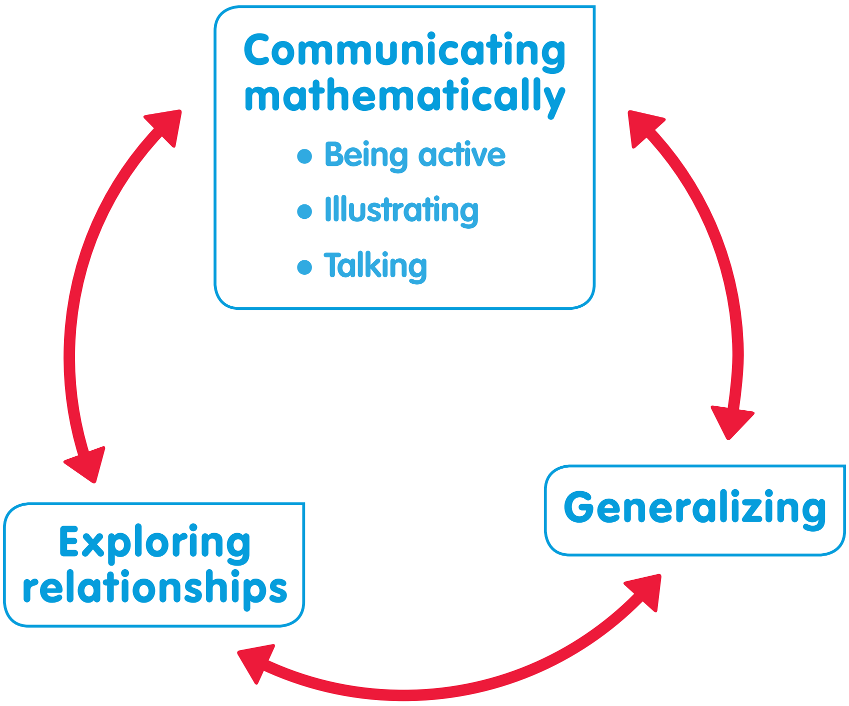
NUMICON

An Introduction



What is Numicon?

Numicon is a distinctive approach to children's mathematical learning that emphasizes three key aspects of doing mathematics: communicating mathematically, exploring relationships, and generalizing.

Communicating mathematically

Doing mathematics involves communicating and thinking mathematically – and these are two sides of the same coin. We think in the same ways that we communicate, and communicate in the same ways that we think. As children learn to communicate mathematically, they learn to think mathematically. This involves them in the following:

Being active: Teaching and learning with Numicon requires children to be active. This does not only mean being physically active (e.g. fitting physical objects together, finding a number on a number line, drawing a shape), but is a requirement reflecting the understanding that mathematics itself is activity: mathematics is something children are learning to *do*.

What this means in practice is that it is always the children themselves who are to do the mathematics. Telling children (or showing, or explaining) 'what to do' can encourage children to be passive. Numicon asks that the children do the mathematics (i.e. both the activity and the thinking). In other words, children actively learn to do mathematics for themselves.

Illustrating: Doing mathematics (i.e. thinking and communicating mathematically) necessarily involves illustrating, because mathematics is about studying relationships between objects, actions and measures, and it is impossible to explore such relationships without some kind of spatial imagery being involved. Even when thinking about just two numbers, one of them 'comes before' the other one on a number line or is said to be 'bigger' or 'higher'; these relations all involve spatial imagery.

Numicon explicitly involves illustrating in every activity. This is how relationships in a wide variety of contexts are both explored and communicated.

Talking: Since doing mathematics involves communicating mathematically (both with others and with ourselves), doing mathematics also involves talking. Talking is an essential aspect of all Numicon activity.

Throughout Numicon, talking means dialogue in which points of view are exchanged between teachers and children, and between children and children. All partners in dialogue will be actively involved, not just passively hearing, or waiting to speak. Such exchanges encourage reflective thinking as children learn to discuss different points of view, both with others and with themselves.

