numicon

Term 1

Numicon 5 Student Book

Collaborative problem-solving activities for Years 6 and 7

OXFORD

Numicon 5 Student Book

Information for Year 6 and 7 teachers

Planning for mathematical progression

This book is designed to be used alongside the Numicon 5 Teaching Handbooks, which can be found on the Teaching Handbooks tab of your *Numicon Online* subscription. The Teaching Resource Handbooks include a suggested order for teaching the activity groups through the year, which the Student Book follows. There are four Student Book pages for each activity group.

Where to find the main teaching

The main teaching to cover before children work on a page from the Student Book is shown at the top of the page. For example, before children work on page 2 of this book, teach Activity 1 from Numbers and the Number System 1.

Using the Student Book

The pages in the Student Book provide further practice and challenging questions to follow up the main teaching activity and deepen the learning. You can use the questions in the Student Book selectively, to meet the needs of children in your class.

Using apparatus alongside the Student Book

The Student Book questions aim to stimulate rich mathematical discussion. Children are encouraged to make use of structured apparatus and imagery in responding to the questions and communicating their ideas.

Where to find answers

You can find complete answers to the questions in this book in *Numicon Student Book 5 Answers,* and also on *Numicon Online*. The answer book also contains an introduction to the Student Books by Dr Tony Wing and a chart to support your planning.

Assessing understanding with the Explorer Progress Books

After completing work on an activity group, you can use the Explorer Progress Book to assess how well children have understood the key learning.

At the end of each 4 page section in the Student Book, you will find a reference to the Explorer Progress Book. For example, page 5 of the Student Book links with pages 4–5 of *Numicon Explorer Progress Book 5a*. You can find the Explorer Progress Books by going to the Online Index on *Numicon Online*.

Tracking children's progress with Numicon Online

You can use assessment evidence from the Explorer Progress Books, and from children's work throughout the activity group as a whole, to record progress on the Numicon 5 Milestone Tracking spreadsheet on *Numicon Online* (on the Oxford Owl website).

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Term 1

Numicon 5 Student Book

Collaborative problem-solving activities for Years 6 and 7

Written by Jayne Campling, Andrew Jeffrey,
Adella Osborne and Dr Tony Wing

OXFORD

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How to use this book

Welcome to the Numicon 5 Student Book.

Coloured sections

You will notice that the sections of this book are coloured red, yellow, blue, green or purple.

Each colour shows a different maths theme:

- Pattern and Algebra
- Numbers and the Number System
- Calculating
- Geometry
- Measurement

In this book you can try out new calculations…

… and new methods for finding answers.

There are opportunities to look for patterns…

…and to think about how different maths ideas are connected.

Practice

These questions help you to practise and explore the new maths ideas you have learned.

Going deeper

These questions give you extra challenge and make you think deeply.

You will need to work with a partner on questions that have this symbol.

When you see this grey symbol you can do these activities in the Explorer Progress Book pages online.

Working with large numbers

Practice

1 Can you write down in words the number of people who are estimated to live in New York City?

2 • Write down a 7-digit number, keeping it secret from your partner.
- Read the number for your partner to write down.
- Compare your numbers. Are they the same?
- Swap over and play again.
- Continue taking turns until you have each written ten numbers.

Going deeper

1 Using each of the digits below once, can you write down a number that lies between 3 286 471 and 4 183 762? Can you explain how you worked out your answer?

1	2	3	4	6	7	8

2 Can you write a set of instructions for reading out 7-digit numbers? Try to write them so that a younger child could follow them.

Visualizing a million

1 teaspoon = about 50 grains of rice

50 teaspoons = 1 cup

4 cups = 1 litre

Practice

1 Can you work out the approximate volume of one million grains of rice?

2 What other things can you think of that take up about the same amount of space (that is, have the same volume) as one million grains of rice?

Going deeper

1 Can you explain how long one million seconds is?

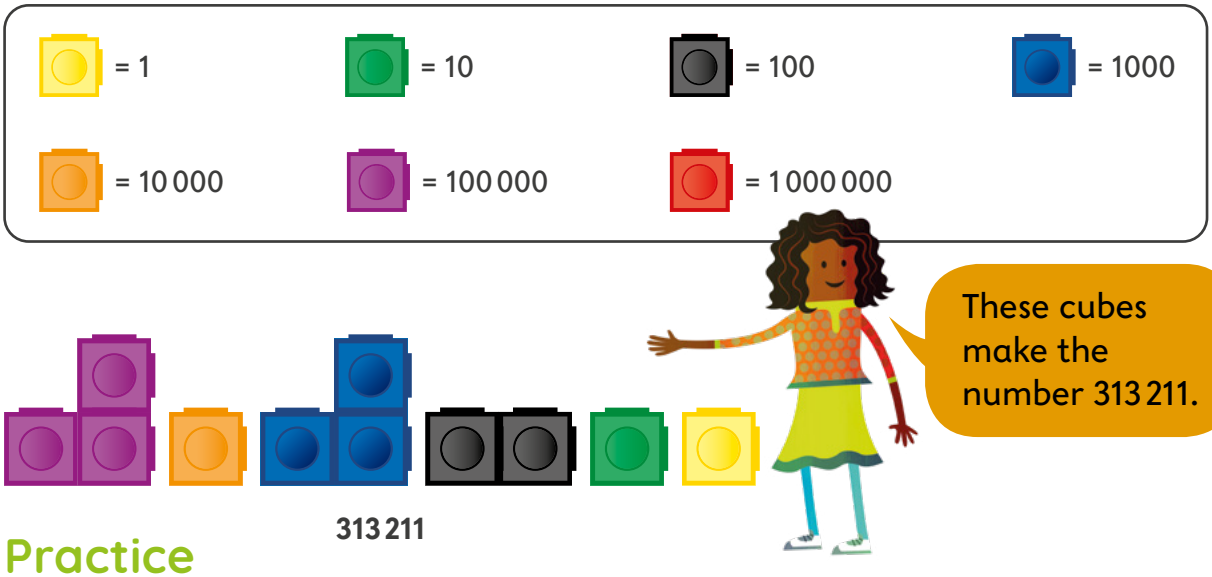
2 Can you work out whether you have reached a million days old?

3 Can you work out roughly how long ago you would need to have been born to be a million days old now?

4 How would you explain to a younger child how big a million is?

5 Can you think of some different ways of showing how big a million is?

Exploring place value



Practice

1 Make a 6-digit number by rolling a dice six times (you can use the numbers you roll in any order). Then make your number with cubes.

Can you count on ten steps of 1000 from your number?

2 Make a 5-digit number by rolling a dice five times (you can use the numbers you roll in any order).

Can you count back from this number in steps of 100? What number will you arrive at if you count back ten steps of 100? Can you explain how you know?

3 Make five 7-digit or 6-digit numbers by rolling the dice and put all your numbers in order.

Going deeper

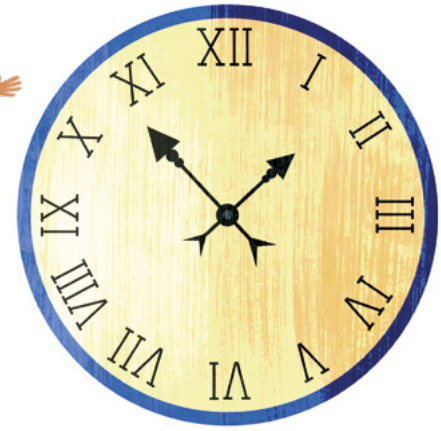
1 Take turns to make 7-digit numbers by rolling a dice five times and then including two zeros.

Can you subtract 90 000 from each number you make?

2 Can you change the game to make it even harder? What would be the hardest subtraction that you could think of? Can you make it easier?

Working with Roman numerals

In Roman numerals,
I stands for 1,
X stands for 10,
L stands for 50,
C stands for 100,
D stands for 500 and
M stands for 1000.



Practice

1 Can you work out what numbers these Roman numerals represent?

a XXXVI b XLII c DCCXX

2 Can you use Roman numerals to write:

a today's date b the date you were born c 4 July 1862?

Going deeper

1 a Which of these two numbers is bigger? Can you explain how you know?

b Can you add these two numbers together?

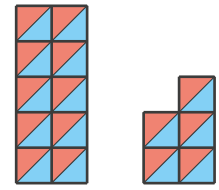
c Can you subtract the smaller number from the larger one?

2 Do you think it is easier to add and subtract with Roman numerals or with the numerals we usually use? Can you explain why, or why not?

