



Numicon

The concrete support for mathematical learning

Overdale Junior School

The Purpose

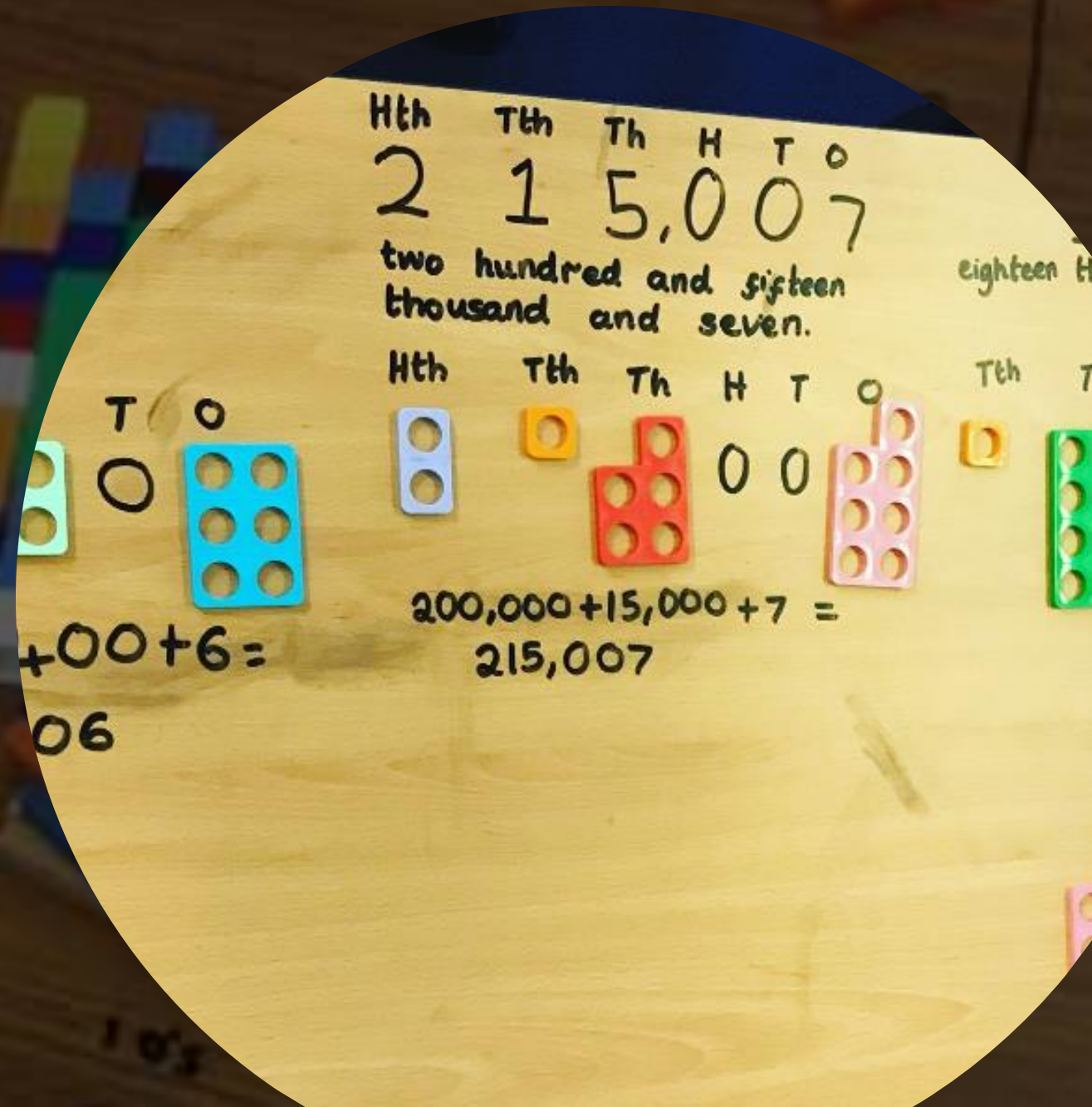
This booklet aims to support teaching staff and parents by providing ideas of how to integrate Numicon into learning.

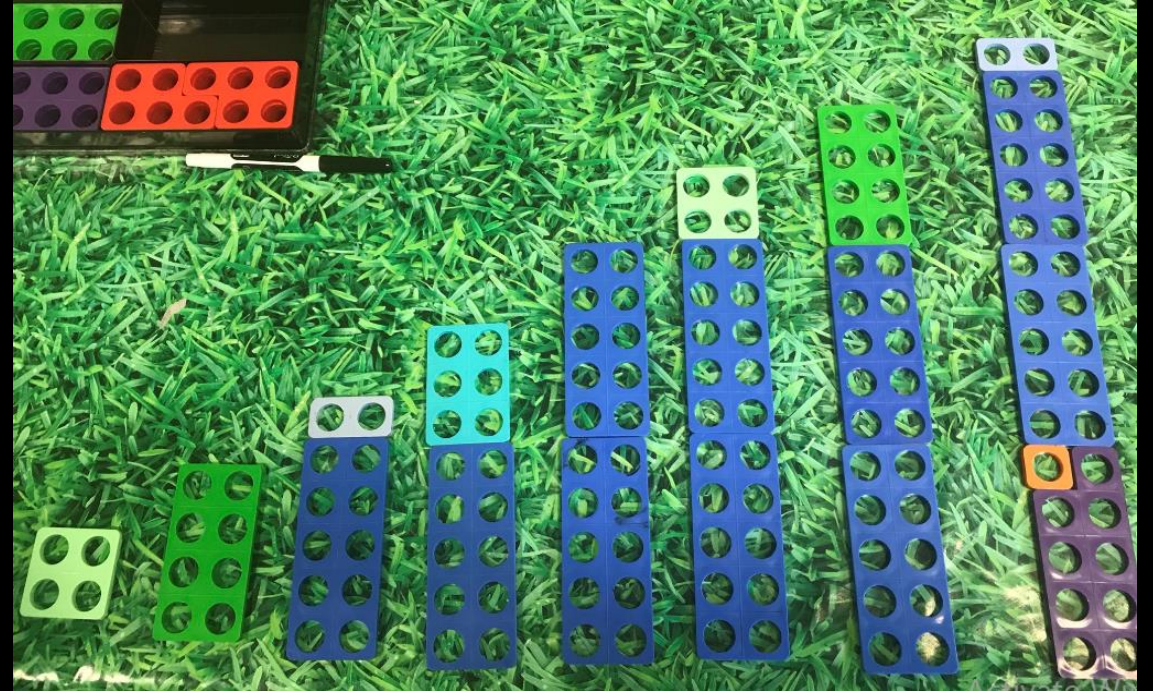
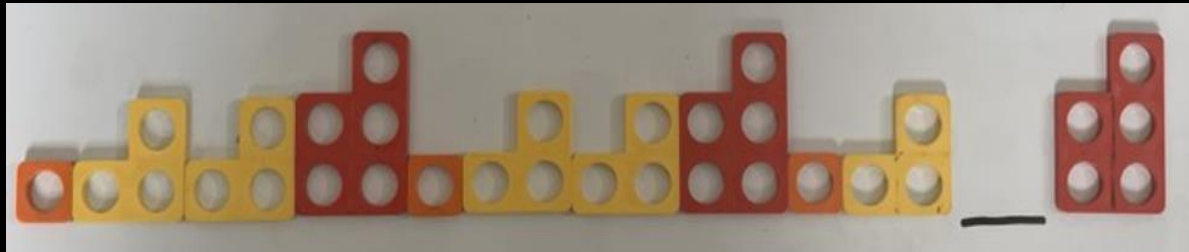
The pictures within this booklet have been collated over years of practise and the ongoing development of a mathematical pedagogy at Overdale Junior School.

The Maths curriculum at Overdale Junior School is centered around the children exploring and reasoning with each other. Numicon has supported this and has helped us to develop confident mathematicians.



Place Value

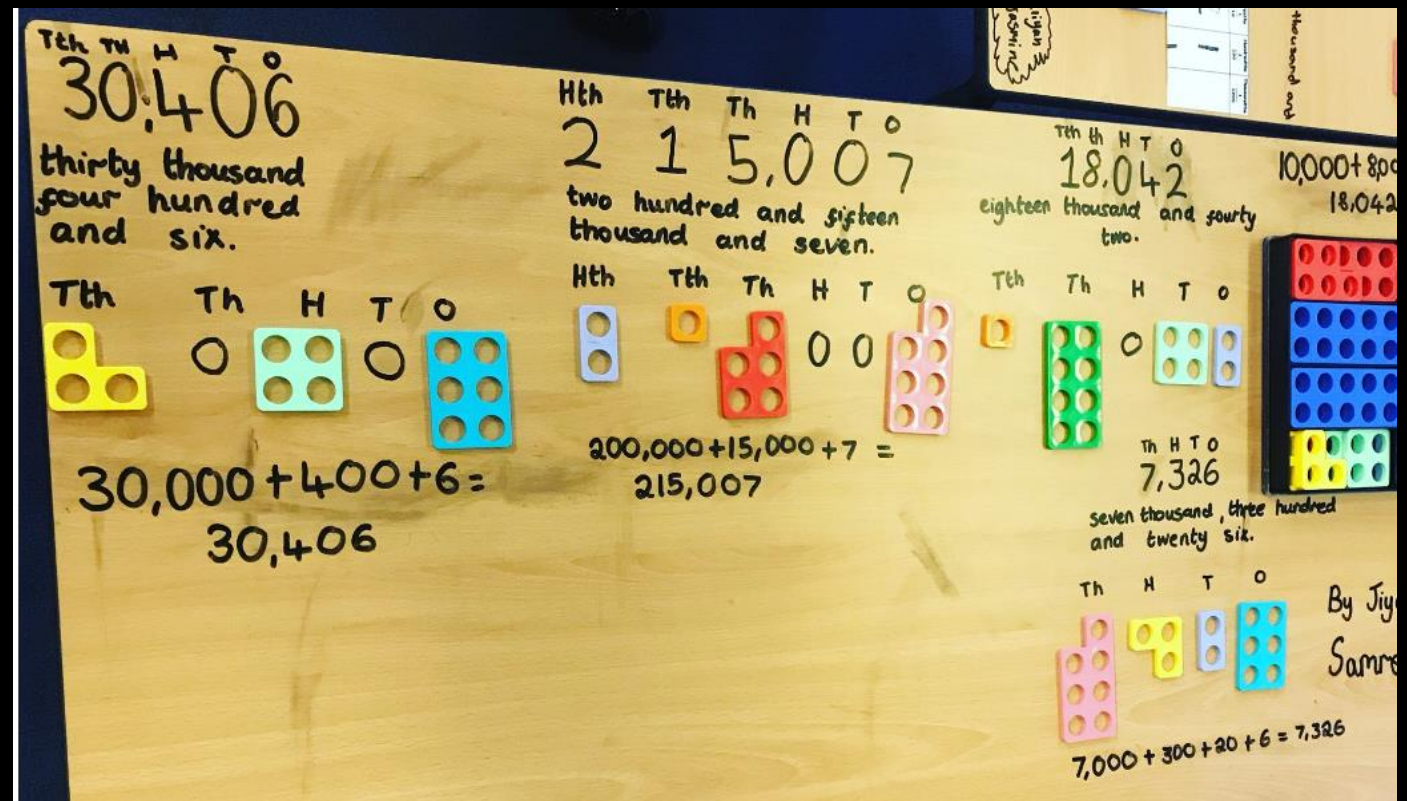




Patterns

Children create and explore repeating patterns. From this, they can then identify "what is missing" or "what is next" in a given sequence.

Children can then apply this idea to practise counting, forwards and backwards, in multiples of a number.

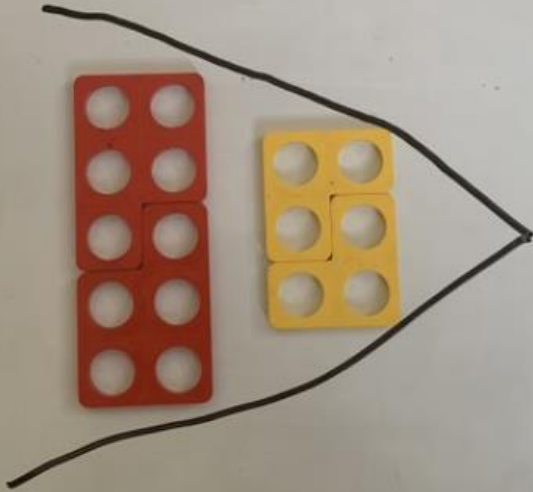


Reading and writing numbers

Children start to develop fluency in representing numbers in different ways. Numicon tiles are used as part of children's practical construction of different numbers.

Children should articulate the value of each digit in a number.

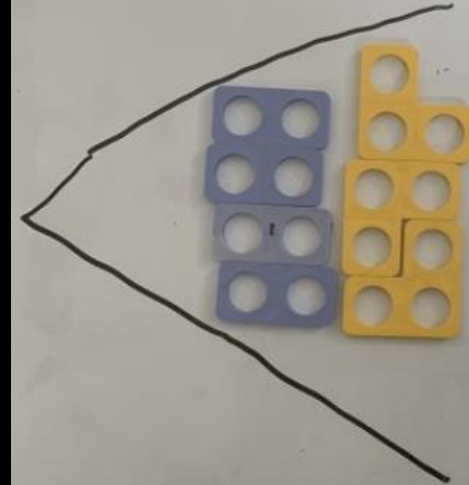
$$2 \times 5 \boxed{>} 3 \times 2$$



$$2 \times 4 \boxed{=} 4 \times 2$$

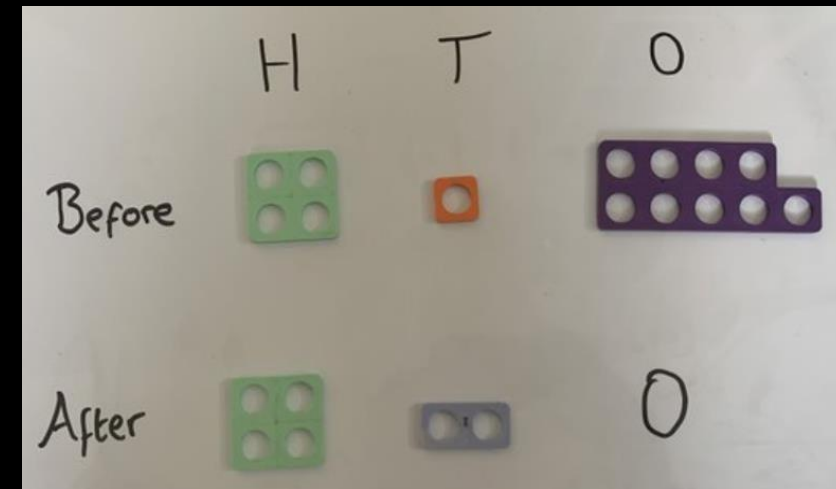
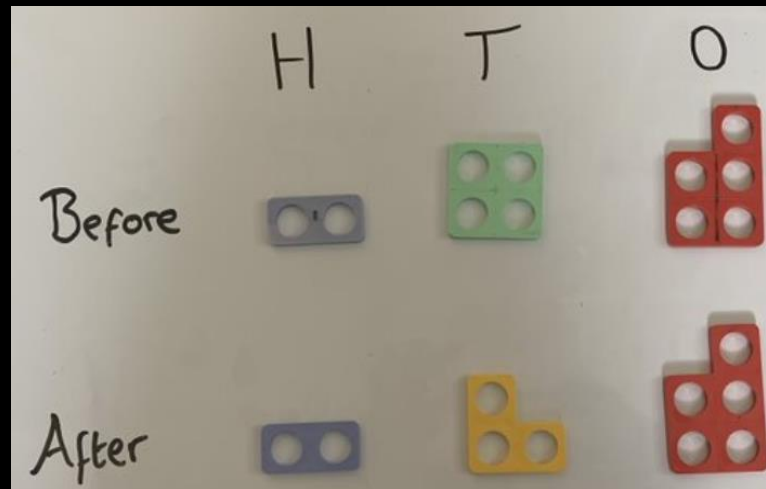
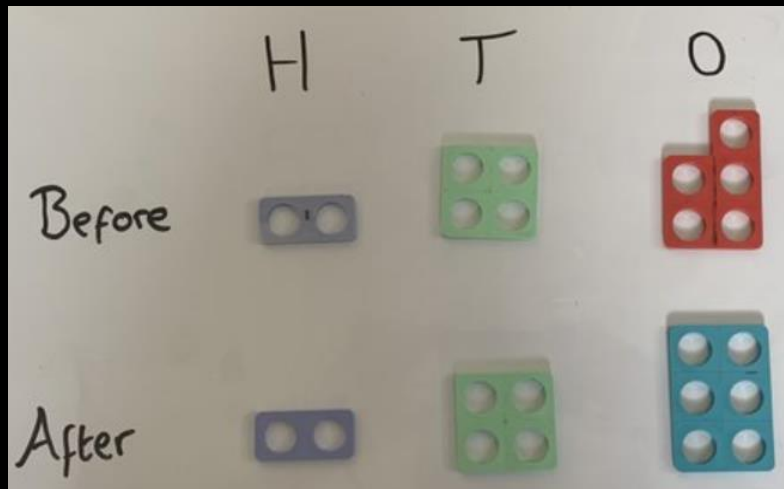