Exploring relationships (in a variety of contexts)

Doing mathematics involves **exploring relationships** (i.e. the structure) in any situation in order to develop some kind of control in that area. Relationships can be between amounts, between positions, between shapes, between things that are varying, between things that are constant, or between any combination of all or any of these things. Fully understanding the relationships in a situation makes it predictable, thus making it possible to find out and/or manage what is needed. The mathematical reasoning children are asked to do is their expression of relationships they perceive.

Numicon ensures that children explore the relationships within a wide range of contexts so that they learn not only *how* to do mathematics, but *when* the mathematics they are learning is useful.

Generalizing

In doing mathematics, exploring relationships and looking for patterns in various situations lead to **generalizing**. It is generalizing that allows us to make whole ranges of new situations predictable.

Numbers are generalizations that we all use to make predictions when calculating. For example, the '6', '2' and '8' in the number sentence '6 + 2 = 8' are generalizations; 6 of anything and 2 of anything will together always make 8 things, whatever they are.

'The angles of a triangle add up to 180°' is a generalization that is often used when doing geometry; 'the area of a circle is πr^2' is another that is used when measuring.

In each of these cases, noticing patterns in relationships allows us to generalize about an infinite number of other, similar situations.

It is because we make and use generalizations continually as we do mathematics that mathematical thinking and communicating will appear to have an abstract character for children if they are not involved in doing the generalizing for themselves.

Communicating mathematically, **exploring relationships** and **generalizing** all come together when *doing* mathematics.

What doing mathematics demands of children

In learning to do mathematics at the age they are asked to, children face a unique challenge in their school curriculum: thinking and communicating about abstract objects.

Pure numbers, such as 6 and 254, are abstract objects and in no other subject are children asked to work with and reason about such things so early. It is not surprising that children can hesitate, be puzzled, get stuck, or take time to work things out for themselves.

Most mathematicians will say that doing mathematics is about looking for patterns in situations. It is, but there is more to it than that. When you find a pattern, it means you have noticed something regular, something that always seems to be happening, and this means that any pattern you see is a generalization of yours. Mathematics deals almost entirely in generalizations.

Which is what numbers are: as generalizations, they are abstract objects. Very early on in mathematics, young children are asked to do things with lots of abstract objects, like those we call '3', and 'ten'. Not '3 pens', or 'ten sweets', or '3 friends'. Just '3', or, even more curiously, the two-digit '10' by itself. Notice how quickly children are asked to add and subtract these abstract objects to and from each other. There are high expectations of every child from the very beginning in mathematics.

The central problem: communicating with, and about, abstract objects

How is it possible to communicate about abstract objects? And, since thinking is communicating with ourselves, how is it possible to *think* about abstract objects?

Notice that 'abstract' does not mean 'imaginary'. We can easily picture imaginary things, such as unicorns and beings from outer space, but abstract things are different. Abstract things include qualities like 'truth', 'intelligence', 'common sense', as well as generalizations such as '6 of anything'. The problem is, as soon as you try to picture '6 of anything', you find you are imagining '6 of something'.

The answer, as Jerome Bruner observed, is that we think about abstract objects with symbols; in the case of numbers, we do it with numerals.

The important thing about symbols is that they do not attempt to show literally what is being talked about; they are simply arbitrary, conventional marks on a page (or spoken words). When what we are communicating about with these symbols is abstract, that absence of a picture is inevitable; how could we possibly picture something that is abstract?

It is easy to picture '3 pens' or 'ten friends', but what might the abstract '3' look like? Or, how about the curious two-digit '10'? Since numerals do not look like the abstract things they 'stand for', how are children to learn to interpret such arbitrary symbols in their thinking and communicating? Doing mathematics – thinking and communicating about abstract things with symbols – is certainly not easy for young children.

How does Numicon help?

Essentially, Numicon does two things. Firstly, Numicon acknowledges that in order to understand what numbers are, children have to generalize. Secondly, Numicon follows Bruner's advice in using children's actions and imagery to prepare for their use of mathematical symbols in their thinking and communicating. In practice, these two things are done at the same time.