3 What can you find out about how the ancient Romans actually did calculations?

Exploring equivalence with fractions

Practice



1 Can you find three different ways to describe the amount of these tiles that is coloured blue? Can you find:

   a an improper fraction

   b a mixed number

   c a proper fraction?

2 a Can you draw an illustration that shows the fraction $\frac{18}{2}$?

   b Can you write $\frac{18}{2}$ in another way?

   c Can you explain how you worked this out?

3 Can you write these numbers in other ways?

   a $\frac{23}{2}$ b $12\frac{1}{2}$ c $\frac{38}{2}$ d $25\frac{1}{2}$

Going deeper

1 Can you draw tiles to show these fractions? You can use any design for the tiles you choose.

   a $\frac{1}{4}$ b $\frac{3}{4}$ c $2\frac{1}{2}$

2 Can you make an illustration that shows the fraction $\frac{9}{4}$?

3 a Can you explain how you did question 2 above?

   b Can you use your method to make an illustration that shows $4\frac{3}{4}$?

4 a Which Numicon Shape would be most useful for illustrating quarters?

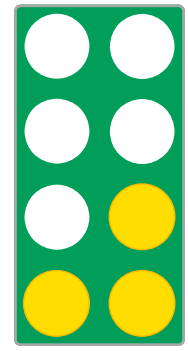
   b Can you explain why, and use some of the Shapes to illustrate $\frac{15}{4}$?

5 a Which number rod would be most useful for illustrating quarters?

   b Can you explain why, and use some of these rods to illustrate $2\frac{3}{4}$?

Converting mixed numbers and improper fractions

Practice



1 Can you write a fraction to go with this illustration?

2 How could you illustrate this same fraction with number rods?

3 a Try to find three ways of illustrating the fractions below.

$\frac{13}{8}$ $\frac{20}{8}$ $\frac{31}{8}$ $\frac{43}{8}$

b What kind of fraction are these?

4 a Can you write the fractions you illustrated in question 3 in another way?

b What kind of numbers have you written?

c Can you explain how you worked these out?

5 Which Numicon Shapes and which number rods could you use to illustrate the following numbers?

$3\frac{1}{6}$ $4\frac{2}{3}$ $2\frac{3}{5}$ $1\frac{4}{7}$

Going deeper

1 Can you write $3\frac{7}{8}$ in two other ways?

2 Explain how you could add together $4\frac{3}{5}$ and $\frac{19}{5}$.

3 Explain how you could subtract $\frac{19}{5}$ from $4\frac{3}{5}$.

Exploring equivalent fractions

The 12 children in Seagulls group found these ways to divide their brownie tray into 12 equal parts.

Dividing a half into sixths

Dividing a third into quarters

Dividing a quarter into thirds

Practice

1 There are 18 children in Gannets group. How many different ways could they divide their brownie tray into 18 equal parts? You can use rectangles to record the ways you think of.

2 How many different fractions could you use to describe:

a $\frac{1}{3}$ of Gannets' tray

b $\frac{1}{2}$ of Gannets' tray

c $\frac{1}{6}$ of Gannets' tray

d $\frac{1}{4}$ of Gannets' tray?

3 How could you use number rods to show $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{9}$ of 18?

Going deeper

1 Can you use any other fractions to describe $\frac{1}{2}$ of anything? Can you explain this?

2 Could you use any other fractions to describe $\frac{2}{3}$ of anything?

3 Do any of the fractions below describe the same amounts?

$\frac{2}{5}$ $\frac{60}{100}$ $\frac{20}{50}$ $\frac{8}{10}$ $\frac{9}{15}$ $\frac{8}{20}$ $\frac{6}{10}$ $\frac{30}{50}$

4 Can you solve these empty box problems and explain how you did this?

a $\frac{5}{8} = \frac{\square}{24}$

b $\frac{3}{9} = \frac{4}{\square}$

c $\frac{\square}{7} = \frac{6}{21}$

d $\frac{3}{\square} = \frac{12}{44}$

Using fractions in everyday situations

I make drinks with 1 cup of cordial for every 5 cups of water. The jug holds 12 cups altogether.

Practice

1 a Can you use number rods or Numicon Shapes to show the proportions of cordial and water in Tia's favourite drink?

b Can you write these proportions as fractions?

2 Can you draw and fill in a table that shows how much cordial and how much water is needed for 1, 2, 3, 4, 5 and 10 jugs of Tia's drink?

Number of jugs	Cordial	Water
1	$\frac{2}{12}$	$\frac{10}{12}$
2	$\frac{4}{24}$	
3		

3 a What patterns can you find in the table you have made?

b Can you see any 'fraction families'?

Going deeper

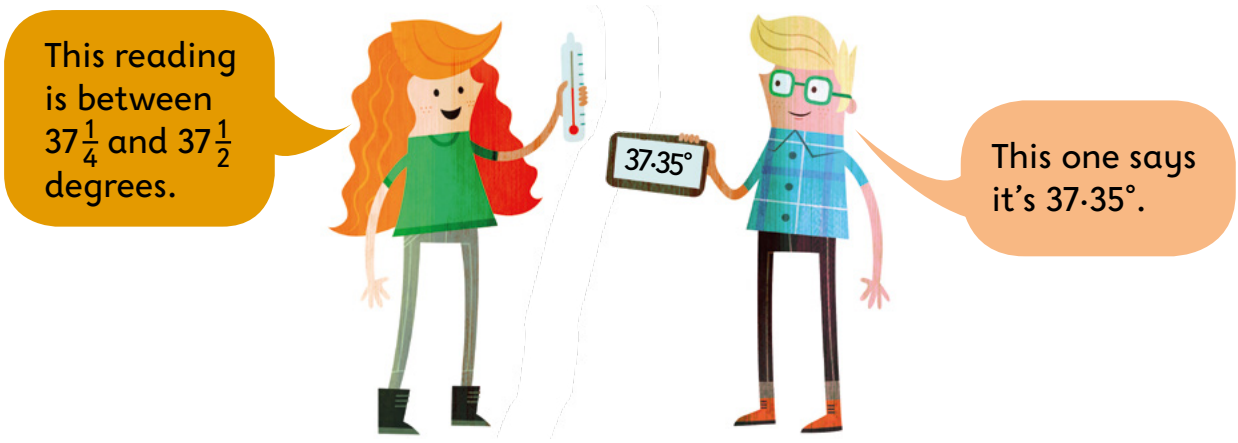
1 Can you illustrate the squash fraction family using Numicon Shapes?

2 Can you illustrate the water fraction family using a number line?

3 Can you illustrate the $\frac{2}{5}$ fraction family using Numicon Shapes?

4 How can you test whether $\frac{3}{7}$ and $\frac{222}{518}$ are in the same fraction family?

Understanding decimals



Practice

1 Does 37·35 lie between $37\frac{1}{4}$ and $37\frac{1}{2}$? Can you explain how you know?

2 How can you write 3 more decimals that lie between $37\frac{1}{4}$ and $37\frac{1}{2}$?

3 How can you write 37·75 as a fraction?

Going deeper

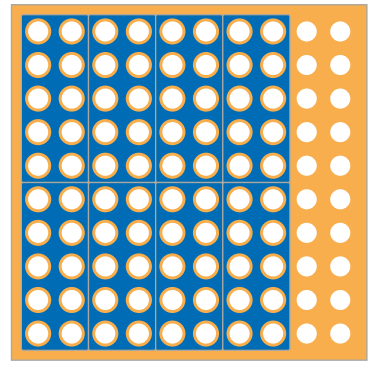
1 Can you explain why some numbers have a decimal point (for example 47·5) and others (for example 84) do not?

2 Can you describe where you have seen decimals being used, outside school?

3 Can you talk about any situations when you have used fractions, outside school?

Converting fractions to decimals

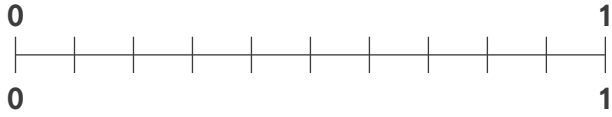
Practice



1 **a** How much of this baseboard is covered in blue?

b Can you write your answer as a fraction and a decimal?

2 Draw a number line like this.



Can you show your fraction and decimal from **question 1b** on it? How did you work out where these numbers go?

3 See if you can make a list of all the proper fractions that have denominators of 2, 4, 5, or 10.

a Can you write them all down in order of size?

b Can you write these fractions as decimals?

c Now can you order the decimals on a blank number line?