$$2 \times 4 \boxed{<} 3 \times 3$$



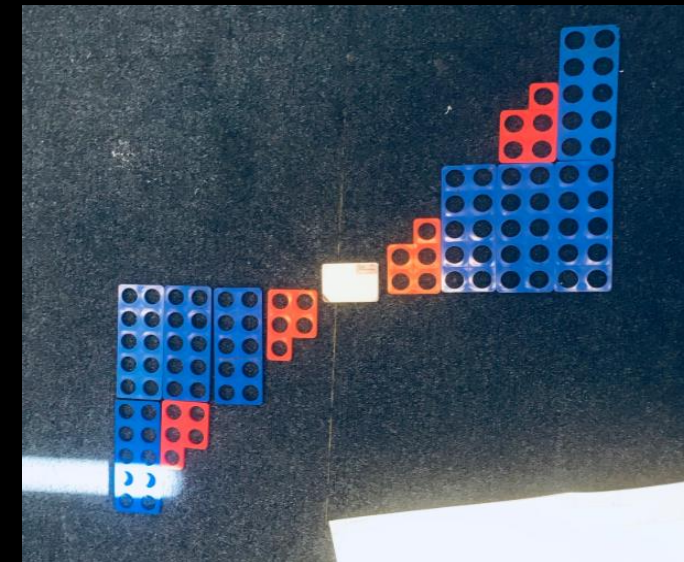
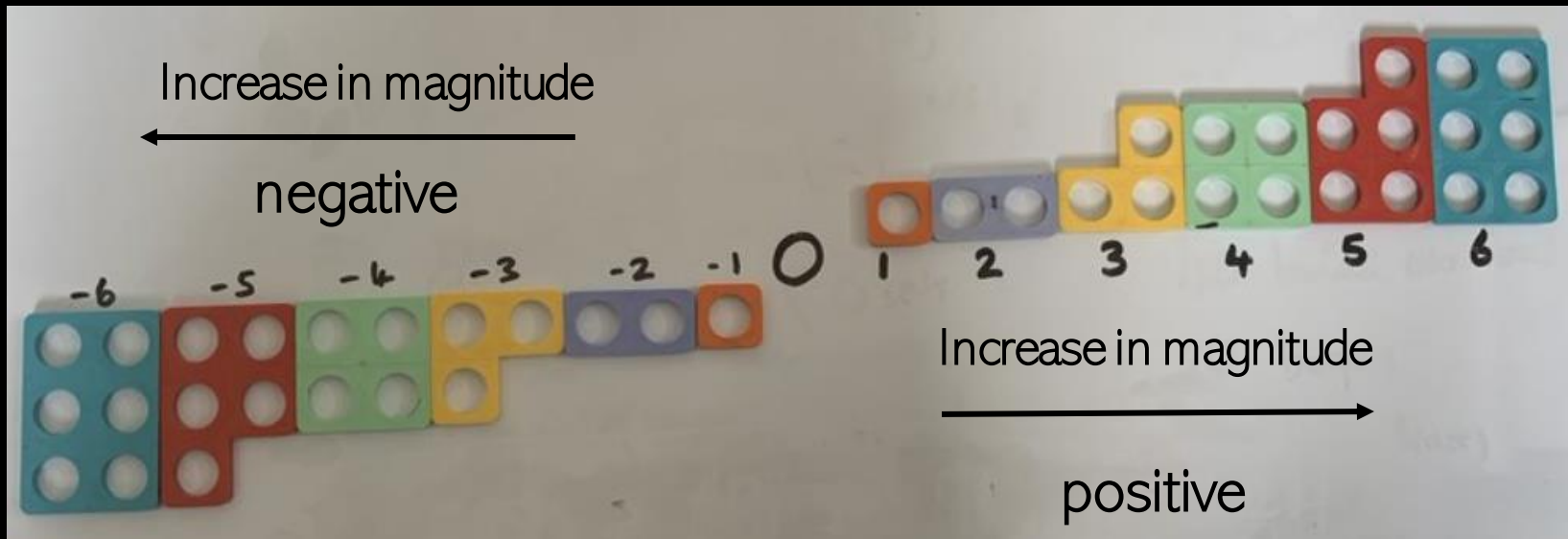
Greater than, less than, equal to.

Children are supported in their use of the $<$, $>$ and $=$ signs in order to compare the magnitude of numbers or calculations.



Adding and subtracting
1, 10 and 100.

Children start to recognise and identify which part of a number is increasing or decreasing in value and articulate how much it has increased or decreased by.

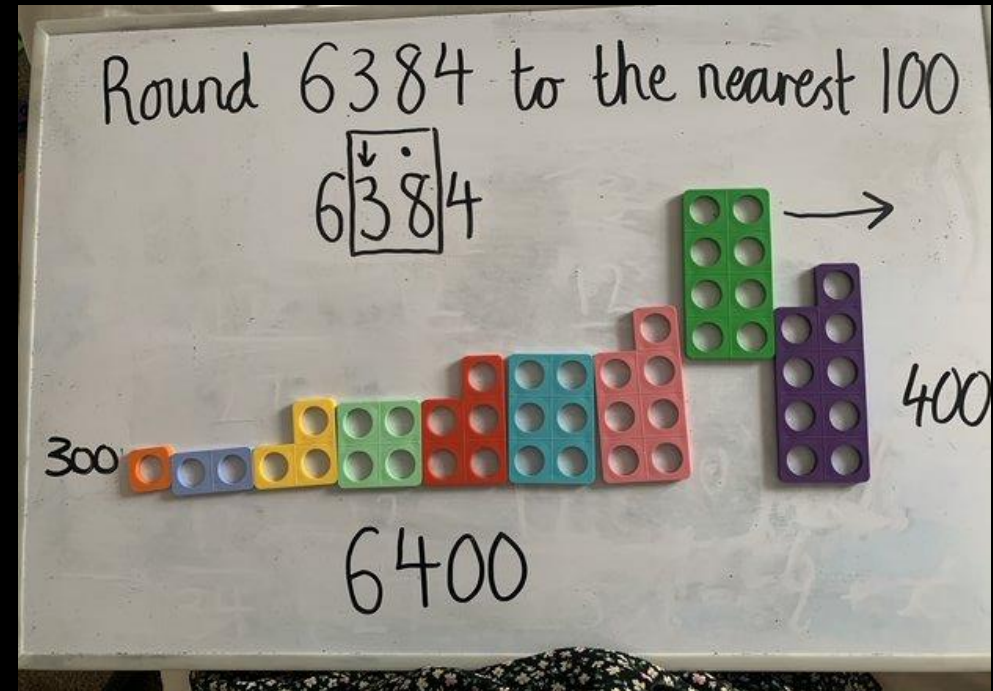
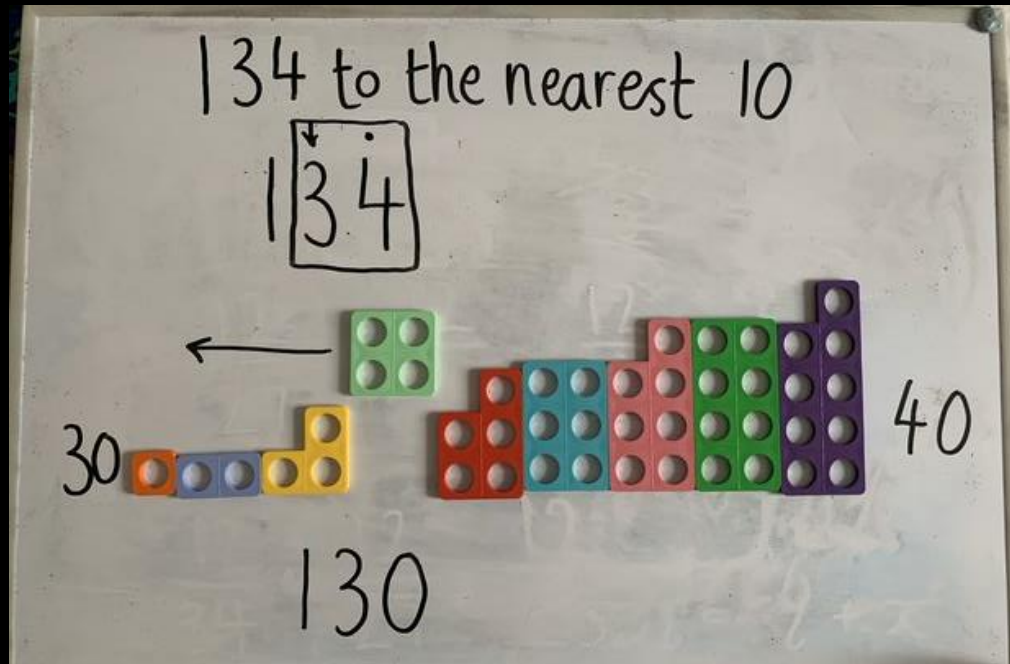


Counting through zero

Children practise counting forwards and backwards through zero by forming the Numicon number line.

Children can use this number line to add or subtract numbers before or through zero e.g. $-5 + 2$ or $2 - 6$.

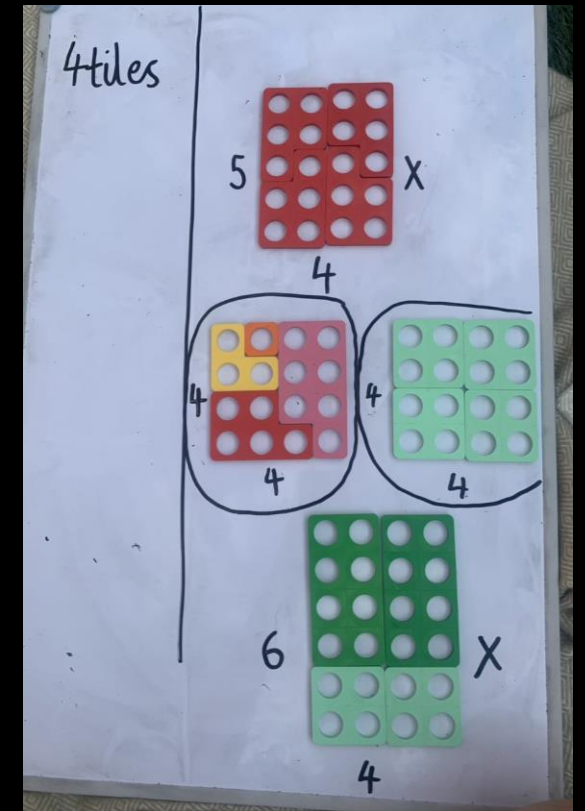
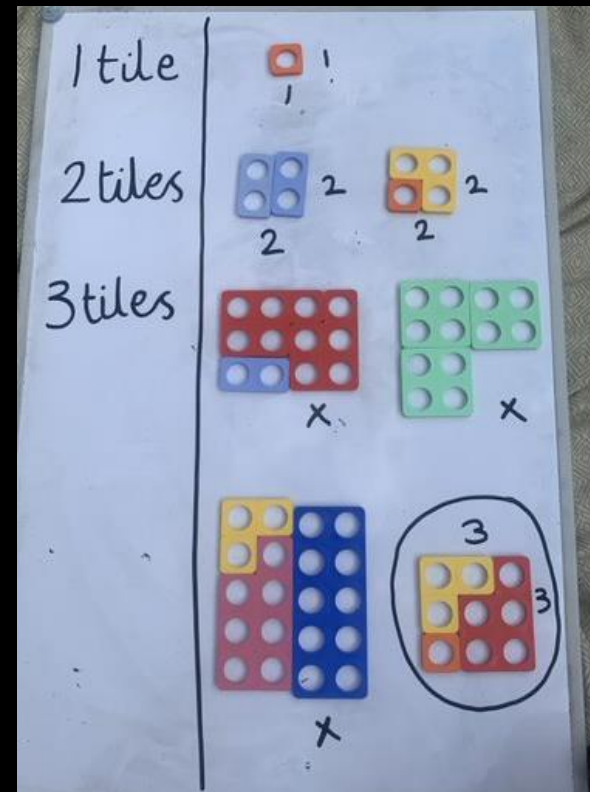
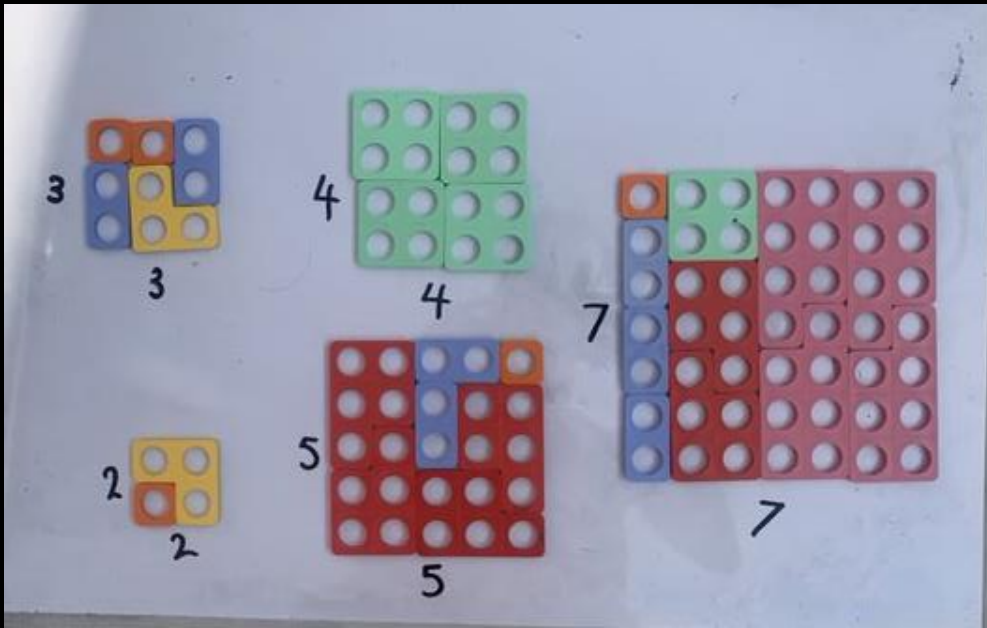
After counting in 1s first, children can then count in multiples of different numbers.