In Bruner's terms, *enactive* and *iconic* representations (action and imagery) are used to inform children's interpretation of the *symbolic* representation (e.g. numerals) that is necessary for communicating their pattern spotting (generalizing). To aid children's necessary generalizing, strong focus is placed upon the use of structured materials.

Generalizing and reasoning – an early years example of teaching with Numicon

Initially, in common with most teaching, Numicon involves a wide variety of everyday objects (such as beads, cubes, pegs and counters, see Fig 1) in order to help children develop their counting, before then introducing the challenges of calculating.

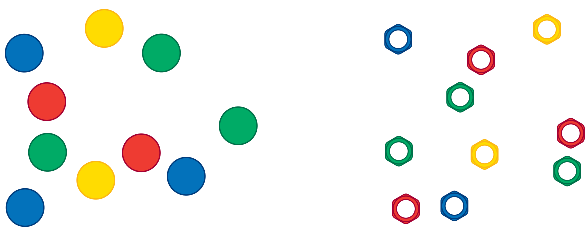
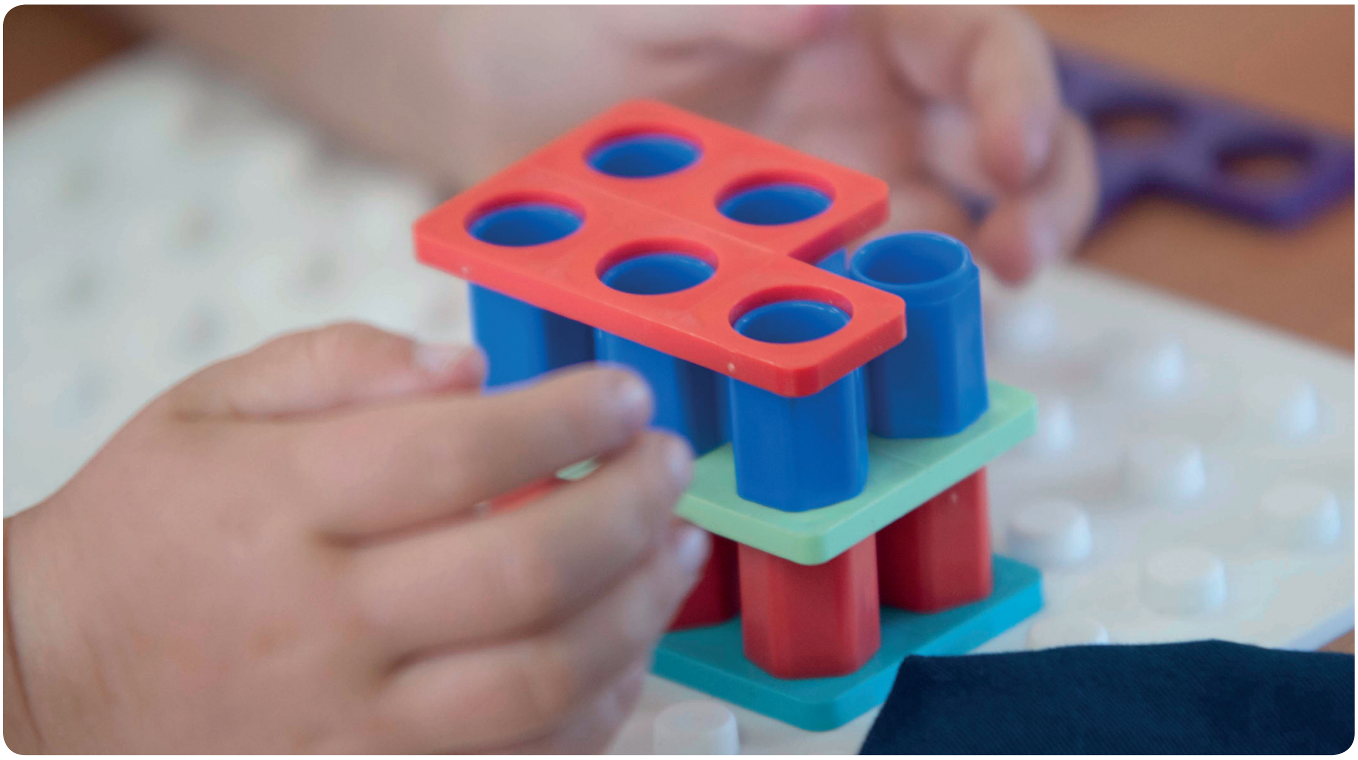