Going deeper

1 **a** How many ways can you find to convert $12\frac{3}{4}$ to a decimal?

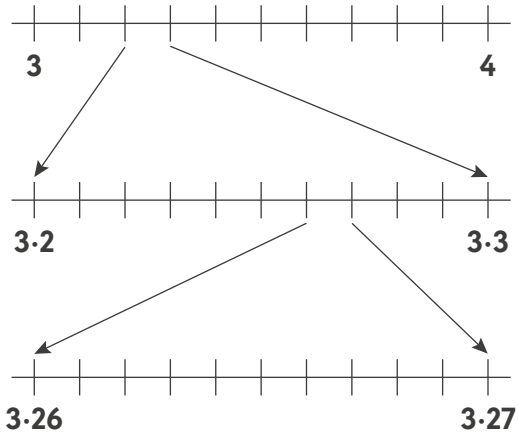
b Which method do you prefer? Can you explain why to your partner?

2 Can you work any of your methods from **question 1** backwards? For example, what is 18·4 as a fraction?

3 Can you find a way to convert $\frac{2}{3}$ into a decimal?

4 Using a calculator, try investigating proper fractions that have denominators of 3 and 9. What happens when you convert these into decimals? Can you explain any of these results?

Thousandths as decimals



Practice

1 Can you explain where 3·264 would be on the diagram above?

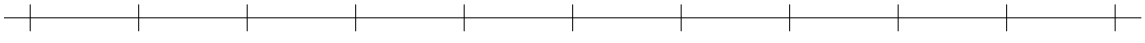
2 Can you draw a set of number lines like these to show where you can find 2·586?

3 How could you write 3·264 as a fraction? Can you explain a good method for doing this?

Going deeper

1 How could you describe how far 0·586 is along a number line from zero?

2 Copy this empty decimal number line. Can you use it to show where you can find 5·289? How will you choose number labels for each end of the line?



3 Draw another empty decimal number line and label the ends '0' and '0·01'. Can you pick a point around the middle of the number line and say what number that is?

Comparing and ordering decimals

Practice

1 Ravi and Tia can't agree whose computer file is bigger. One way of comparing two numbers like this is to use base-ten apparatus.

Does this apparatus help you to decide which number is bigger?

2 a Can you find another way to work out which of the numbers below is bigger than the other?

2·343 and 2·398

b Can you explain your method?

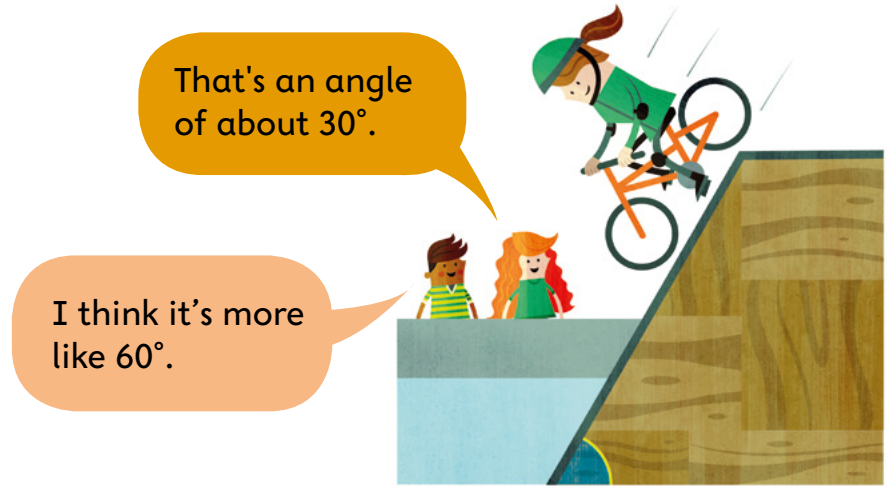
3 Four more children checked their document size in megabytes (MB) and these were: 2·81 MB, 3·147 MB, 3·009 MB and 2·908 MB. Can you put these files and Ravi and Tia's files above, in order of size? Which were bigger than 2·9 MB?

Going deeper

1 Which is bigger, $\frac{38}{100}$ or $\frac{364}{1000}$? Can you explain how you know? Can you write your answer using the signs '<' and '>'?

2 Which number is bigger, 0·02741 or 0·02729? Can you explain why you think so?

Measuring angles in degrees



Practice

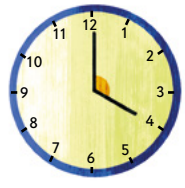
1 **a** Which estimate is closer, Ravi's or Molly's? How do you know?

b Can you say what kind of angle it is? How do you know?

2 Can you work out the size of each angle marked below, in degrees?

a 

b 

c 

d 

3 **a** Can you estimate the size of these angles, by sketching each one?

90° 45° 30° 120° 20° 70° 160° 225°

b Can you explain how you made your estimates?

Going deeper

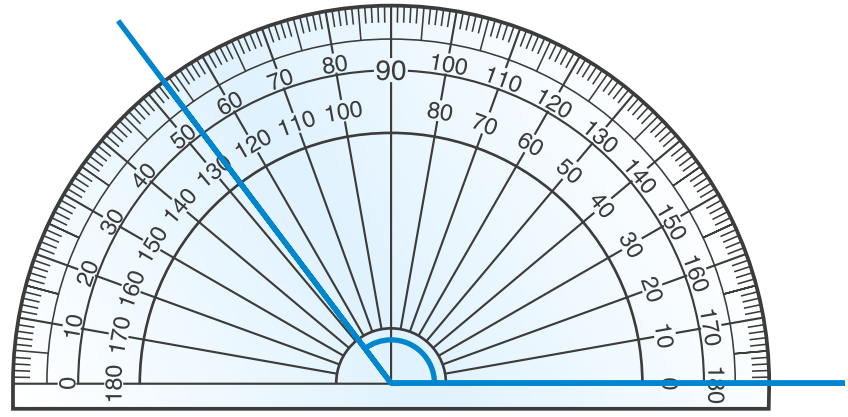
1 Can you work out what angle the **hour hand** of a clock will turn through, in degrees, between:

a 12 noon and 12:30 p.m. **b** 12 midnight and 6:15 p.m.?

c Now compare the position of the **minute hands**. What is the angle between the minute hands in the two positions? Work this out for each pair of times.

Measuring angles with a protractor

Practice

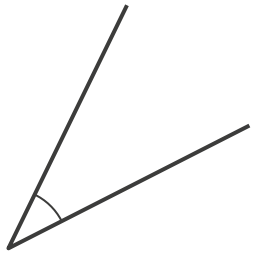


1 **a** Can you say what the angle shown is, in degrees?

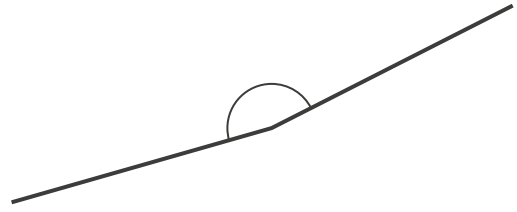
b Can you explain how to measure an angle with a protractor?

2 For each angle, can you say what type of angle it is (acute or obtuse), estimate its size and then measure it with a protractor?

a



b



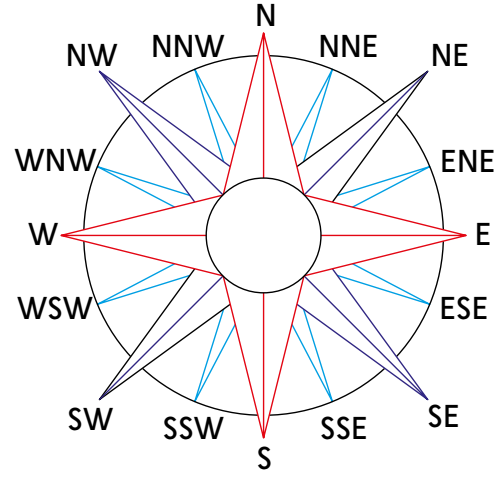
3 For each angle in question 2, sketch the angle, mark a reflex angle on your sketch and label it with its size. How did you work out these sizes?