Rounding

Once children have identified the significant number and decider, they should draw a box to isolate the two digits.

Children create a number line between the two closest multiples of ten, hundred, thousand etc and can articulate which their decider is closest to.



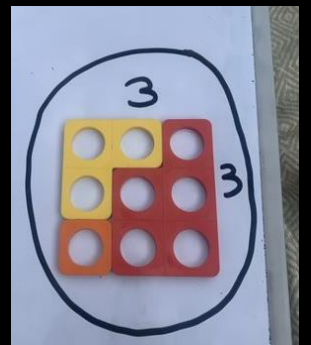
Square numbers

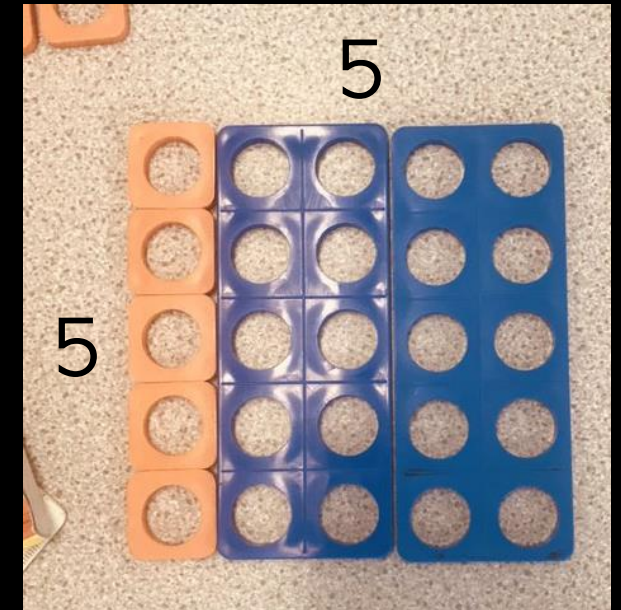
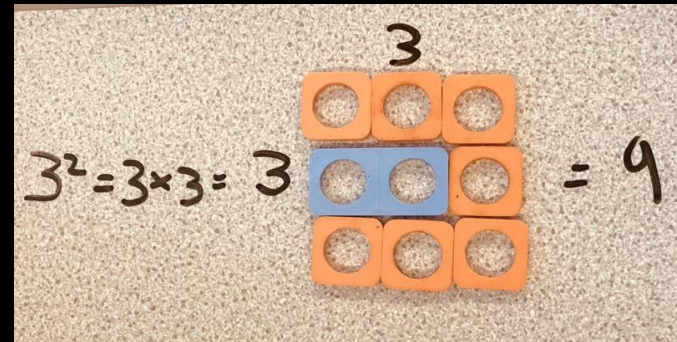
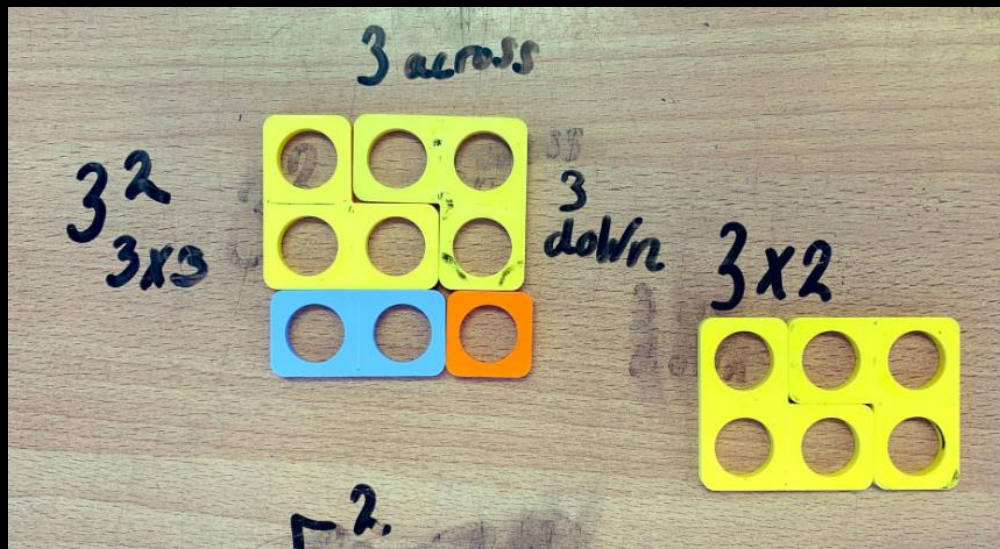
Part 1

Children explore making squares out of Numicon tiles.

Which tiles have they used to make the square number? Can they only use 1 tile, 2 tiles, 3 tiles etc. What do they notice?

Which numbers/tiles are they using to create their squares. Is there one solution?



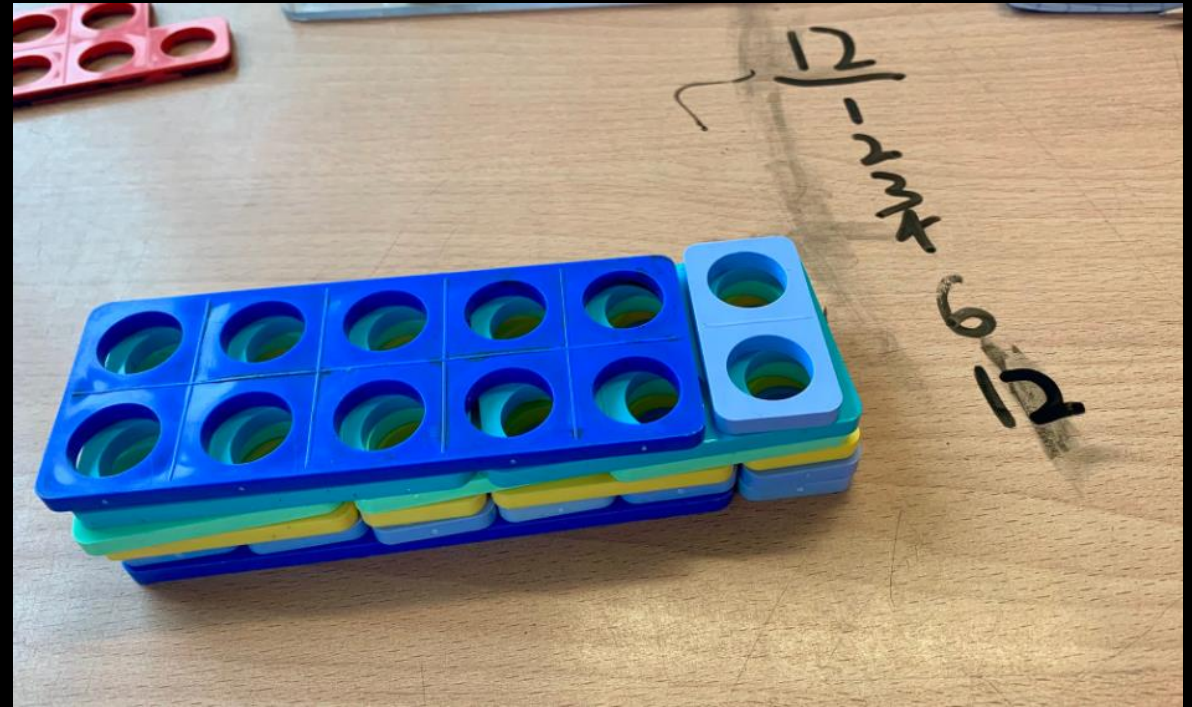
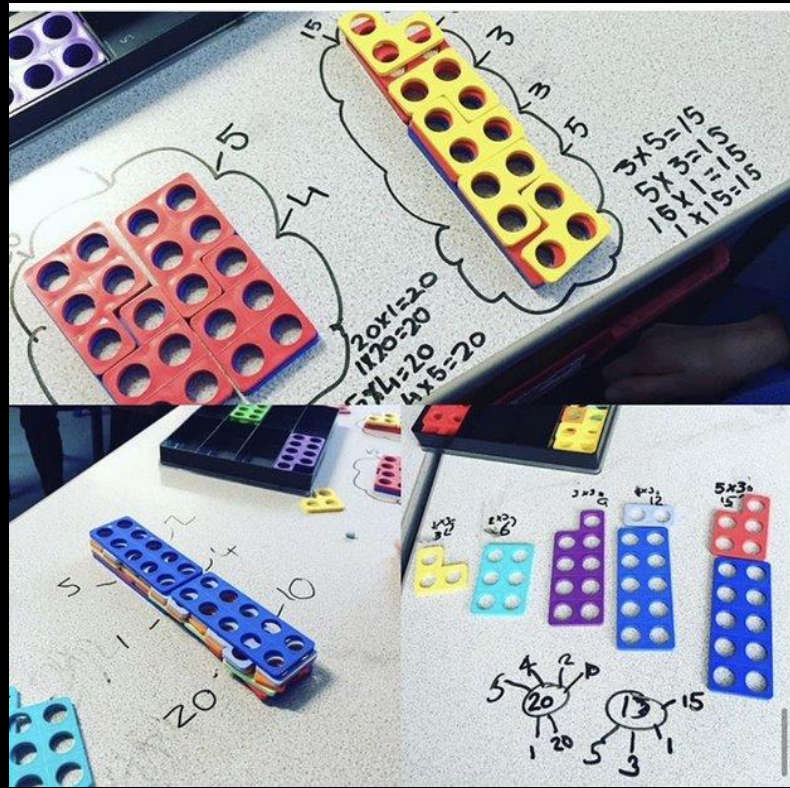


Square numbers

Part 2

Children continue to build squares using Numicon to help understand the difference between multiplying by 2 and squaring.





Children to articulate $3 \times 2 = 6$ and $3 \times 3 = 9$. When building the squares, children should notice that 3×2 does not make a square.



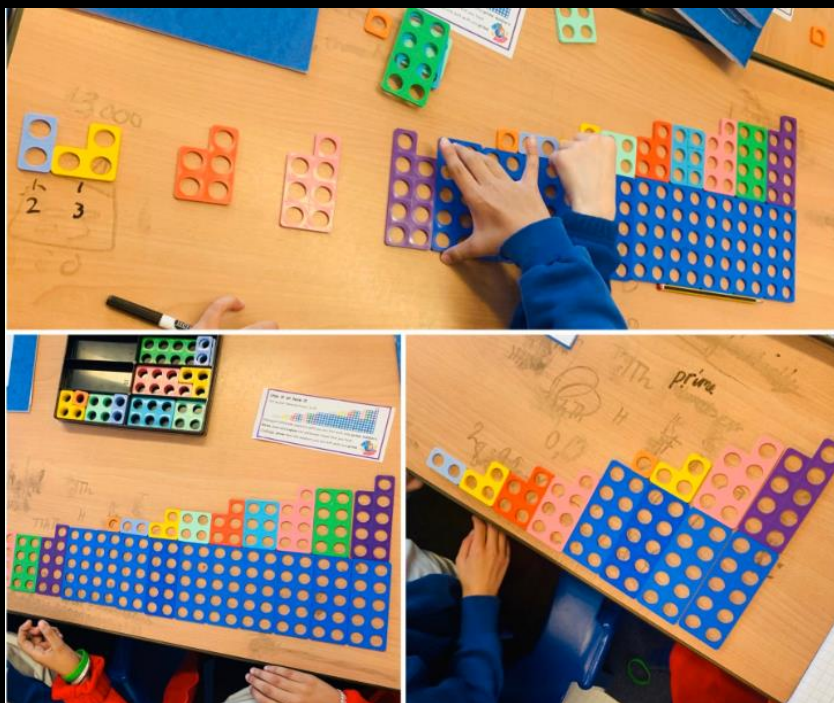
Factors and factor pairs

Children derive multiplication and division facts to find factors.

Children create "factor sandwiches", working systematically to find factors until they reach their starting number.

Factors		2 numbers multiplied to reach another number	
1		6	
2		1	6
3		2	3
6		3	2
		6	1

Factors of 6 = 1, 2, 3 and 6
Factor pairs = 1 and 6, 2 and 3



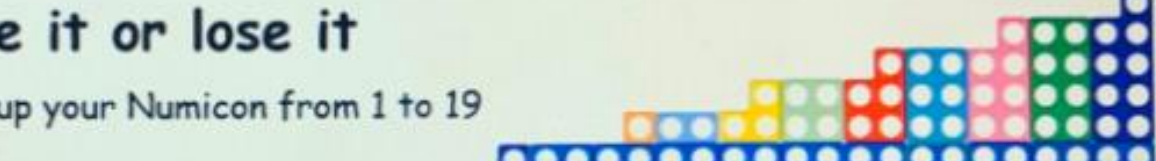
Prime Numbers

Children build on their knowledge of factors and multiples to identify prime numbers.

Children can isolate numbers that they know to be prime and recognise patterns e.g. prime numbers are odd (except for 2 which is the only even prime).

Use it or lose it


Set up your Numicon from 1 to 19

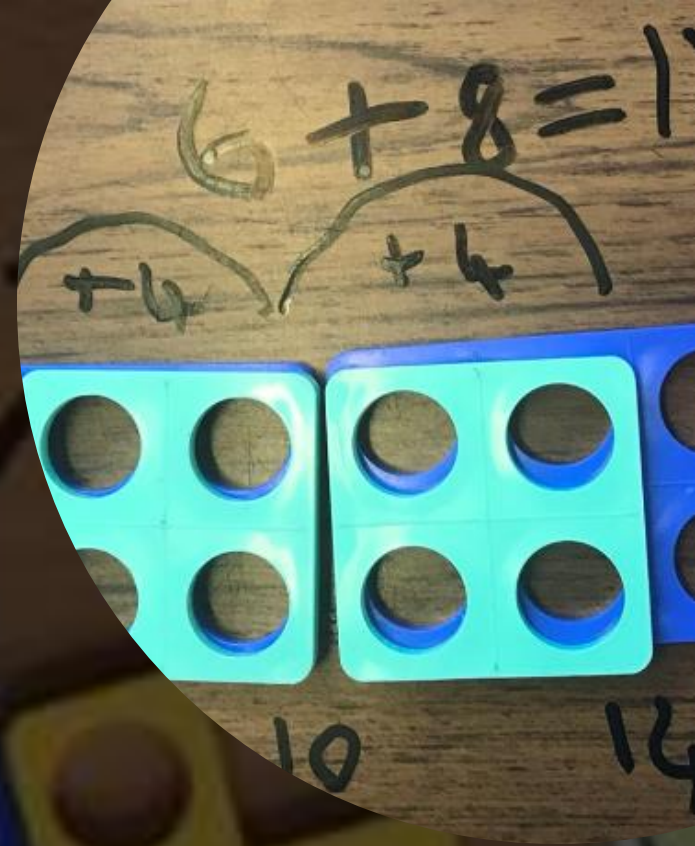
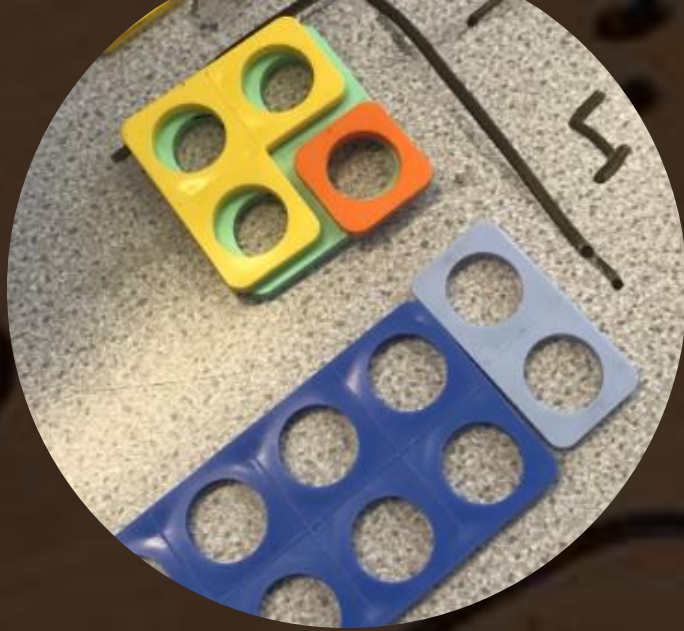


Disregard different numbers until you are left with only **prime** numbers.