Fig 1



Importantly, Numicon also introduces sets of structured materials in which individual pieces have *regular* physical relationships with each other, for example, Numicon Shapes and number rods, see Fig 2. Children explore the physical relationships within these structured materials by, for example, ordering pieces, comparing them, combining them physically to make others.

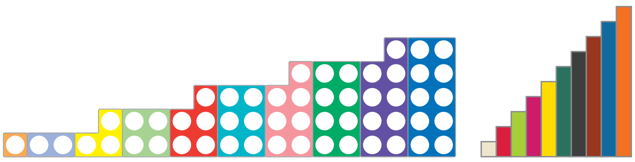


Fig 2

Thus, as children work with loose collections (beads, cubes) and with structured materials (Numicon Shapes and number rods), they are able to combine being active with physical objects and images as illustrations in their talking and thinking about numbers of things.

Numerals are introduced in association with loose collections of objects and with structured materials as children work; thus, symbolic representation appears alongside children's ongoing action and imagery with physical objects.

Number lines (e.g. Fig 3) are introduced to further illustrate the ordering that is evident within the structured materials, and to reinforce the associated ordering of numerals.

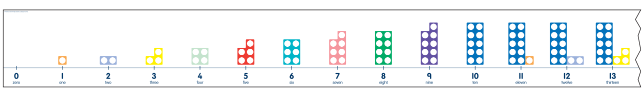


Fig 3

Importantly, loose collections of objects are arranged into the regular patterns of the Numicon Shapes; individual number rods are found to be equivalent in length to different multiples of the smallest 'one' rod (e.g. Fig 4).

Fig 4

Through doing these activities, children learn that *any* collection of loose objects can be arranged into Numicon Shape patterns that can then be 'read'. Children learn to 'see' how many objects are in a collection without counting; numbers of things begin to 'take shape' visually, in organized ways.

It is now possible to prepare children for further generalizing about numbers through exploring relationships between numbers of things. Children's mathematical thinking and communicating continue to develop through their being active with the objects they are given, and in illustrating their talking about what they see and do.

Children generalize that any collection of loose objects can be arranged into Numicon Shape patterns. They also generalize that any number of 'one' number rods may be exchanged for (will be 'as long as') one or more of the larger number rods. Thus *any* number of loose objects can be converted into (is equivalent to) one or more Numicon Shapes or number rods.

In realizing that any collection of loose objects can be arranged into Numicon Shape patterns, and that any number of 'one'

cubes is equivalent to one or more number rods, children become able to 'see the general' in any particular illustration; Numicon Shapes and number rods can be used to illustrate, in an organized way, any numbers of any kinds of things.

Numicon Shapes and number rods themselves may now be used to explore and to communicate about number relationships in general. They have become communication mediators in discussions about numbers and their relationships.

As a particular example, the Numicon Shape that has three holes fits together physically with the Shape that has five holes. The result 'forms the same shape as' (is equivalent to) the Shape that has eight holes, see Fig 5.

Similarly, the number rod worth three ones, combined end-to-end with the rod worth five ones, are together as long as the rod worth eight ones, see Fig 6.

When laid end-to-end along a number line or number track, the '3 rod' and the '5 rod' together reach the position marked '8' on the line.

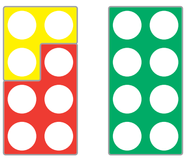


Fig 5

Fig 6

From these actions, and with these illustrations, children are able to generalize that: three *anythings* and five *anythings* together will *always* make eight things.

This generalization can be expressed using the numeric and verbal symbols: 3 and 5 together are equivalent to 8.