Going deeper

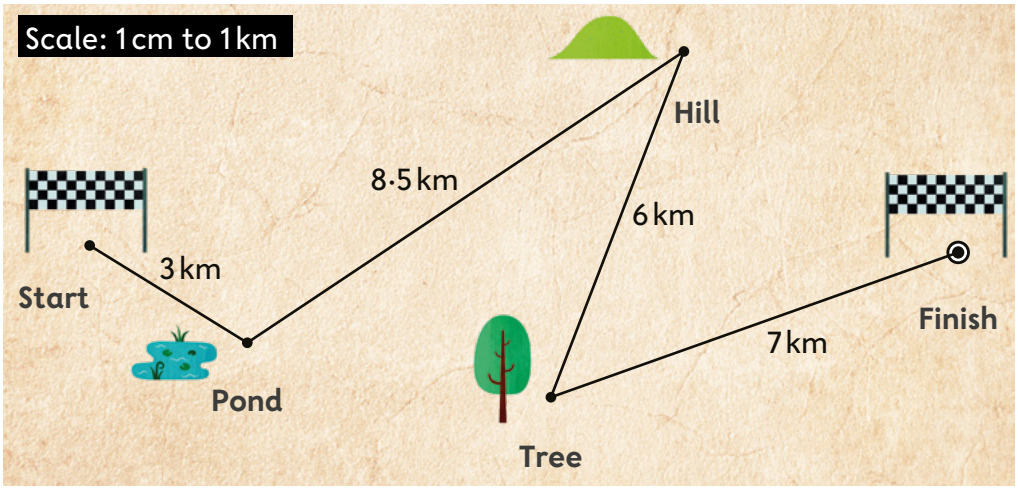
1 **a** Adam is facing west. Can you say what bearing (angle clockwise from north) he is on?

b Adam now turns anti-clockwise to face north-north-east. Can you work out how many degrees he has turned through?

c Can you work out what his new bearing is?



Measuring and drawing angles



Practice

1 Ella is orienteering. She begins at the start, facing the pond. Can you use the map and a protractor to write a set of instructions for her to follow? Give the distances she should run and the turns she should make.

2 Can you draw two lines with an angle between them of:

a 45° b 100° c 108°?

Going deeper

1
- Draw a horizontal line 5 cm long using a ruler.
- Draw a second line, also 5 cm long, joined to the end of the first line. Use a protractor to make the angle between the lines 120°.
- Continue adding lines, each 5 cm long and joining at an angle of 120° to the previous line, until you end up back where you started.
- What shape have you drawn?

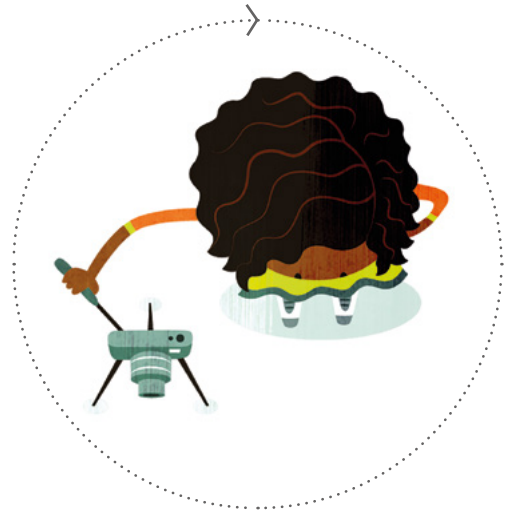
2 Repeat question 1 but this time use an angle of 135°. What shape have you drawn this time?

3 Compare the shapes you drew in questions 1 and 2. Can you explain what difference the size of the angle makes?

Finding missing angles

Practice

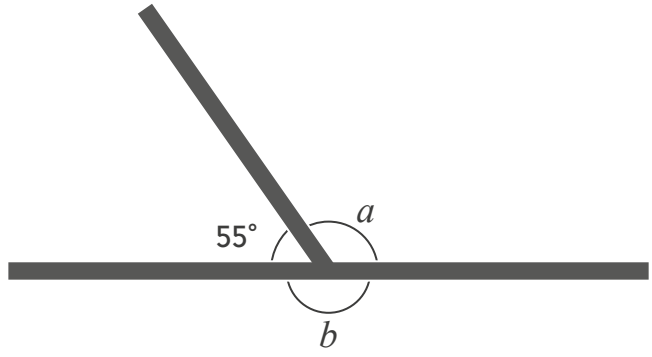
1 Tia is recording the view through her camera as it turns in a full circle.



a How many degrees will the camera turn through to make a full circle?

b If it has turned through 90° so far, how much further does the camera need to turn to complete a circle?

2 The diagram below shows a junction where a side road meets a main road at an angle of 55 degrees. Can you work out the sizes of the missing angles in the diagram? How do you know?



Going deeper

1 A security camera outside a museum is fixed on a pole and has been programmed to turn clockwise continuously. It pauses every time it points north-west, south-south-east and south-west. The camera's starting position is north-west.

What three angles does it turn through, between pauses? Can you draw a diagram showing these angles? How did you work out the answer?

Developing adding and subtracting

Practice

1 Three children swam 65 lengths of a swimming pool in total between them. They each swam at least 18 lengths, but no more than 25 lengths. How many lengths might they each have swum?

2 Can you find five different solutions for each of these empty box problems?

a ☐ + 580 + ☐ = 940 b ☐ + ☐ + 345 = 724

3 Can you solve each of these empty box problems?

a 645 − 251 + ☐ = 646 b 58 + 76 = 53 + ☐

Going deeper

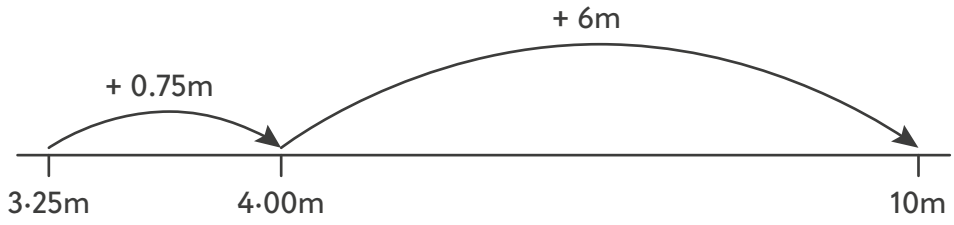
1 Can you explain why the number sentence 438 – 276 = 498 – 336 has to be true, without working out either subtraction calculation?

2 Look at your answer to question 1. Can you find nine other combinations of three whole numbers between 18 and 25 that total 65? You can use any of the numbers more than once. Do you think you have now found all the possibilities? Can you explain why?

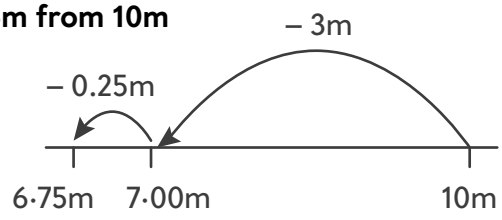
3 What do you think is the quickest way to work out the following empty box problem: 89 − ☐ = 80 – 49?

Adding and subtracting in problem solving

Adding on from 3·25m to 10m



Subtracting 3·25m from 10m



Practice

1 Rui buys 6 different colours of rope. Each coil of rope is 10 metres long. How much rope is left on each coil after he has cut off the lengths below?

a Purple: 3.25m **b** Red: 5.6m **c** Green: 7.35m

d Yellow: 6.20m **e** Blue: 3.49m **f** Orange: 1.72m

2 Kiana is doing a 90 km sponsored cycle ride from Kaiaua to Waikino Station. She takes breaks at Waitakaruru (after 23·1 km), at Thames (after 52·8 km), and at Paeroa (after 70·4 km).

a When she is at Waitakaruru, how far does she still have to go?

b When she is at Thames, how far does she still have to go?

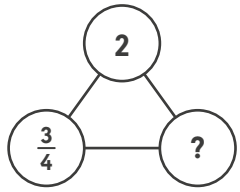
c When she is at Paeroa, how far does she still have to go?

Going deeper

1 If we know that two video clips together last 12 minutes 15 seconds, and that one of them lasts 5 minutes 54 seconds, what else can we work out? How many different number facts can you write out from this information?

2 What do you think are the best ways of doing the calculations for question 1? Can you explain why in each case?

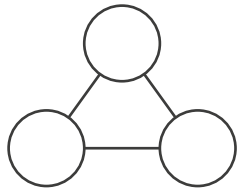
Adding and subtracting fractions



Practice

1 Can you complete the number trio above? Can you now write four different number sentences using just these numbers?

2 Copy this blank number trio onto a whiteboard or into your exercise book. Write a whole number in the top circle and a fraction in one of the lower circles. Can your partner calculate what must go in the third circle? Swap roles and play again. Repeat four times.