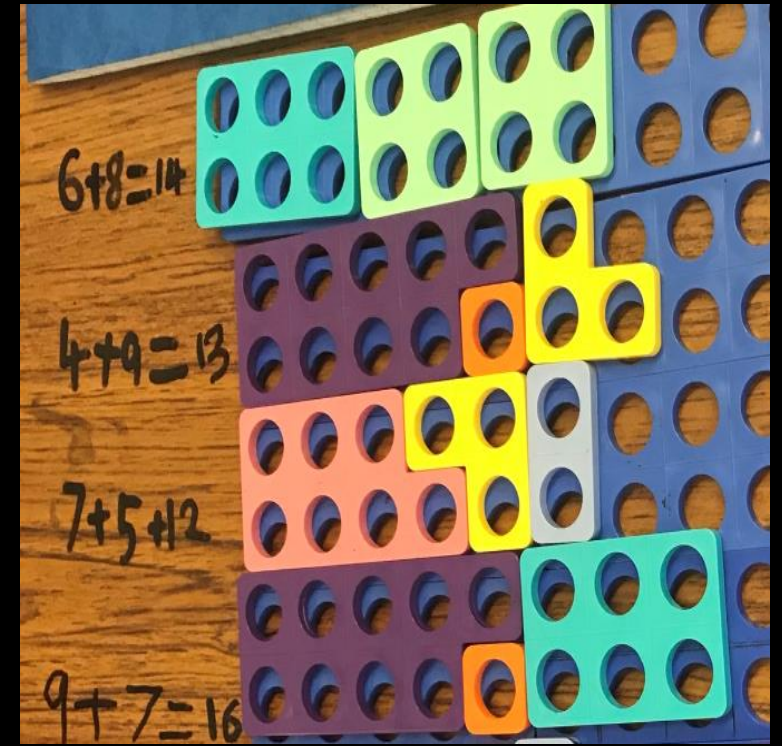
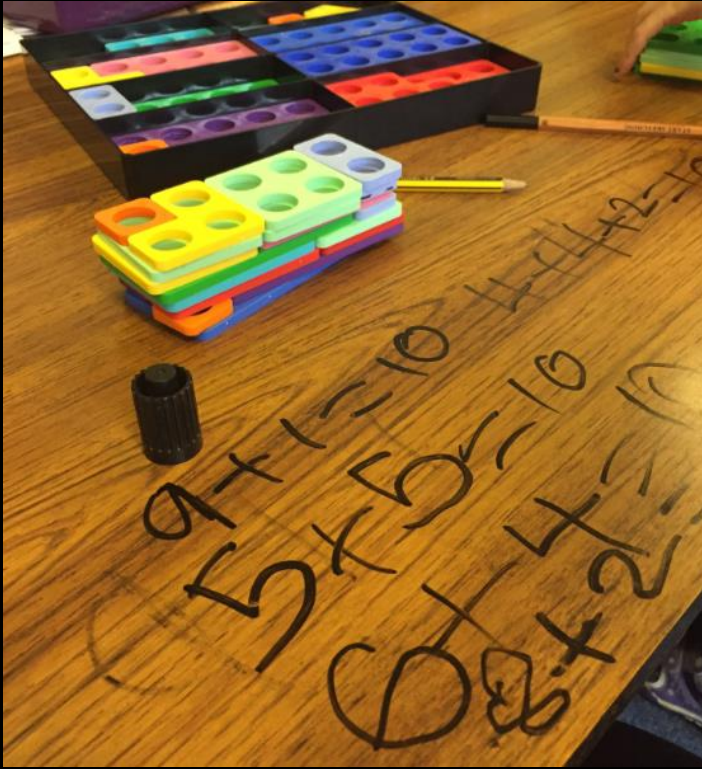
Write down and **explain** the different steps that you took.

Challenge: *prove* that the numbers you are left with are **prime**.





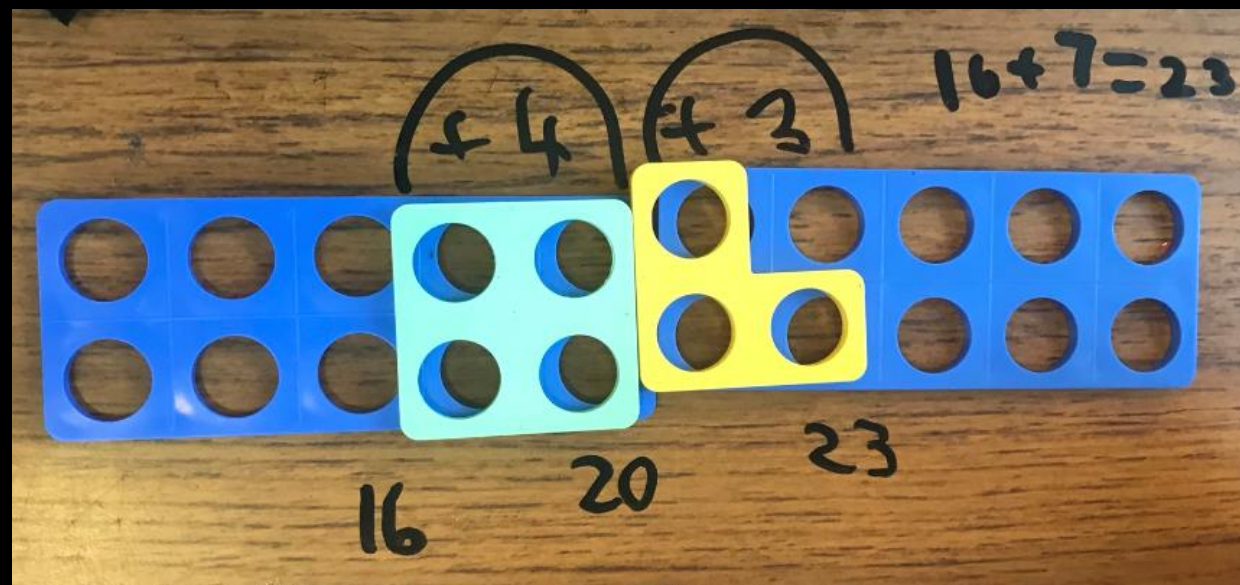
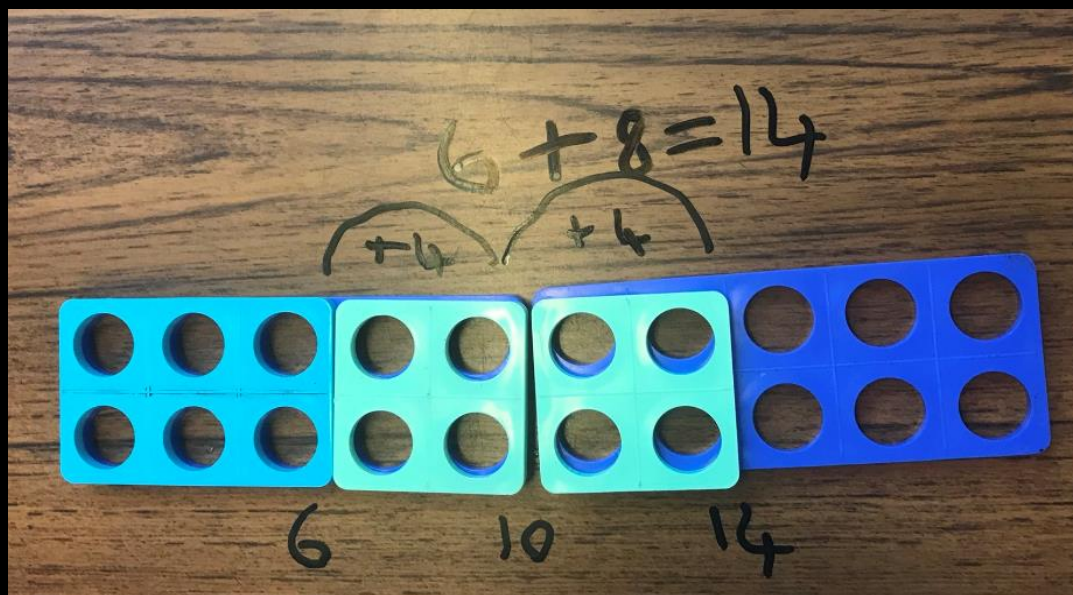
The Calculation Policy



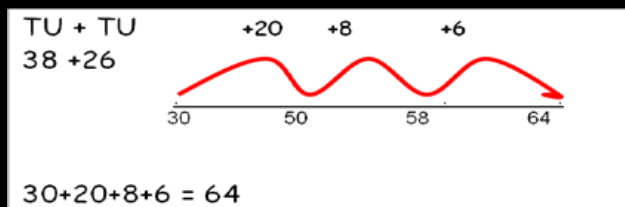
Addition – Step 1

Explore number bonds to 10 so children are confident at recalling them.

Children to then add two one-digit numbers together which bridge 10. They should partition the second digit to make the next 10 and then add on what is still remaining.



Addition – Step 2

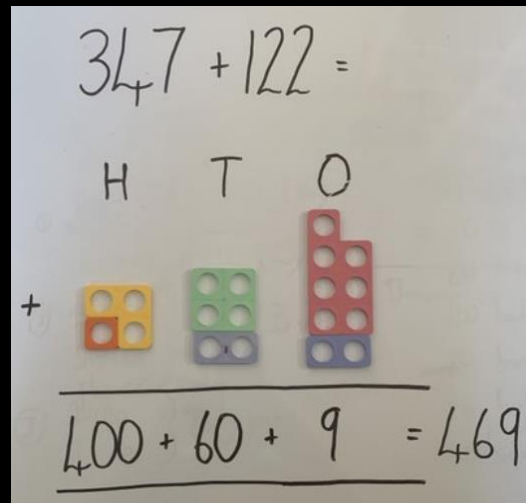
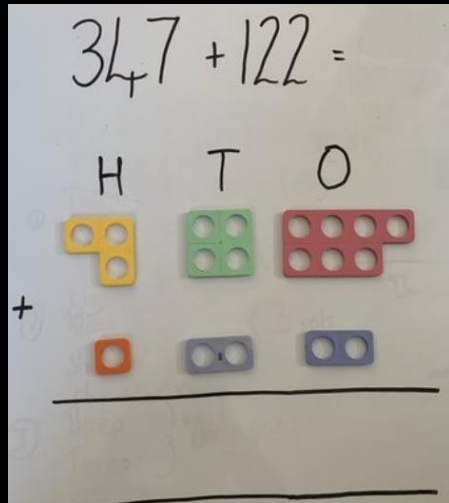


For this step, children apply their knowledge of partitioning.

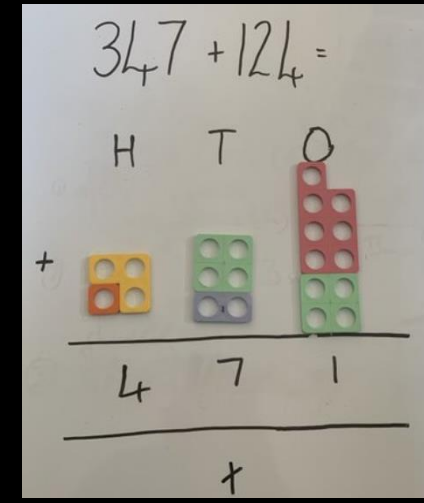
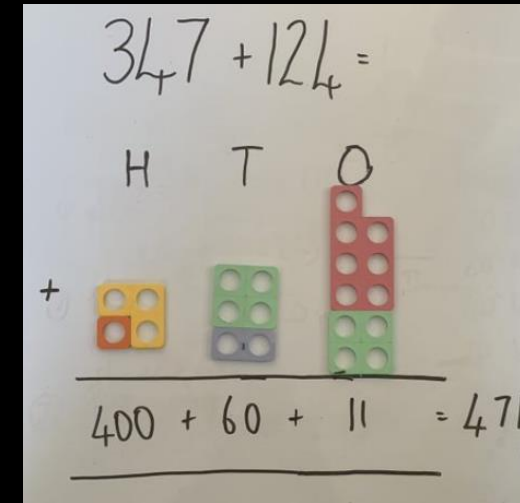
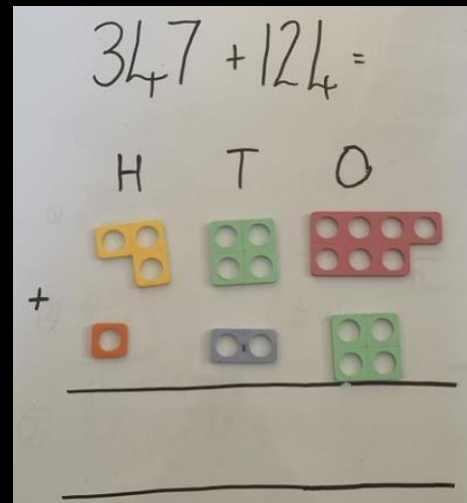
If adding a one-digit number, children will bridge the ten by partitioning.

If adding a two-digit number, children partition both numbers (addends) into tens and ones (adding the tens first and then the ones). This supports the children in developing a mental calculation strategy and lays the foundations for a more abstract, written method of addition.

3.1 - No bridging.



3.2 - Children to practise bridging the next ten.