Later on, using further actions and further illustrations, children become able to interpret and use the further symbols '+' and '=' to express their generalization thus:

$$3 + 5 = 8$$

Importantly, at this stage children will have begun to use number words (one, two, three) as *nouns* instead of as adjectives (two sweets, three pencils) in their talking.

With their use of number words as nouns, numbers as *abstract objects* have now appeared in children's mathematical thinking and communicating, referred to with *symbols*.

Such generalizing and use of symbols can now be exploited further. If 'three of *anything*' and 'five of *anything*' together always make 'eight *things*', then:

3 tens + 5 tens	=	8 tens
3 hundreds + 5 hundreds	=	8 hundreds
3 millions + 5 millions	=	8 millions

or

30 + 50	=	80
300 + 500	=	800
3,000,000 + 5,000,000	=	8,000,000

Such is the power of generalizations, and of the symbolic notation that children can by this stage use to think and communicate mathematically about them.

Progressing from such early beginnings

The foregoing example also illustrates how Numicon supports the teaching of children's subsequent mathematics.

In a wide variety of contexts, children are offered opportunities to be active and to illustrate their talking about any (and all) relationships they are exploring between both physical and abstract objects. They do this in order that they may generalize and, as a consequence, think and communicate about their generalizations with the conventional symbols of mathematics: in other words, that they may *do* mathematics.

Of course, there is much more beyond this first example. Children have many other relationships to explore, further generalizing to do, and thus many more conventional symbols to learn to interpret and use. However, wherever it is used, the approach is essentially the same. *Enactive* and *iconic* representation are used to inform children's interpretation of the *symbolic* representation that is necessary for thinking and communicating about their constantly developing pattern spotting (generalizing).

The going gets harder as children progress in their number work: place value, ratios, fractions, decimals and negative numbers will all challenge children's capacity to generalize onwards from these early beginnings. However, such advanced work is equally possible with the same approach of actions and illustrations building towards generalizing, and a consequent use of symbols to think and communicate about the generalized, abstract mathematical objects created. This is how the symbolism of mathematics becomes meaningful.

In their work on geometry, the generalizations that children reach in their mathematical thinking and communicating happen more gradually as they progress towards being able to reason about invented abstract objects such as 'any triangle' and (later) 'any polygon'.

It is impossible to draw the abstract object 'any triangle' in the same way that it is impossible to imagine 'six of anything'. As soon as you draw a triangle, however you have chosen to draw it, you have drawn a particular one; it does not matter whether you draw one that is right-angled, isosceles, equilateral, or scalene, what you have drawn is not a general triangle – it is a particular one, e.g. Fig 7.

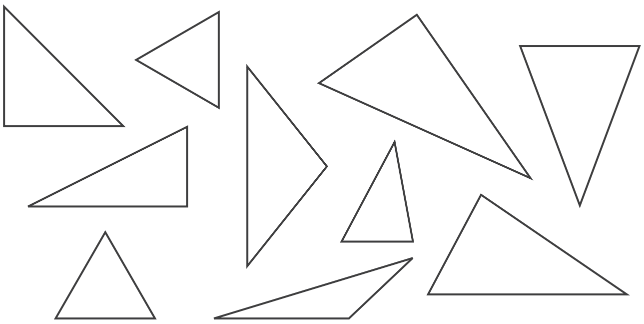


Fig 7

However, as with generalizing about numbers, in doing geometry, much exploring of relationships with action and imagery (enactive and iconic representation) prepares children for reasoning meaningfully about 'any triangle' with symbolic representation (words and symbols).

In this example, as children physically construct and transform many varieties of triangles with dynamic materials, what they see before their eyes is how lengths of sides may vary endlessly as angles change, how angles may vary endlessly as lengths change and yet, despite all the variety they see, the shapes they are creating all turn out to be triangles.

Dynamic mental imagery deriving from these physical experiences then allows children to speak and to think of 'any triangle' as they imagine an infinitely flexible closed 2D shape with three straight sides.

By stressing the straightness of the sides, that there are always exactly three sides and that the sides fit together to 'close' the shapes they make, and by ignoring the constantly changing lengths of sides and sizes of angles, children become able to imagine 'any triangle' and to reason about it with words and symbols.