3 Can you work out what number goes in the empty box below?

$3\frac{2}{5} + 2\frac{4}{5} = 3\frac{3}{5} + \square$

Going deeper

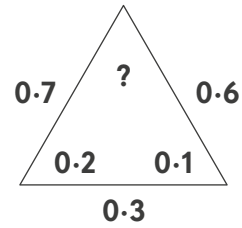
1 What do you think are the best ways of calculating the answers to complete any number trios? Can you explain why?

2 What do you think is the best way of calculating the answer to this empty box problem? Can you explain why?

$4\frac{5}{7} - \square = 2\frac{3}{7} - 1\frac{4}{7}$

Adding and subtracting decimals

Practice



1 Can you find the missing number in this triangle?

2 Can you copy and complete these adding grids?

a

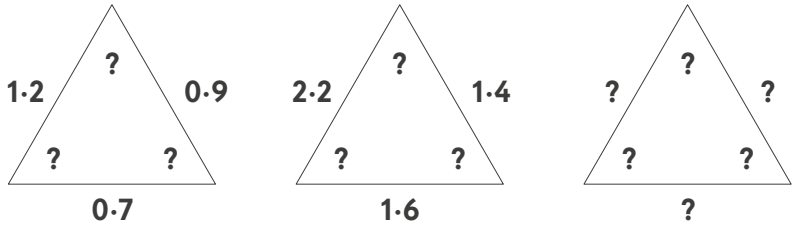
+	2·4	1·3
5·1		
4·7		

b

+		
	5·7	4·5
	10·1	8·9

3 Can you find another solution for the grid in question 2b?

Going deeper



1 Can you suggest one way to complete each triangle above?

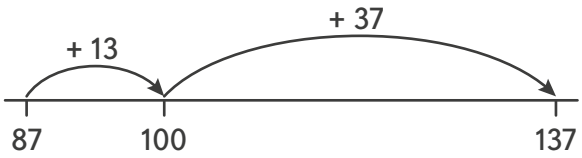
2 In a magic square, each row, column and diagonal adds up to the same total. Can you copy and complete this magic square?

0·7	0	
	0·4	0·6
0·3		0·1

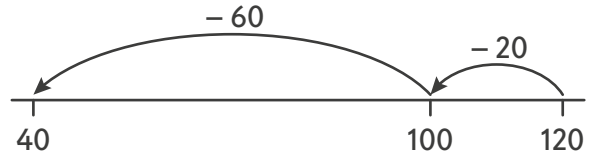
3 Can you explain the quickest way to find the missing number below?

7·35 − 3·53 + ▢ = 7·36

Using bridging strategies



87 + 50



120 – 80

Practice

A: 170, 210, 340, 470, 520, 650, 6940, 7830

B: 47, 69, 85, 96, 389, 475, 580, 675

1 Choose four pairs of numbers, one from each list above, and either add them or subtract a number from List B from a larger number in List A.

2 Can you find one way to complete each of the calculations below?

a 195 + 75 = 195 + ☐ + ☐ = 200 + ☐

b 174 – 80 = 174 – ☐ – ☐ = 100 – ☐ = 94

c 360 – 85 = 360 – ☐ – ☐ = 300 – ☐ = 275

d 1600 + 540 = 1600 + ☐ + ☐ = 2000 + ☐ = ☐

Going deeper

1 80 + 57 = 87 + 50, but does 80 – 57 = 87 – 50? Can you explain?

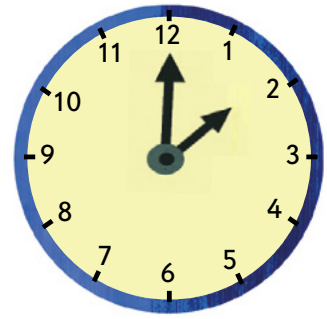
2 What is the best way to work out how far you need to jump from any 2-digit number to 100?

3 Which methods would you choose to do the following calculations?

a 76 + 84 = ☐ b 154 + 132 = ☐ c 193 + 57 = ☐ d 293 + 198 = ☐

e 368 – 207 = ☐ f 240 – 122 = ☐ g 325 – 75 = ☐ h 476 – 97 = ☐

Bridging with time



Practice

1 Molly's favourite TV programme lasts 45 minutes. If it began at the following times, when would it end?

a 2:00 p.m. b 1:45 p.m. c 3:30 p.m.

d 4:25 p.m. e 6:50 p.m.

2 Another programme lasts 50 minutes and ends at these times. What time did it begin?

a 1:15 p.m. b 5:30 p.m. c 3:40 p.m.

d 2:35 p.m. e 8:10 a.m.

3 Charlotte arrived back home at 4:10 p.m. after a journey of two and a quarter hours. When did she set off?

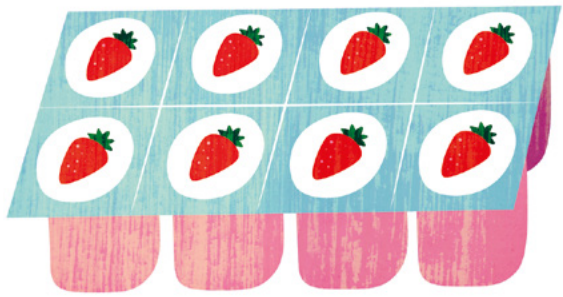
Going deeper

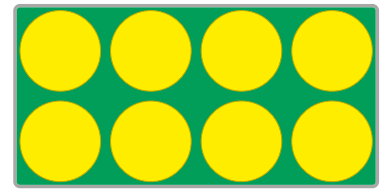
1 Jay has to get a meal ready for 1:30 p.m. The meat will take 1 hour 50 minutes to cook, the potatoes take 55 minutes, the carrots 40 minutes, the broccoli 6 minutes and the gravy 25 minutes. Can you make a list of the times when Jay has to start cooking each ingredient?

2 Tia's morning in school lasted 2 hours 45 minutes, and the afternoon was 1 hour 20 minutes long. She adds these times together using a column method and gets the answer 3 hours 65 minutes. Is she correct?

```
  2 : 45
+ 1 : 20
-------
  3 : 65
-------
```

Bridging with fractions





Practice

A: $3\frac{3}{8}$ $4\frac{1}{2}$ $2\frac{1}{4}$ $5\frac{1}{8}$

B: $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$

1 a Can you write and solve three different adding calculations by pairing a fraction from List A with a fraction from List B?

b Can you write and solve three different subtracting calculations by pairing a fraction from List A with a fraction from List B?

2 Can you make two adding and subtracting questions with the answer $3\frac{2}{5}$?

Going deeper

1 What do you think is your best method for taking $\frac{7}{8}$ away from the mixed numbers in List A above? Can you explain why you prefer this method?

2 What is a good method for adding $\frac{3}{4}$ to the mixed numbers in List A?

3 What do you think is the best way of solving the problem below? Can you explain why?

$$\frac{5}{6} + \frac{13}{6} = 1 + \square$$

Bridging with decimals

Practice

1 Hazel has grown 0·8 cm during the first term, and she is now 131·3 cm tall. How tall was she at the beginning of term?

2 Write and solve three adding and three subtracting calculations by pairing a number from List A with a number from List B. (Avoid negative numbers.)

A: 14·3, 12·1, 6·4, 25·2

B: 2·9, 4·8, 5·7

3 Can you make two adding and subtracting questions with the answer 2·6?

Going deeper

1 What is the best method you can find for taking away 2·9 from the numbers in List A above?

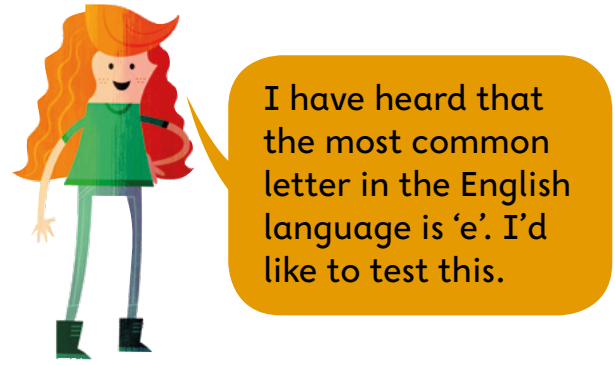
2 What is a good method for adding 4·8 to the numbers in List A?

3 Can you find two different ways of solving this empty box problem? Does it help if you think about money? Can you explain?