Addition – Step 3

347 + 122 =		
300	40	7
+100	20	2
<hr/>		
400	60	9 = 469

Children are to be shown the expanded column method using partitioning.

$$2453 + 4532 =$$

$$\begin{array}{r} 2453 \\ + 4532 \\ \hline 6985 \end{array}$$

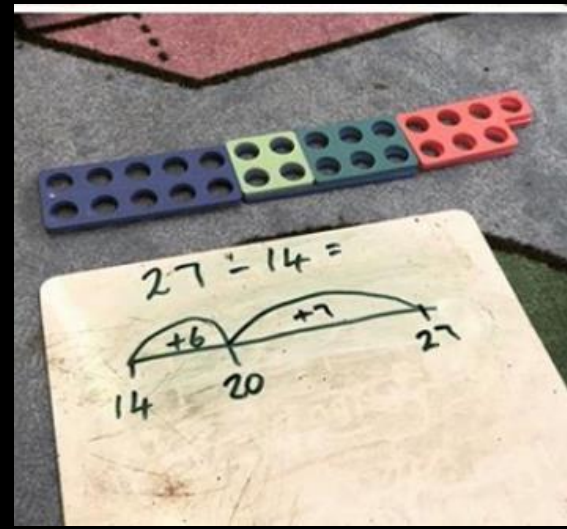
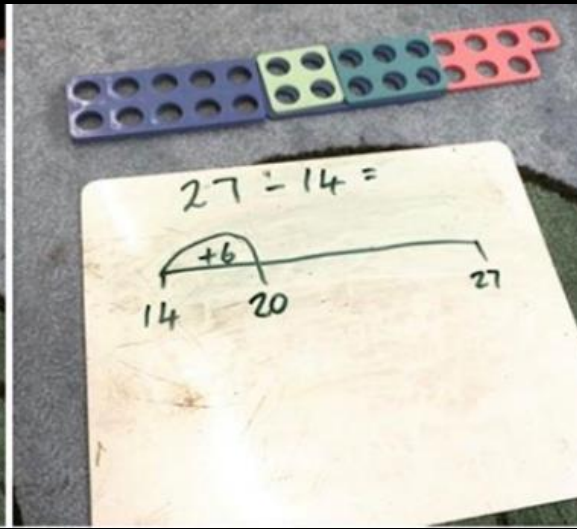
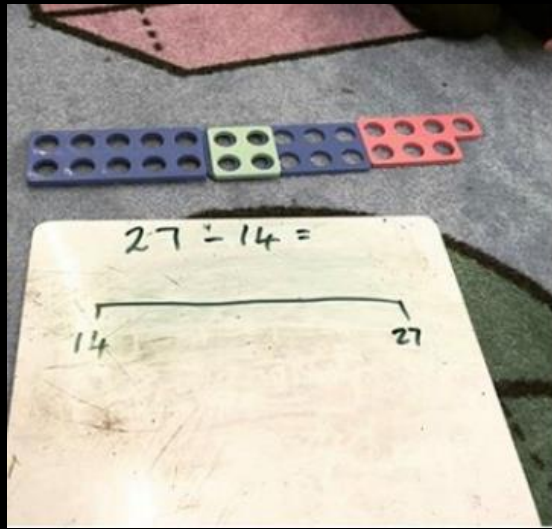
$347 + 122$ <p>From 347</p> $\begin{array}{r} 347 \\ + 122 \\ \hline 469 \end{array}$	<p>Then, with carrying</p> $\begin{array}{r} 159 \\ + 264 \\ \hline 423 \\ 11 \end{array}$
<p>Same number of decimal places</p> $\begin{array}{r} 78.5 \text{ km} \\ + 54.6 \text{ km} \\ \hline 133.1 \text{ km} \\ 11 \end{array}$	<p>Then, different number of decimal places</p> $\begin{array}{r} 124.9 \\ + 7.25 \\ \hline 132.15 \\ 11 \end{array}$

$$\begin{array}{r} 2654 \\ + 1238 \\ \hline 3892 \end{array}$$

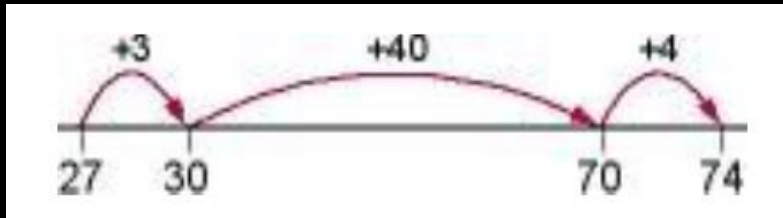
Addition – Step 4

Children are to develop their understanding of the compact column addition method in this step.

Children move towards working with decimals.



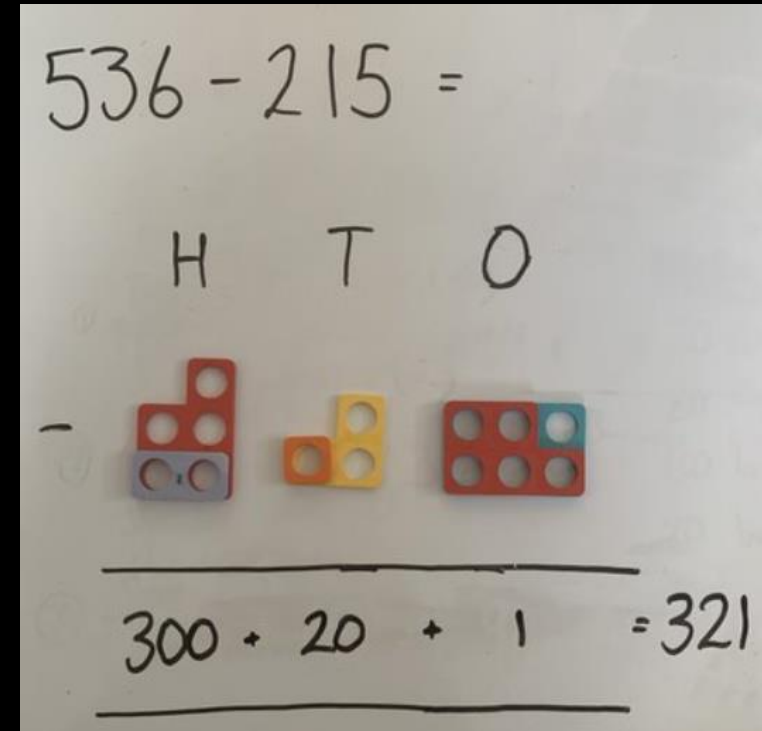
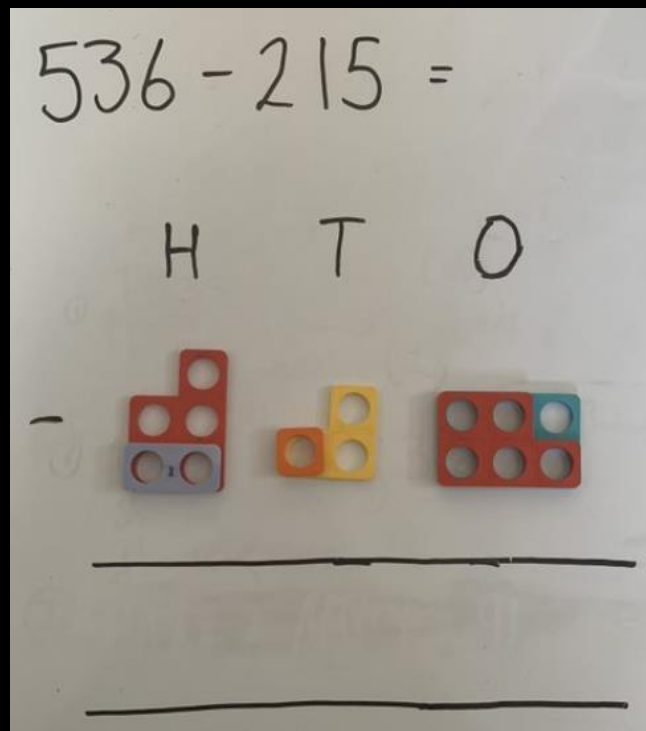
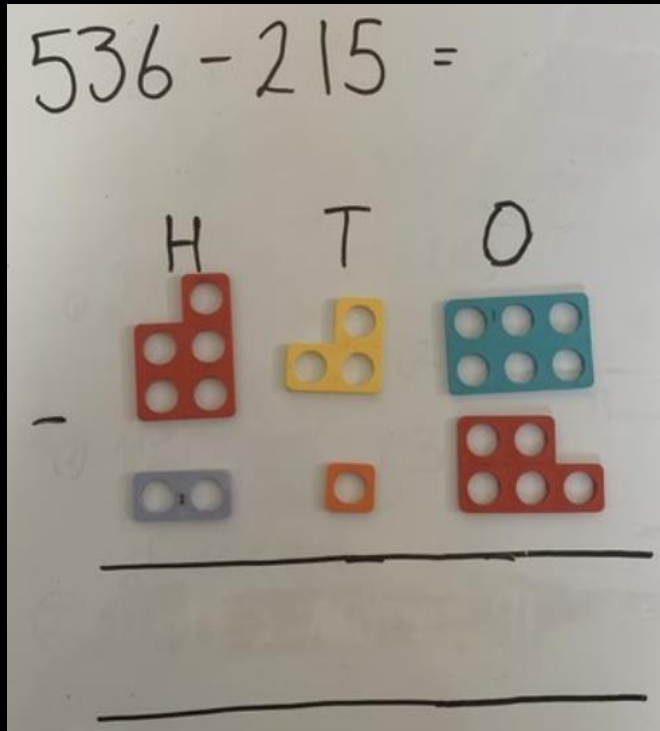
Subtraction – Step 1



Children are taught to find the difference by counting on.

Children are to add to the next ten first.

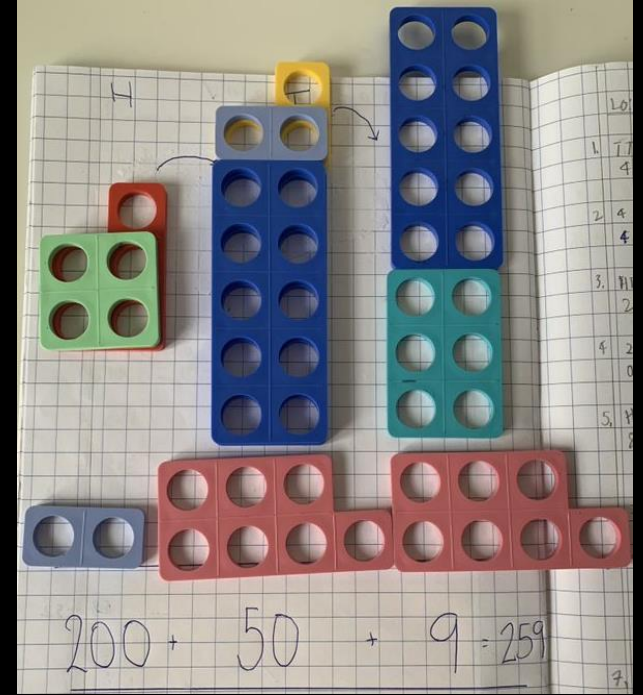
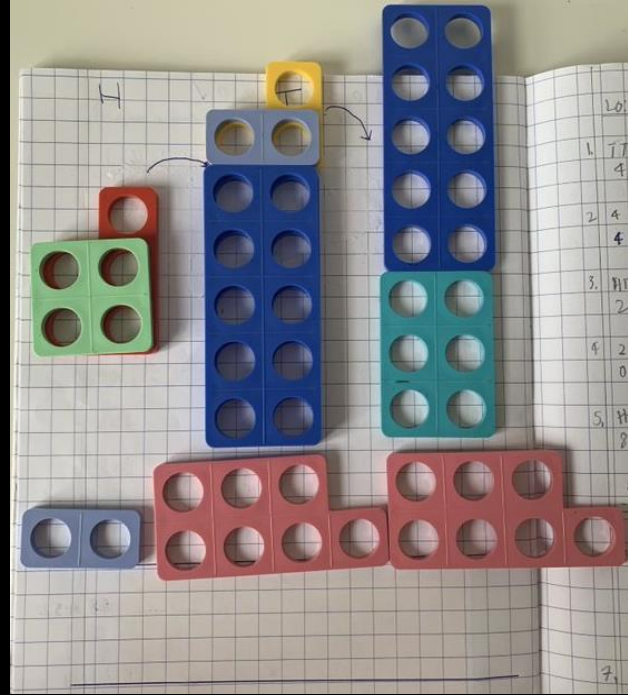
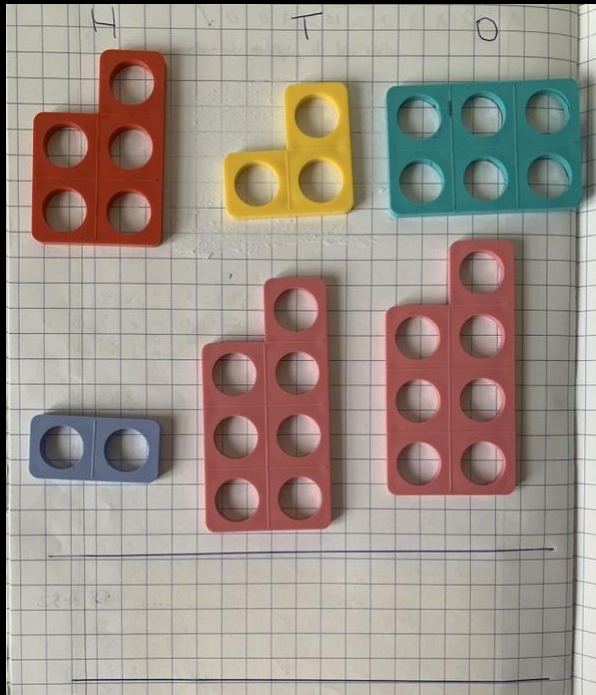
This step helps to support mental strategies taught and develops the transition into a formal written method.



Subtraction – Step 2

$$\begin{array}{r}
 536 - 215 \\
 500 \ 30 \ 6 \\
 - 200 \ 10 \ 5 \\
 \hline
 300 \ 20 \ 1 = 321
 \end{array}$$

Introduce children to the formal column method without exchanging. Partition the numbers eg hundreds, tens and ones.

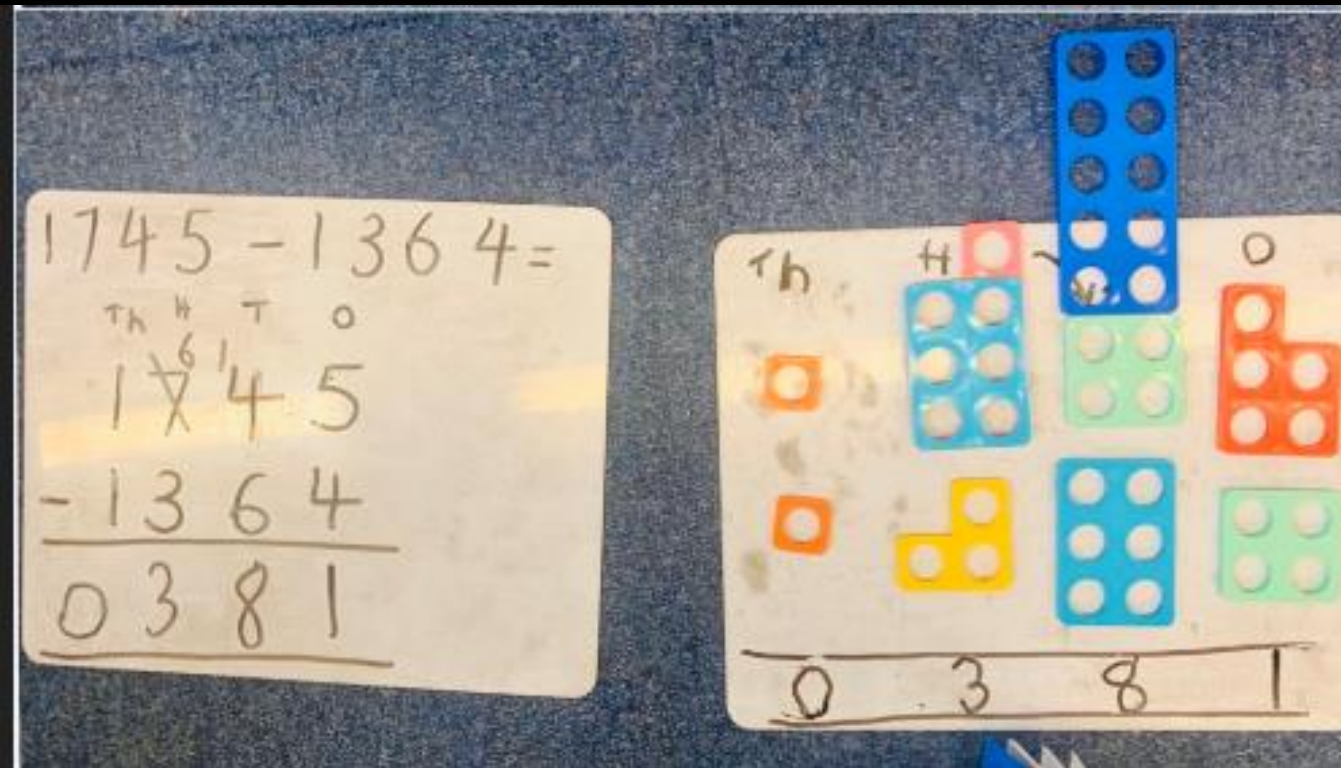


Subtraction – Step 3

Children are shown how to exchange using partitioning in this part of the calculation method.