Then, having generalized to a stage when their mental imagery allows them to imagine and speak meaningfully about 'any triangle', children are in a position to further generalize that, for example, 'the angles of any triangle add up to 180°' through further action, imagery and reasoning with symbols.

In the same way that generalizing about numbers is approached, exploring relationships with action and imagery in geometry prepares children for reasoning meaningfully about abstract mathematical objects with symbols.

We reach all the generalisations of mathematics, and the symbols we use to express them, through being active and illustrating our talking about the relationships we are exploring.

Doing mathematics in the world – solving problems

Of course, *doing* mathematics in the everyday world is not simply about making generalizations and using symbols. Crucially, it also involves making *use* of generalizations to solve problems in particular situations.

For example, the generalization '4 x 25 = 100' allows us to predict that the perimeter of a square of side 25 cm will be 100 cm, that the area of a field measuring 4 m by 25 m will be 100 m^2, and that if you save £25 a week for four weeks you will have £100. It can also be very useful to help calculate that:

$$36 \times 25 = (9 \times 4) \times 25 = 9 \times (4 \times 25) = 900$$

Children need to be able to readily connect the generalizing that they do with mathematical symbols with everyday world situations in which those generalizations are useful.

In their more general problem solving, children learning, for example, *how* to divide one number by another, need to learn *when* that operation is useful.

The Numicon teaching materials organize activities into groups based around mathematical themes. For example, in the *Number, Pattern and Calculating 1 Teaching Resource Handbook*, 'Introducing the subtracting symbol' (Calculating 1) and 'Adding more than two numbers' (Calculating 8), each activity group is introduced with, or involves a context in which that mathematics is useful.

The activities in 'Introducing the subtracting symbol' begin with the context of a number of children leaving a play park, and the activities of 'Adding more than two numbers' include shopping situations in which more than two items are being bought. In both activity groups, it is clear to children that the situations are familiar and relevant to their experiences.

In the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*, a group of activities entitled, 'Making pictures, shapes and patterns' encourages children to connect their own pattern-making with everyday designs in fabric, wallpapers, tiling, wrapping papers, and so on.

In these ways, children's mathematics is introduced, wherever possible, within contexts in which that mathematics is useful. Such contexts help children to 'see the point' of the mathematics they are learning, and prevent mathematics becoming a series of answers to problems they have never had.

Flexibility, fluency, and persistence

Being able to do mathematics effectively also includes being able to remember basic things such as those generalizations known as 'times tables' and number 'facts'. There are many practice activities built into the Numicon written materials that encourage children to develop a regular familiarity with basic facts.

Even more important to effective functioning is flexibility in mathematical thinking and communicating. Three kinds of flexibility are especially useful.

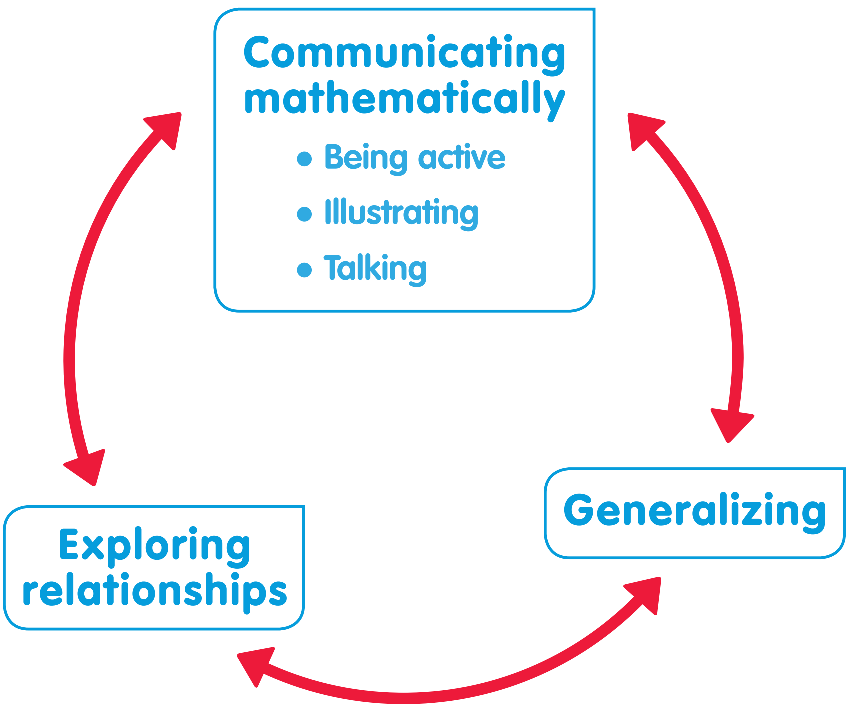
Through being active, children are able to 'invert' their actions; put more simply, this is about 'doing and undoing'. As children have found 'how many' objects there are in a collection by physically grouping them into tens (and hundreds), they are later readily able to 'partition' numbers ('undo' their groupings) when calculating with symbols. As children can combine and separate Numicon Shapes and number rods physically ('do' and 'undo' their actions), they are able to connect adding and subtracting as inverse operations and to check a subtracting calculation by adding.