7·6 − 2·99 = ☐

Everyday estimating

Practice

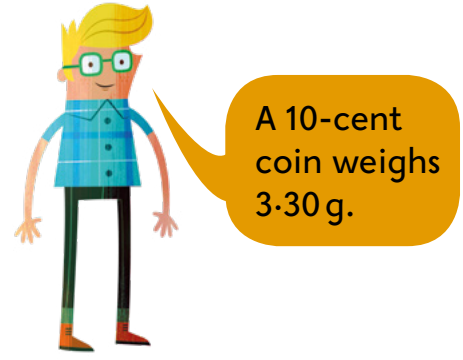


1 **a** Roughly how many times do you think the letter ‘e’ is used in a page of your reading book?

b Approximately how many times do you think the letter ‘e’ is used in the whole book? How could you estimate this?

2 Roughly how many times do you think the letter ‘t’ is used in a page of your reading book? Do you think that it would be used more or less often than ‘e’?

3 How much do you think 1000 10-cent coins would weigh? Discuss how to estimate this with your partner.

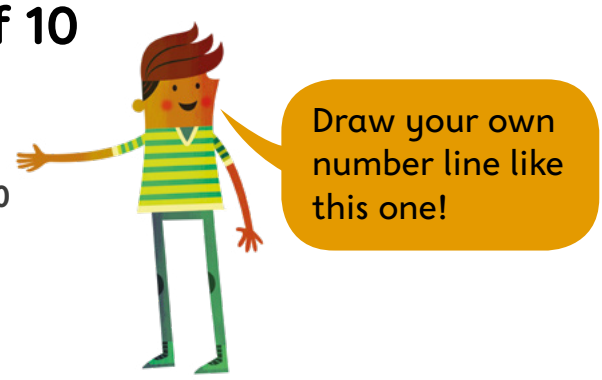


Going deeper

1 What strategies would you recommend to estimate the number of tables in the school?

2 Chris is at a school fair, trying to guess how many lollies are in a big glass jar. Can you explain a good strategy for him to use to your partner?

Rounding to multiples of 10



Practice

1 Can you show where you think 24 242 should be on your number line?

2 Rounding to the nearest 100, can you find one number that would round up and one that would round down to each entry on the number line? Make each of your numbers at least 30 away from the target.

3 Use scales to find the mass of one of your shoes, to the nearest 100 g.

4 Can you use a tape measure to measure the width of your table. How wide is it, to the nearest 10 mm?

5 The distance around the equator is 40 075 km. What is this to the nearest 1000 km?

6 Which of the numbers below is closest to 10 000?

10 030, 10 012, 9990, 9980, 10 019

Going deeper

1 Can you round 1 738 562 to the nearest:

a 10 **b** 100 **c** 1000

d 10 000 **e** 100 000?

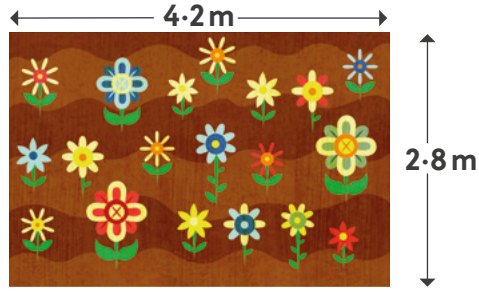
Can you explain the best way of rounding numbers like these?

2 The size of the crowd at a festival is 45 000 to the nearest hundred.

a What is the smallest number that the crowd could be?

b What is the largest number that the crowd could be?

Rounding with decimals



Practice

1 Ava wants to make the flower bed above. Can you work out what the area will be to the nearest square metre?

2 Seeds are bought in bags of 50 g, and should be spread at the rate of 2 g per square metre. Roughly how much of the bag will be left over after Ava has sown the seeds?

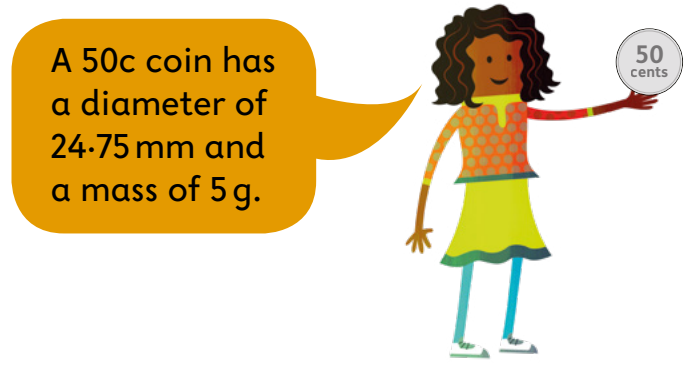
3 Ava wants to put edging around her flower bed, which comes in packs of 1·5 m for $3·69. How much edging do you think she will have to buy? How much do you think it will cost, to the nearest $1?

Going deeper

1 Josh said that he would rather be given a column of $2 coins as tall as himself than a collection of $1 coins weighing as much as himself. He is 140 cm tall and weighs 35 kg. A $2 coin is 2·7 mm thick and a $1 coin weighs 8 g.

Estimating roughly, do you think he made the right choice? Can you explain?

2 If you lined up 1 km of 50c coins, roughly what would their mass be?



Estimating calculations

2	3	5	7	8	9
120	210	290	420	490	630

Practice

1 Look at the table above. Can you write a calculation using one number from the top row and one from the bottom row that will produce the answer closest to 150? Explain how you know you are correct.

2 Can you estimate the answers to the calculations below?

a 4653 + 490 = ☐

b 8322 − 510 = ☐

c 25 × 85 = ☐

d 3199 ÷ 38 = ☐

Going deeper

349, 303, 4509, 455, 798, 1003, 29, 12, 5, 20

1 Can you choose pairs of numbers from the list above to write calculations that will produce a number between:

a 300 and 500

b 1000 and 1500

c 2000 and 5000?

2 Choosing any numbers from the list above, can you write a calculation that will produce the answer 600?

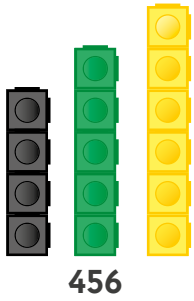
3 Choosing pairs of numbers from the list above, can you write a calculation to produce:

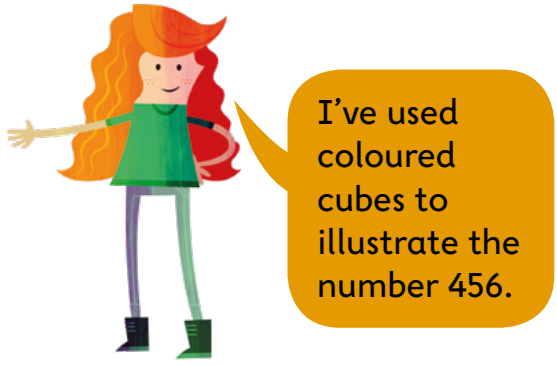
a the largest possible number

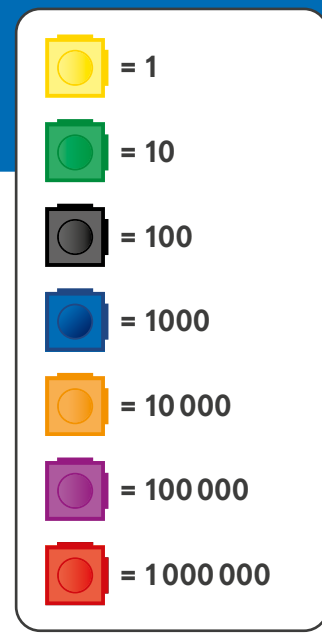
b the smallest possible number?

Explain how you know you are correct to your partner.

Further adding and subtracting







Practice

1 Molly adds 1 black cube, 4 green cubes and 3 yellow cubes onto the number she made above. Can you write this as an adding calculation? What new total has she made?

2 Next she adds 4 yellow cubes. What is the new total?

3 Then she adds 4 black cubes. What is the new total?

4 Now she adds 1 purple, 3 orange and 2 blue cubes. What is the new total?

5 Now she takes away 10 000. Can you write this as a subtracting calculation? How much is left?

6 Finally, she takes away 4 green cubes. What is the final total?

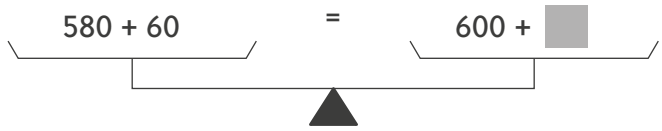
Going deeper

1 If you start with 12 631 and you end up with 11 831, how much did you take away? How can you work this out in your head?