Then $536 - 277 =$

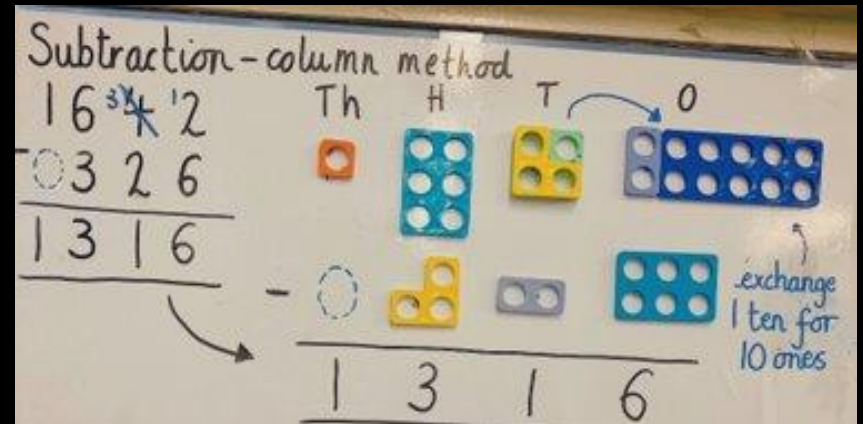
400	120	1
- 500	30	6
<u>200</u>	<u>70</u>	<u>7</u>
200	50	9 =



$$\begin{array}{r} 21 \\ 137 \\ - 29 \\ \hline 108 \end{array}$$

Then...

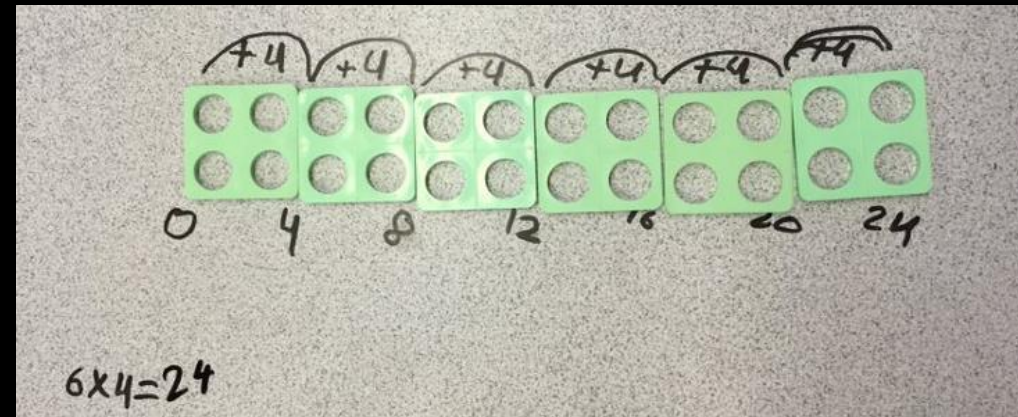
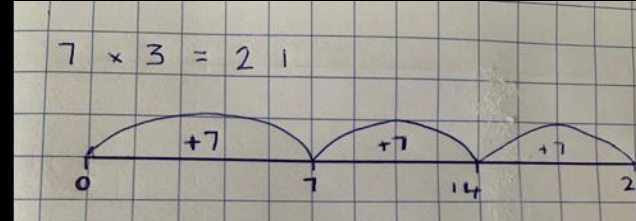
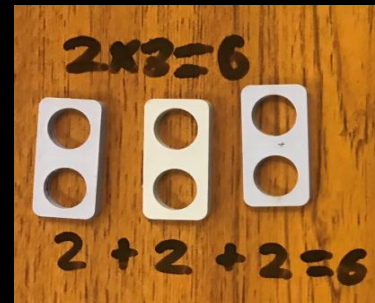
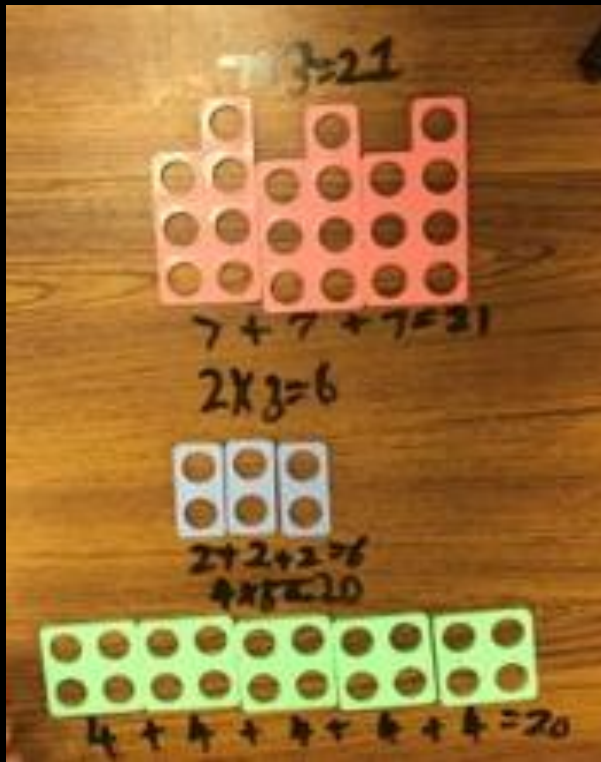
$$\begin{array}{r} 1 \\ 421 \\ 536 \\ - 277 \\ \hline 259 \end{array}$$



Subtraction – Step 4

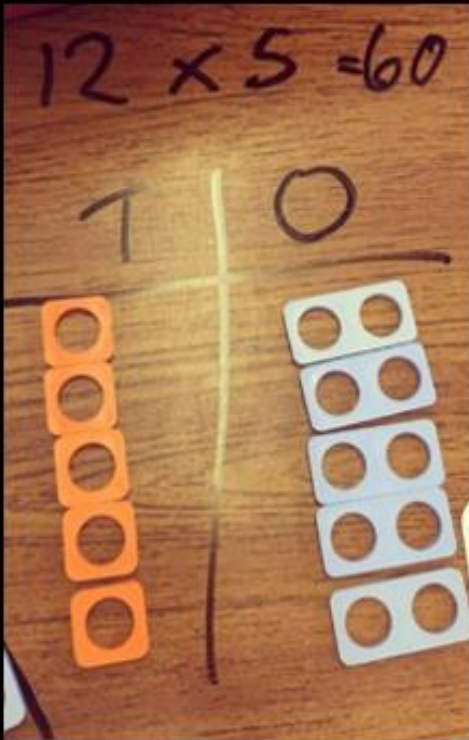
Children are introduced to the compact column method.

Children need to understand that $3-6$ does not equal a positive number so they need to exchange from the next column.



Multiplication – Step 1

Children begin to multiply by using repeated addition before moving onto more formal written methods. This supports the mental calculation strategies children use.



Abstract method

12×5 becomes
 $10 \times 5 + 2 \times 5$

$10 \times 5 = 50$
 $2 \times 5 = 10$

$50 + 10 = 60$

$$24 \times 14 = 336$$

x	20	4	
10	200	40	=240
4	80	16	=96

$$\begin{array}{r} 240 \\ +96 \\ \hline 336 \end{array}$$

124×26 becomes

	1	2	
	1	2	4
x		2	6
	7	4	4
2	4	8	0
3	2	2	4
1	1		

Answer: 3224

Multiplication

Step 2 – partitioning

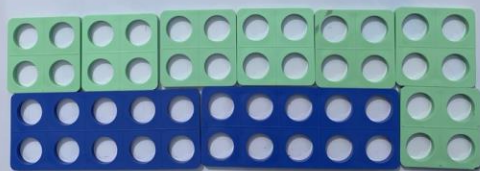
Step 3 – grid method

Step 4 – column method

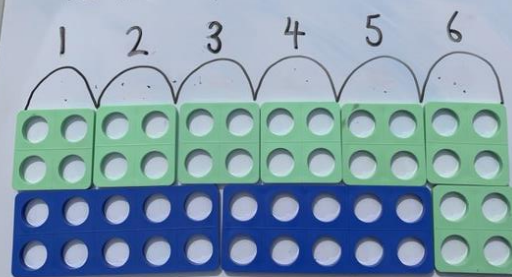
Children use knowledge of place value to partition multiplicand. Children add products together.

Alongside children's developing recall of multiplication facts through times tables, children are then introduced to the formal written methods.

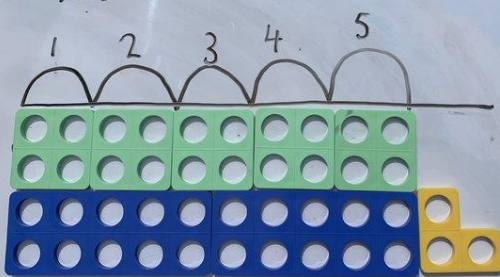
$$24 \div 4 =$$



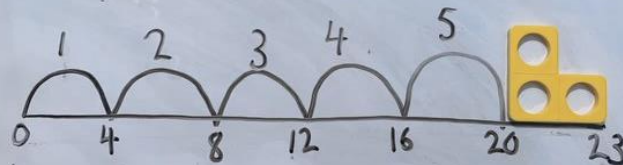
$$24 \div 4 =$$



$$23 \div 4 =$$



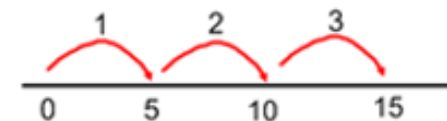
$$23 \div 4 = 5 \text{ r } 3$$



Grouping using number line

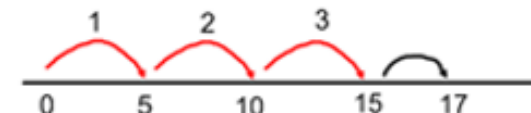
$$15 \div 5 = 3$$

$$15 \div 5 = 3$$



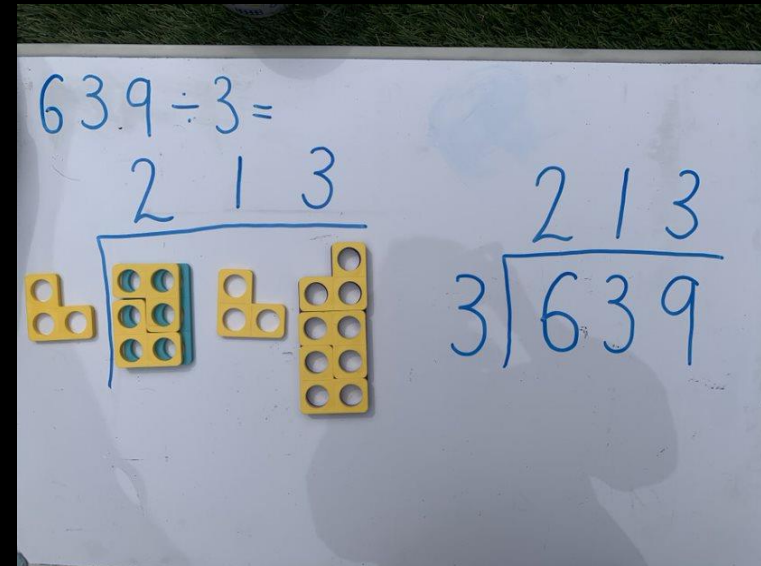
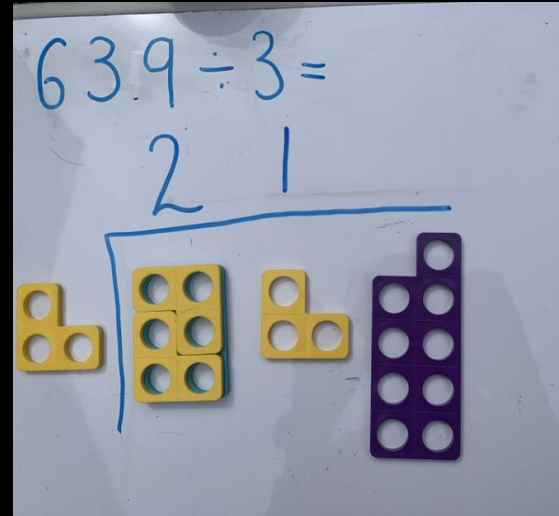
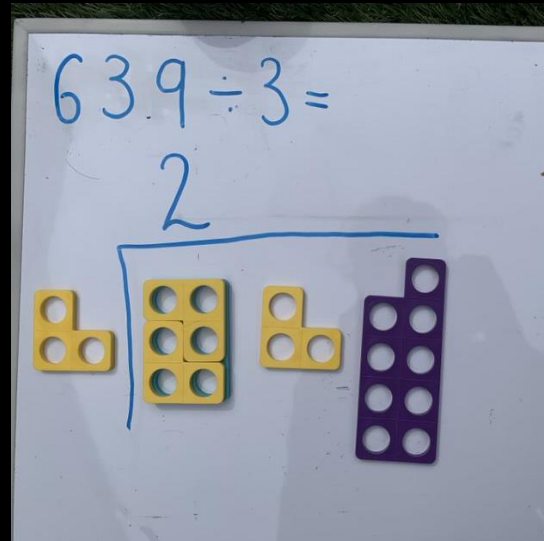
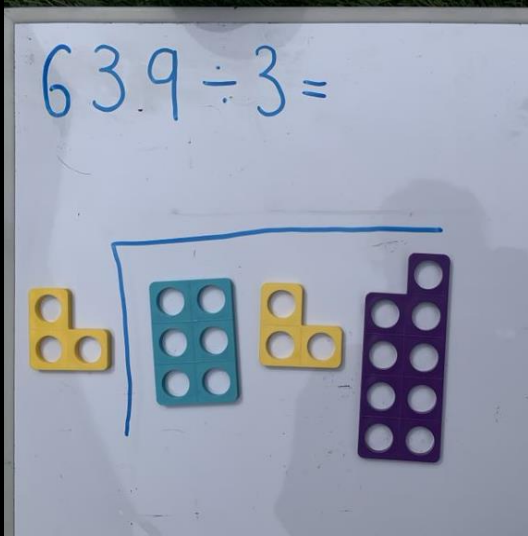
Finding a remainder $17 \div 5 = 3 \text{ r } 2$