The introducing and encouraging of a variety of ways of calculating means that children are able to choose methods of calculating that suit the particular numbers involved, rather than adopting a standard method for any calculation. Who would want to subtract 1998 from 4673 using a column method, just because the numbers are big?

As Numicon approaches children's use of mathematical symbols with prior actions and imagery, if children become 'stuck' or hesitant while working with symbols, it is always possible for them to recall and return to the supporting activity and illustration from which their original generalizing arose. For example, if they are working on prime numbers, using symbols, they can immediately return to actively illustrating how factors 'go into' their various multiples physically with Numicon Shapes and with number rods. They can follow a similar pattern with any other kind of number relationship. This flexibility is one of movement backwards and forwards between Bruner's *enactive*, *iconic* and *symbolic* forms of representation in their thinking and communicating.

Finally, because Numicon continuously emphasizes communicating mathematically, it is possible to stress to children that all is not lost when they feel 'stuck'. The thing to do, always, when they 'don't know what to do' in a situation is to communicate.

Being active, illustrating and talking about the relationships we are exploring *is* doing mathematics. Persistence – an invaluable quality when doing mathematics – comes from continuing to communicate, with yourself and/or with others, whenever (for the moment) you cannot see where you are going.



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Numicon Overview Chart

YEAR GROUP	TEACHING RESOURCES	ASSESSMENT	ACTIVITIES FOR HOME	APPARATUS	ONLINE SUPPORT
Y1 Number, Pattern and Calculating	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Books x 3	• Explore More Copymasters	Starter Apparatus Pack A	Numicon Online Online planning and assessment support
Y1 Geometry, Measurement and Statistics	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Book x 1	• Copymasters are included within the Teaching Resource Handbook		
Y2 Number, Pattern and Calculating	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Books x 3	• Explore More Copymasters		
Y2 Geometry, Measurement and Statistics	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Book x 1	• Copymasters are included within the Teaching Resource Handbook		
Y3 Number, Pattern and Calculating	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Books x 3	• Explore More Copymasters	Starter Apparatus Pack B	
Y3 Geometry, Measurement and Statistics	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Book x 1	• Copymasters are included within the Teaching Resource Handbook		
Y4 Number, Pattern and Calculating	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Books x 3	• Explore More Copymasters		
Y4 Geometry, Measurement and Statistics	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Book x 1	• Copymasters are included within the Teaching Resource Handbook		
Y5 Number, Pattern and Calculating	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Books x 3	• Explore More Copymasters	Starter Apparatus Pack C	
Y5 Geometry, Measurement and Statistics	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Book x 1	• Copymasters are included within the Teaching Resource Handbook		
Y6 Number, Pattern and Calculating	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Books x 3	• Explore More Copymasters		
Y6 Geometry, Measurement and Statistics	• Teaching Resource Handbook • Implementation Guide	• Explorer Progress Book x 1	• Copymasters are included within the Teaching Resource Handbook		