2 If you start with 133 003, which of the numbers in the list do you think are the hardest to take away? Can you explain why?

4, 40, 400, 4000, 40 000

Balancing with adding and subtracting



Practice

1 Can you find the missing number by balancing the calculation above?

2 Choose pairs of numbers from the list. For each pair, write one adding and one subtracting balancing calculation. For example, 379 − 169 = 380 − 170 , and 379 + 169 = 400 + 148.

379, 288, 169, 236, 187, 455

3 Can you solve the empty box problems below without working out the actual totals or differences?

a 695 + 40 = 700 + ☐ b 695 − 40 = 700 − ☐

4 Can you use balancing to make the calculations below easier? For example, 287 + 63 = 300 + 50.

a 589 + 43 = ☐ + ☐ b 632 − 197 = ☐ − ☐

Going deeper

1 a Write an adding calculation that you can make easier using balancing. Now write an adding calculation that you think is **not** made easier using balancing.

b Can you explain the difference? Why does balancing help with the first calculation, but not the second?

2 When does balancing make subtracting easier? Can you explain why?

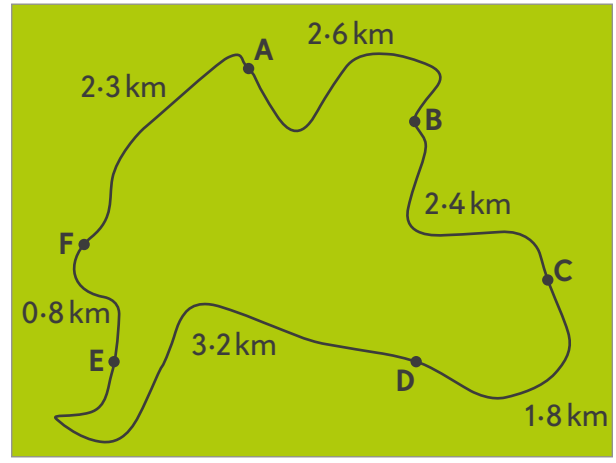
Adding and subtracting decimals by partitioning

Practice

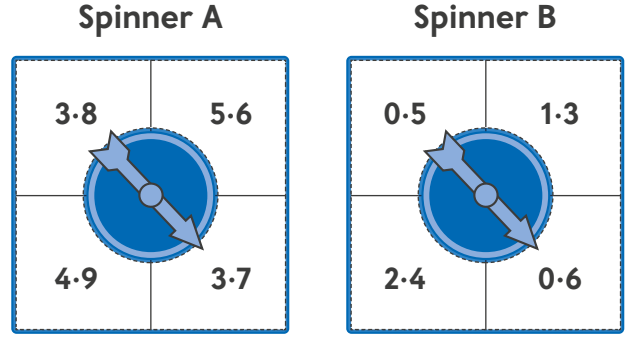
1 Sam is cycling around this trail and starts at point A.

a How far will Sam have cycled when he reaches each of the points B, C, D, E and F?

b How far will he cycle altogether?



2 How many subtracting calculations can you write using one number from Spinner A and one number from Spinner B?
What are the answers?



Going deeper

1 Can you explain which is your best method for:

a adding 3·8 and 2·4

b subtracting 2·4 from 3·8?

2 a Which way do you prefer to find the total of 5·2 and 2·4?

b What about subtracting 2·4 from 5·2?

3 Can you explain anything about connections between the number pairs and the methods you prefer in questions 1 and 2?

Adding and subtracting in columns

Practice

1 Can you think of other ways of adding 167 and 156?

	1	6	7
+	1	5	6
	3	2	3
	1	1	

2 Can you think of other ways of subtracting 386 from 484? Which method do you think is best?

	$^{3}\not{4}$	$^{17}\not{8}$	$^{1}4$
−	3	8	6
		9	8

3 Lily and Hiro each chose a different item from the menu. Lily paid the total bill using her bank card. The total was less than $13.

a Can you work out three different pairs of items they could have chosen?

b For each of these, how much money would Lily have left in the bank if she started with $16·23?

Salad	$9·85
Quiche	$6·99
Cookie	$3·45
Muffin	$4·49
Energy ball	$2·89
Brownie	$5·15

Going deeper

1 Can you solve these problems by using a written column method?

a 2326 + ☐ = 5798

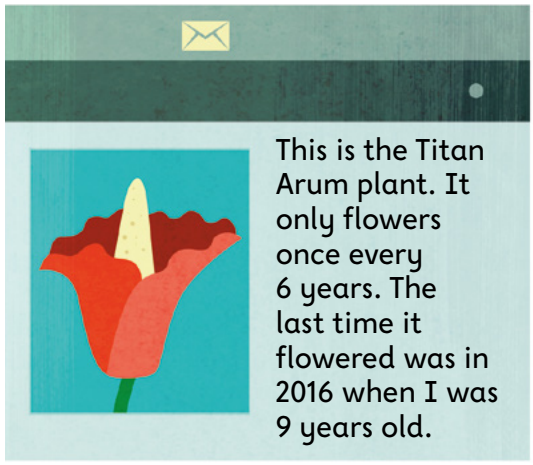
b ☐ + 1786 = 4000

c ☐ − 4553 = 2345

d 7803 − ☐ = 4719

Sequences and patterns

Aanjay lives in Sumatra in Indonesia.
He has sent an email to the class.



Practice

1 Can you work out the years when Aanjay's Titan Arum flowered in the past 35 years? Write them down as a sequence.

2 Leap years happen every four years and 2016 was a leap year. Can you work out the next five years which will be leap years?

3 If you start with the year you were born, can you make a sequence by adding 9 years repeatedly?

4 Can you work out the missing numbers in this sequence?

3231, ☐, ☐, 3267, ☐, 3291

Going deeper

1 Can you make up another problem like question 3 above, for a partner to solve?

2 If 123 is the first number of a regular sequence, and 168 is the last number, what are the possible numbers that could appear regularly spaced between these two?

3 How would you explain to a friend how to work out the answers to a question like question 2 above? Can you make up a sequence problem of your own?

Decimal sequences

Practice

1 What number values could you give to these green and orange rods?

2 If the orange rods each had the value '1', what value would you give to the green rods? Write these out as a sequence of decimal numbers.

3 Can you write a 0·3 sequence, starting at 0·5? Can you calculate when the first whole number will occur, before you reach it?

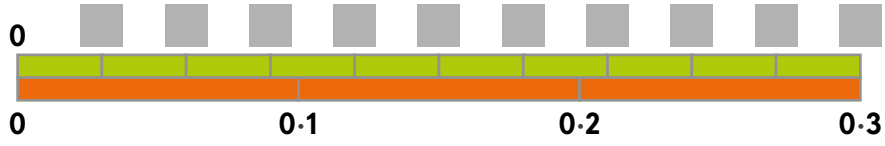
4 Can you carry on this sequence?

5·3, 5·7, 6·1, 6·5, ...

5 Try to make up the beginnings of some more decimal sequences for your partner to carry on with. You can make them as difficult as you like.

Going deeper

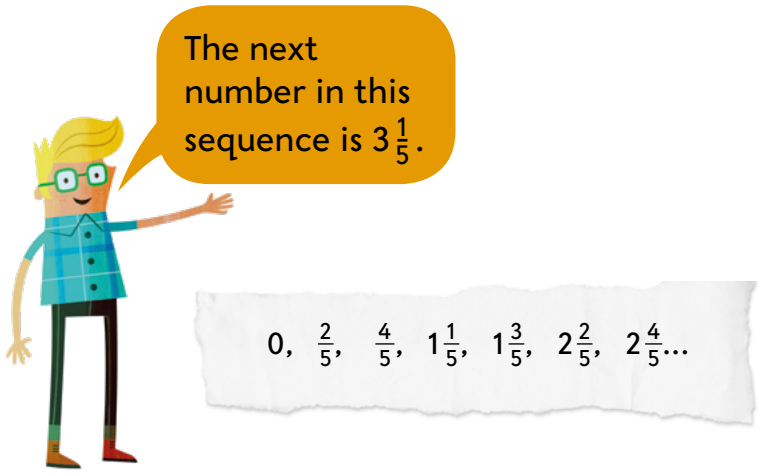
1 Jan made a different decimal sequence. Which numbers could go in these empty boxes?



2 How could you use number rods to illustrate the sequence below? Can you explain how you worked out how to do this?

7·49, 7·56, 7·63, 7·7, 7·77, 7·84, ...