$$17 \div 5 = 3 \text{ r } 2$$



Division – Step 1

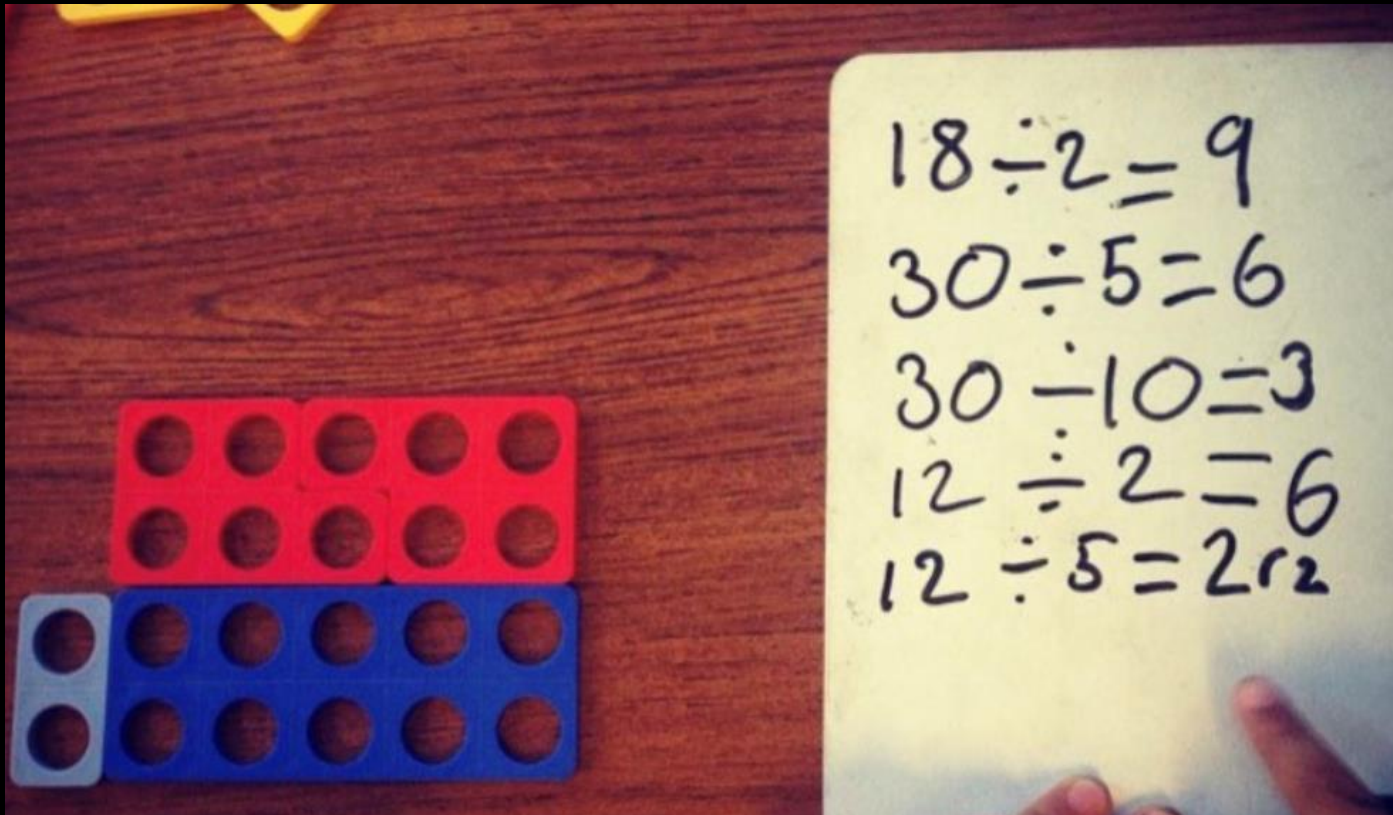
Children begin to explore division using a number line. Children practice constructing a number line using Numicon. Children are supported in seeing when a number can be grouped equally, but also when there is a remainder.



Division – Step 2 'goes into' method

Children progress on from the number line method but can still use the same strategies to group and share. They are introduced to this written method without needing to perform an exchange.

Step 3 - Exploring remainders

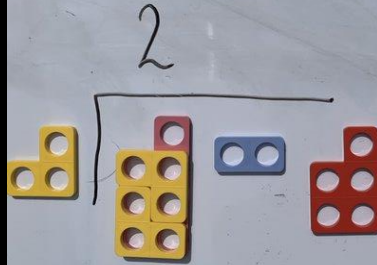


Children refamiliarize themselves with the concept and value of a "remainder" before they practice them in a formal way.

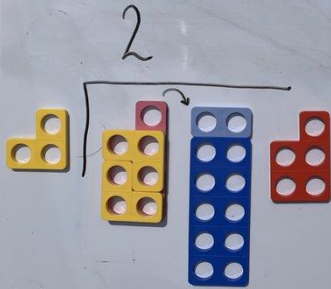
$$725 \div 3 =$$



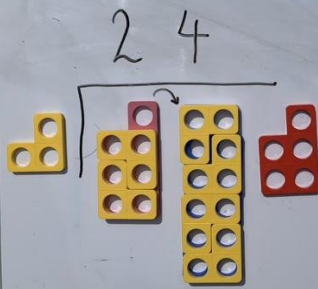
$$725 \div 3 =$$



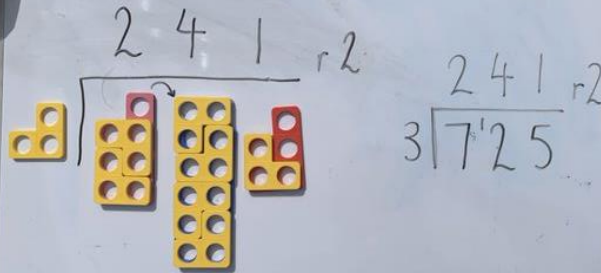
$$725 \div 3 =$$



$$725 \div 3 =$$



$$725 \div 3 = 241 \text{ r}2$$



Formal 'Goes Into' Method

$$\begin{array}{r} 14 \text{ r} 2 \\ 5 \overline{) 72} \end{array}$$

Then

$$\begin{array}{r} 164 \text{ r} 3 \\ 6 \overline{) 987} \end{array} = 164 \frac{3}{6}$$

With decimals: Use formal method

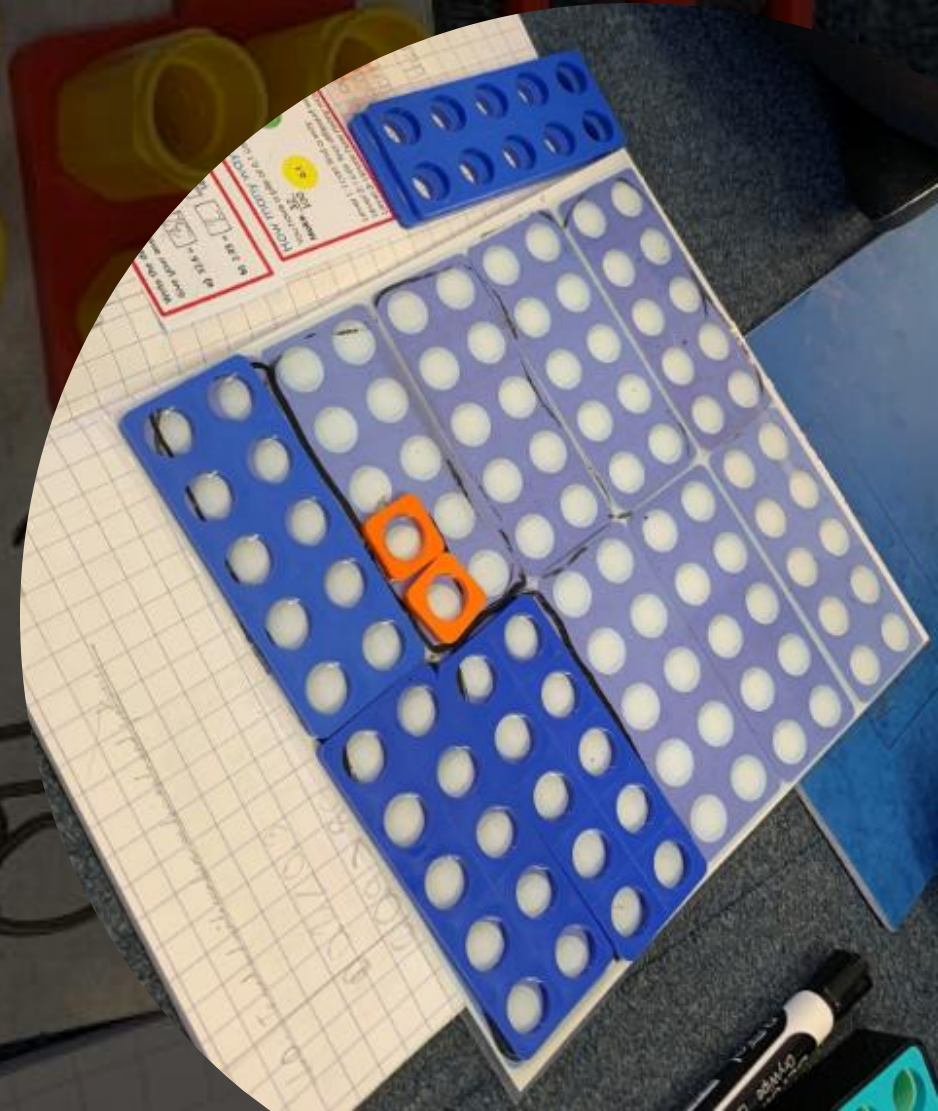
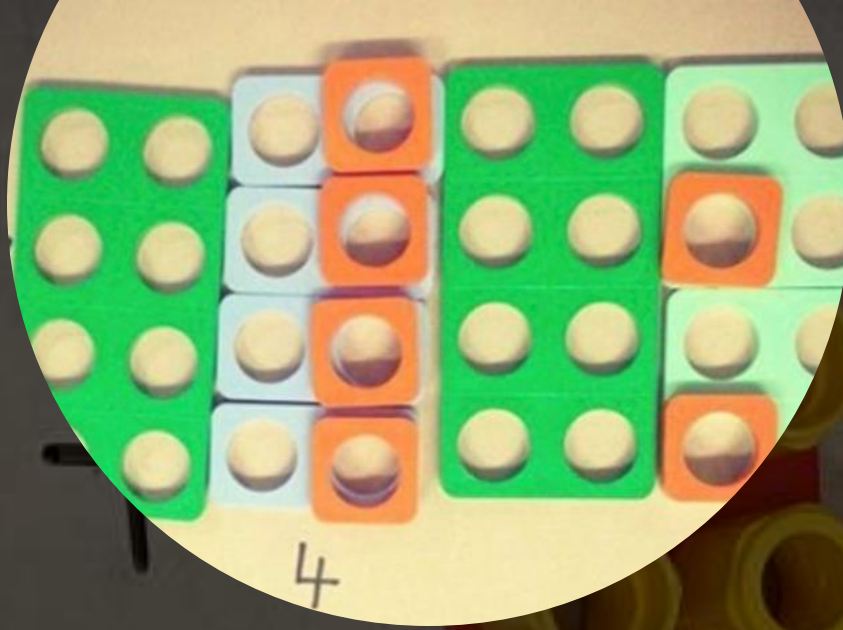
$$\begin{array}{r} 12.5 \\ 7 \overline{) 87.5} \end{array}$$

More able could express answer to calculation as 12.5, or remainder 3.5, discussing that 3.5 left over is half of the 7 that they were dividing the number by.

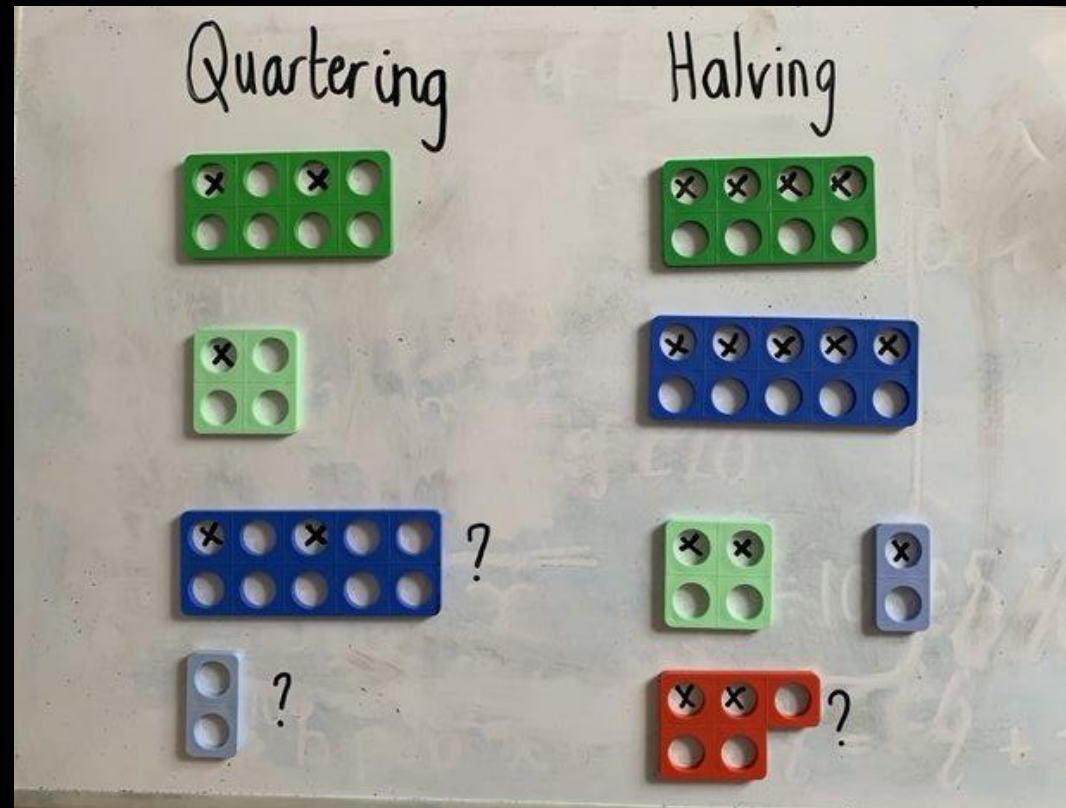
Division – Step 4

Children apply understanding of remainders and must perform an exchange. Numicon is used to exchange one hundred for 10 tens, one ten for 10 ones, 1 one for 10 tenths etc. This is modelled and practiced alongside the formal method.

