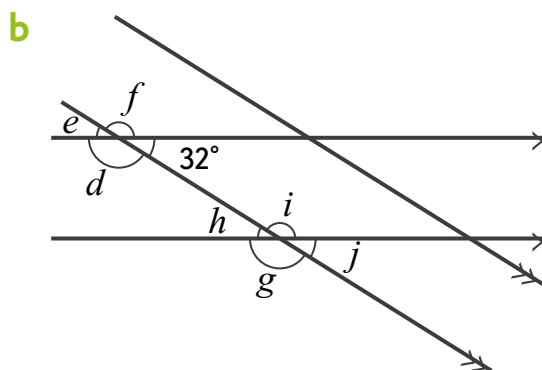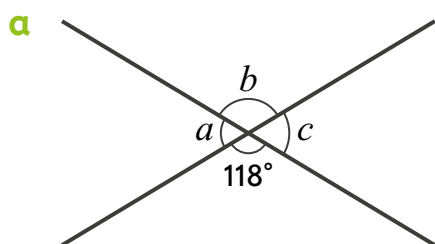
Finding missing angles

108°

I know angle ABC is 108° because the angles of a regular pentagon are 108°.

Practice

1 ABC forms an isosceles triangle, so can you calculate the other two angles in the triangle ABC? (angle BAC and angle BCA)

2 How could you calculate angle CAE?

3 Can you work out all the missing angles in the diagrams below?

a

b

....

Going deeper

1 Can you draw the pentagon and five-pointed star above and then label the size of all the other angles? (Do not measure the angles with a protractor.)

2 Sketch a rhombus and show the diagonals within it. If one of the interior angles in the rhombus is 116°, can you label all the other angles within your drawing?

3 Create a missing angles problem for your partner, giving them the fewest number of angles they need to solve it.