Fraction sequences



Practice

1 How many different ways can you think of to illustrate the fraction $\frac{2}{5}$?

2 Can you use any of these different ways to illustrate the fraction sequence above?

3 Can you show these fractions as steps along a number line?

4 Do any of these illustrations help you to count on and back in fractions?

5 Can you continue counting on in the following sequences?

a $\frac{2}{3}$, $1\frac{1}{3}$, ... b $\frac{5}{6}$, $1\frac{2}{3}$, ... c $\frac{3}{8}$, $\frac{3}{4}$, ...

Going deeper

1 a Can you work out what the tenth term of the following sequence will be, without working out the first nine terms?

0, $\frac{7}{9}$, $1\frac{5}{9}$, ...

b How do you know your tenth term is correct? Can you check? Can you explain to a partner how you worked this out?

Connecting sequences with equivalent fractions

$\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}$ ...

Practice

1 Can you carry on writing out this sequence of fractions?

2 What do you think is the easiest way to continue writing out this sequence?

3 What can you say about **all** of these fractions?

4 Try writing out fraction families, in the easiest way that you know, for each of these fractions:

a $\frac{1}{3}$ b $\frac{2}{7}$ c $\frac{5}{8}$ d $\frac{3}{5}$

Going deeper

1 How do we know that lists like these really do show fractions of the same families? How can you test them?

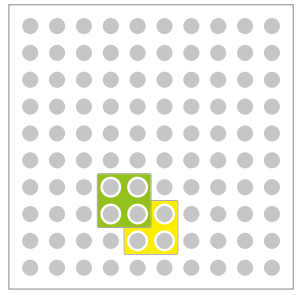
2 Can you think of other ways to generate fraction family lists?

3 If you were explaining what fraction families are to a friend, how could you use a number line to help illustrate what you are saying?

4 Could you use Numicon Shapes to explain why different members of a fraction family are definitely in the same family?

5 Is there a quick way to work out the tenth member of the $\frac{3}{8}$ family, without working out the first nine members as well? Can you explain your method?

Transformations



Practice

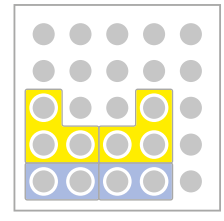
1 How many symmetrical shapes can you make on a baseboard using just one yellow and one light green Numicon Shape?

2 How many ways can you find to add a third Numicon Shape to the board to create a symmetrical figure? You can use **any** of the Numicon Shapes.

3 Mia wants to use a symmetrical pattern of tiles in her new bathroom. Can you create a design for the tiles using three types of Numicon Shapes on a baseboard? Swap with your partner, and find the line of symmetry on each other's design.

Going deeper

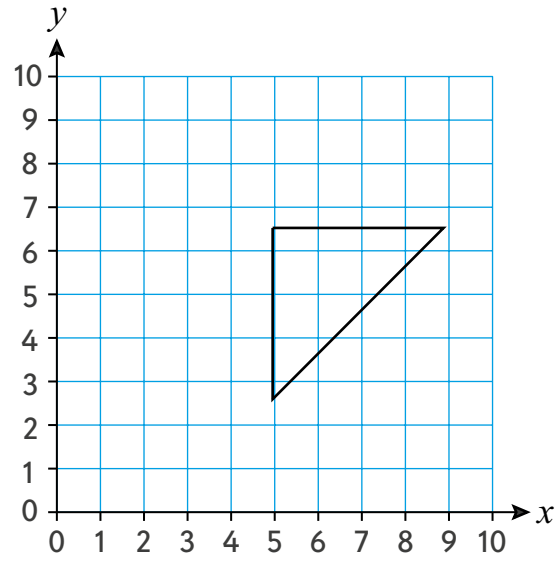
1 How many symmetrical figures can be created by using two 2-shapes and two 3-shapes on a 5 × 5 section of a baseboard? Here is one possible arrangement:



2 Take turns to draw a symmetrical pattern onto a folded piece of squared paper, without your partner seeing. Can you describe the pattern in words for your partner to draw? Afterwards, show them the original pattern to check.

Reflections on a coordinate grid

Practice



1 Copy the coordinate grid with the triangle shown here. Can you reflect the triangle in three different ways, using each of the triangle's sides as a line of symmetry?

2 What is the same about all of the new triangles you have drawn? What is different? Explain to a partner.

3 If you think of the four triangles as a single shape and draw around the outline, what shape have you drawn? Can you find its line of symmetry?

Going deeper

1 Take turns to choose a shape and plot it onto a coordinate grid. Give your partner the shape's coordinates and choose a line for them to reflect the shape in. Ask your partner to plot the reflected image.

2 The 'Clear Reflections' window cleaning company is looking for ideas for a logo using their initials (CR) and two different reflections.

Using a coordinate grid with axes from 0 to 10, and the letters C and R, create a symmetrical design for the logo. List the coordinates for your design.

Describing translations using coordinates

I drew a rectangle with vertices (1,8), (1,10), (4,10) and (4,8).

Ben

I drew a rectangle with vertices (10,7), (10,9), (7,9) and (7,7).

Molly

I drew a rectangle with vertices (4,4), (7,4), (7,2) and (4,2).

Ravi

I drew a rectangle with vertices (1,2), (4,2), (4,4) and (1,4).

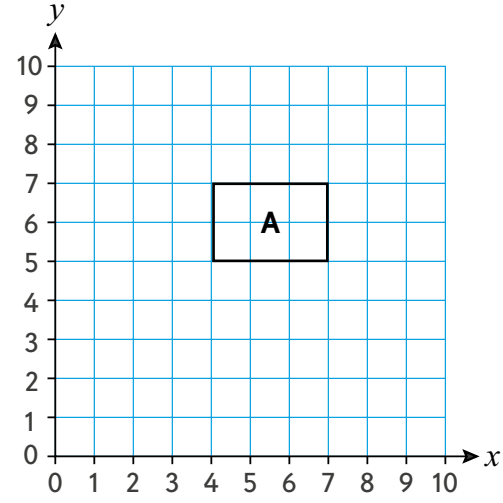
Tia

Practice

1 a Copy rectangle A on a coordinate grid (with axes from 0 to 10). Mark on the rectangle for each child.

b How would you translate rectangle A on to each of the children's rectangles?

2 How would you translate each of the children's rectangles onto rectangle A?



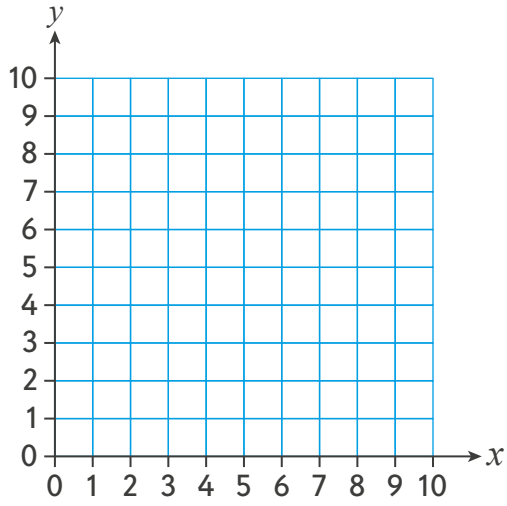
Going deeper

1 If you combine three of the translations listed below, one of the children's rectangles above will translate onto rectangle A.
Can you work out whose rectangle it is?

Right 4, down 2	Left 16, up 3
Right 10, down 7	Right 6, up 7

2 Can you describe any reflections you can see in your diagram from question 1?

Exploring translations on a coordinate grid



Triangle A has coordinates (2,6), (2,4) and (5,4)

Triangle B has coordinates (7,6), (7,4) and (10,4)

Triangle C has coordinates (1,9), (1,7) and (4,7)

Triangle D has coordinates (3,10), (3,8) and (6,8)

Triangle E has coordinates (1,3), (1,1) and (4,1)

Triangle F has coordinates (7,10), (7,8) and (10,8)

Practice

1 Draw the triangles on a coordinate grid like the one above.

2 Find out which triangle would map onto which other triangle using these translations:

a 5 right

b 6 up

c 2 right, 1 up

d 4 left

e 6 right, 3 up

f 6 left, 3 down

g 6 left, 7 down

Going deeper

1 Where would you draw the coordinates of a new triangle (G) so that each of the translations below would map it to one of triangles A–F?

2 left	1 left, 3 up
7 up	4 right, 3 up

2 Draw a design on a coordinate grid and give your partner instructions for a translation. Can your partner plot the new position of the design? How can you check their answer? Swap roles and repeat three times.