Fractions, Decimals and Percentages

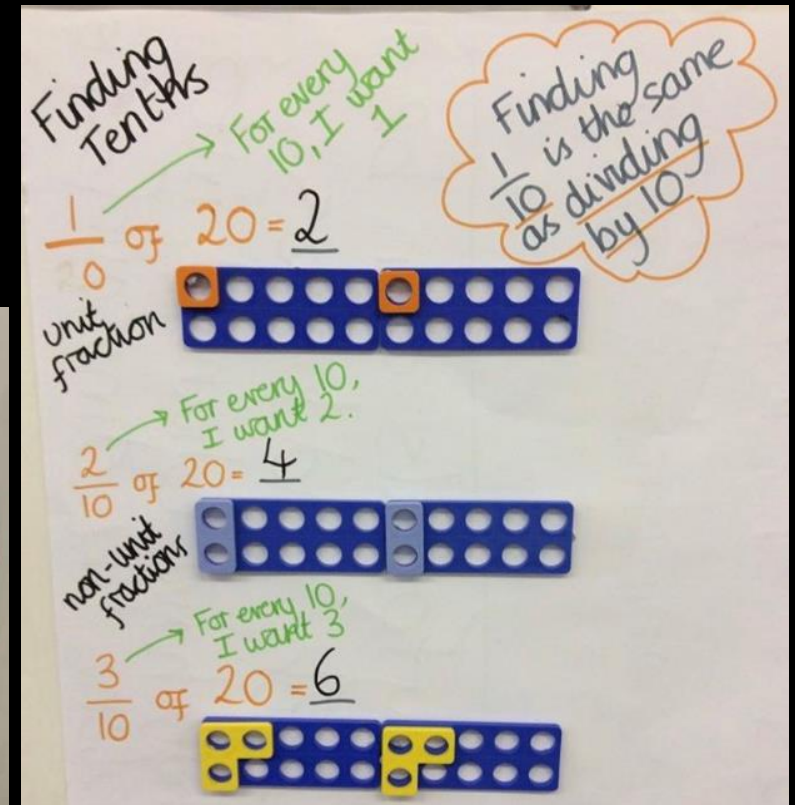
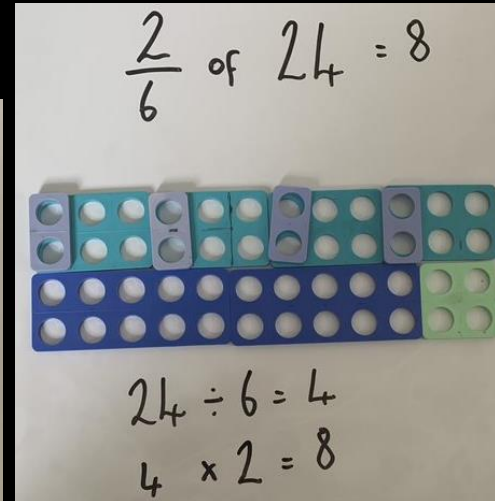
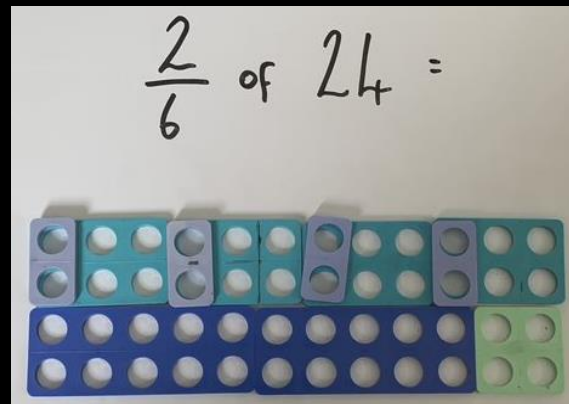
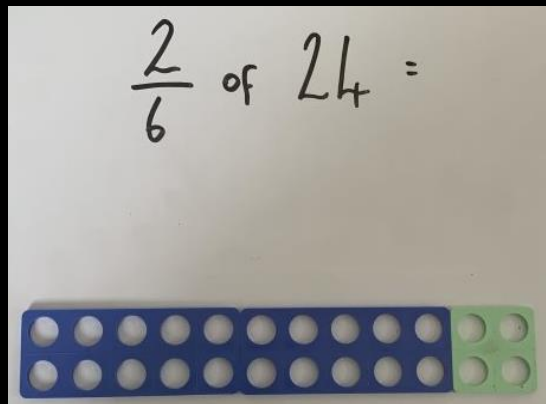
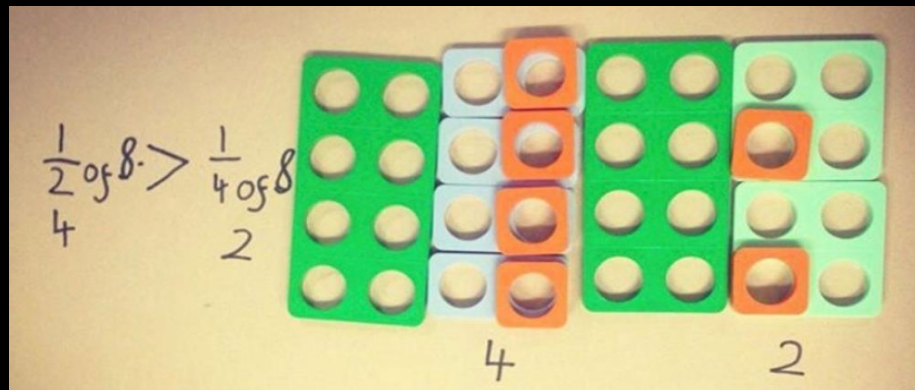


Halves and Quarters



Children explore halving and quartering numbers where they are left with a whole number (halving an even number).

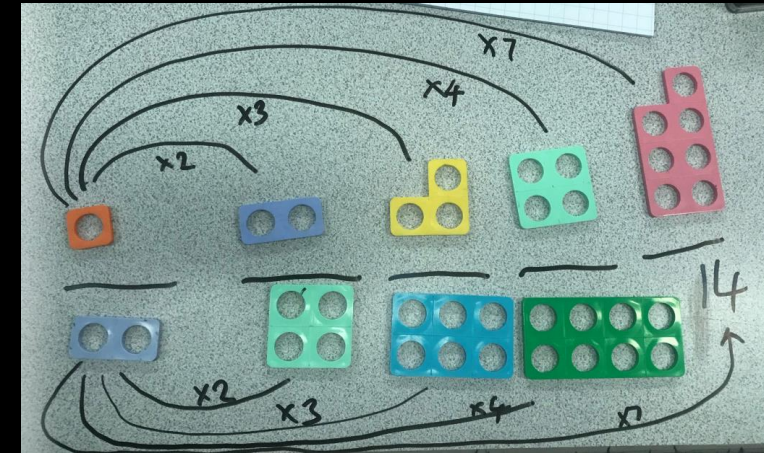
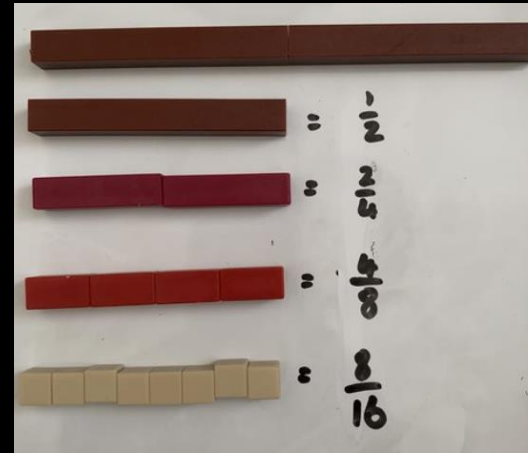
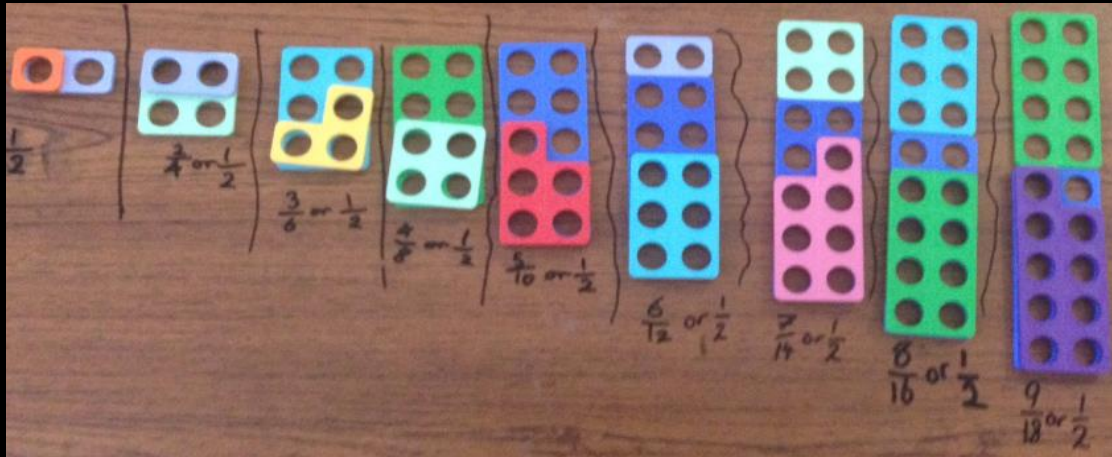
Before moving on to finding fractions of amounts, children are given opportunity to discuss halving and quartering, and misconceptions such as "you cannot halve an odd number" can be addressed.



Fractions of amounts

Children begin to explore the idea of "for every group of ____, I want ____".

As they progress, children start to divide the amount by the denominator, giving one tenth, quarter, third etc. When the numerator is greater than one, they place the numerator tile on top of each denominator tile. This represents the multiplying stage.



Equivalent Fractions

Children explore the concept of equivalent fractions.

Although this helps children work on the rule "Whatever you do to the top, you do to the bottom", it is vital that children understand that the whole has not changed in size.

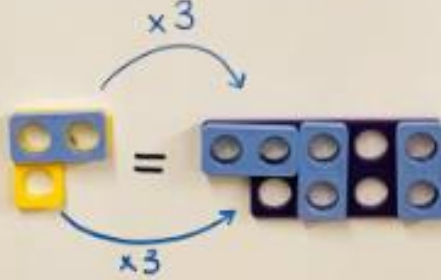
Comparing Fractions with different denominators

"which is greater?
which is less? Are they equal?"

$$\frac{2}{3} \boxed{>} \frac{5}{9}$$

$$\frac{2}{3} \xrightarrow{\times 3} \frac{6}{9}$$

"we must make them the same
before we can compare!"

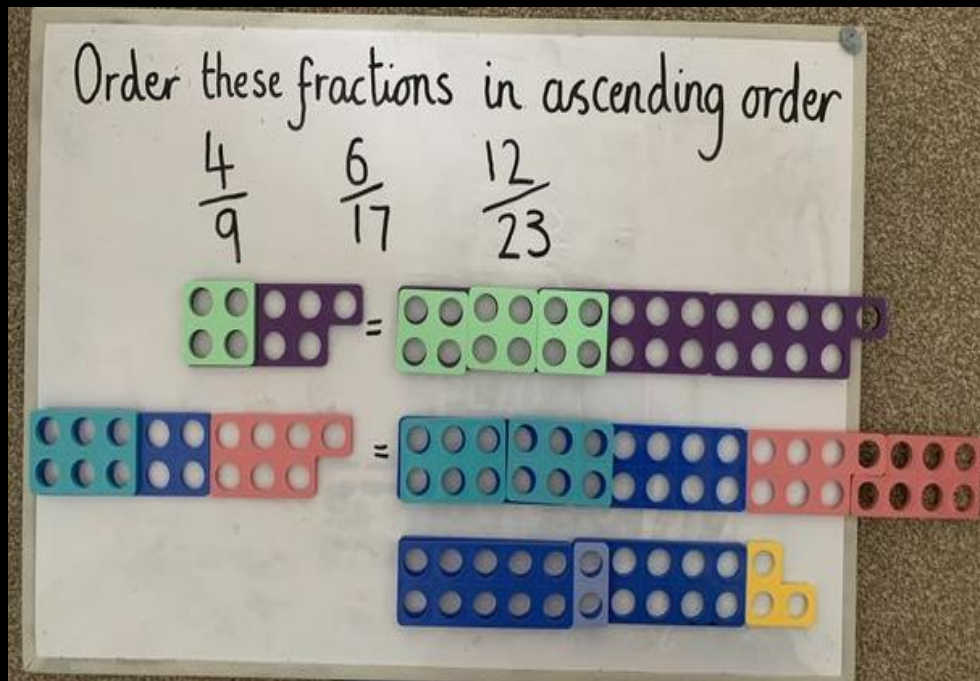


$$\frac{6}{9} > \frac{5}{9} \quad \text{so} \quad \frac{2}{3} > \frac{5}{9}$$

Comparing and ordering fractions

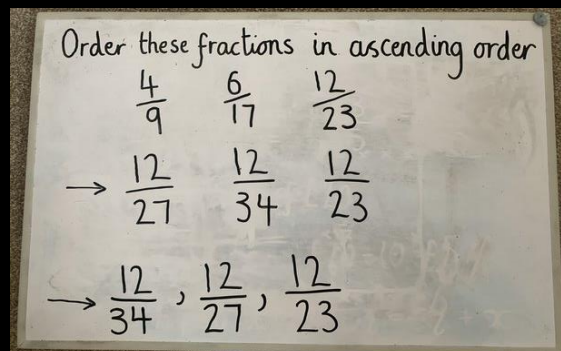
Building on from equivalent fractions, children use the rule to make the denominators equal.

They can then compare fractions more easily and identify which is worth less or more.



Children realise that finding the lowest common multiple of the denominators is tricky, however, finding the lowest common multiple of the numerator is more efficient.

Context for this example. It is key that we don't always give the context at the start.



Pizza and friends

12 pizzas between 27 friends

12 pizzas between 34 friends

12 pizzas between 23 friends

In which scenario would you receive the smallest amount of pizza?

Comparing and ordering fractions

Providing a context to a question allows children to realise that making the numerator the same can give you the correct answer.

Adding fractions

If denominators are different, find equivalent fractions to make them the same.

e.g. $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$

So $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

$$\frac{3}{5} - \frac{4}{10} =$$

$$\frac{3}{5} = \frac{6}{10}$$

Adding and Subtracting fractions

Once the denominators are equal, children can then add or subtract fractions with more consistency and identify that only the numerator is added or subtracted.

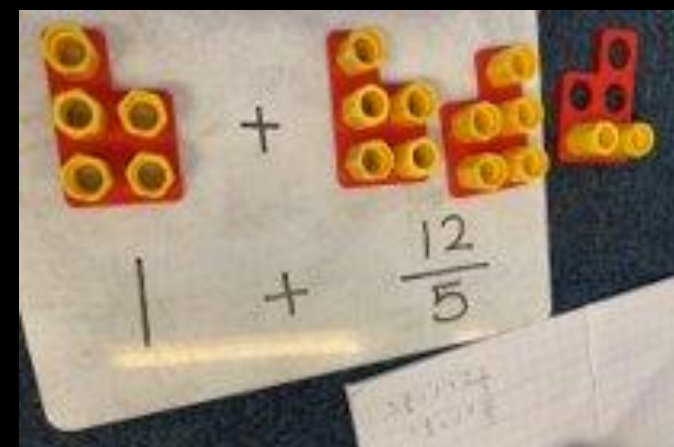
Improper fractions \longleftrightarrow Mixed numbers

How many whole groups of $\frac{5}{3}$ can be made in 5? 1 with 2 left over.

$1 \frac{2}{3}$

whole $\frac{2}{3}$ proper fraction

$1 \times 3 = 3$
 $3 + 2 = 5$ thirds



Converting improper fractions and mixed numbers

Children start to see how fractions can be more than 1 whole. They can describe fractions both as a number of wholes, but also a number of parts.

As they progress to the written method, the link between fractions and division is cemented.

Multiply fractions by integers

$$3 \times \frac{2}{3}$$



whole
number
e.g. $2 = \frac{2}{1}$
 $3 = \frac{3}{1}$

$$\frac{3}{1} \times \frac{2}{3} = \frac{3 \times 2}{1 \times 3} = \frac{6}{3} = 2$$

Multiplying fractions by whole numbers

Children create the fraction, and then replicate e.g. $3 \times \frac{2}{3}$

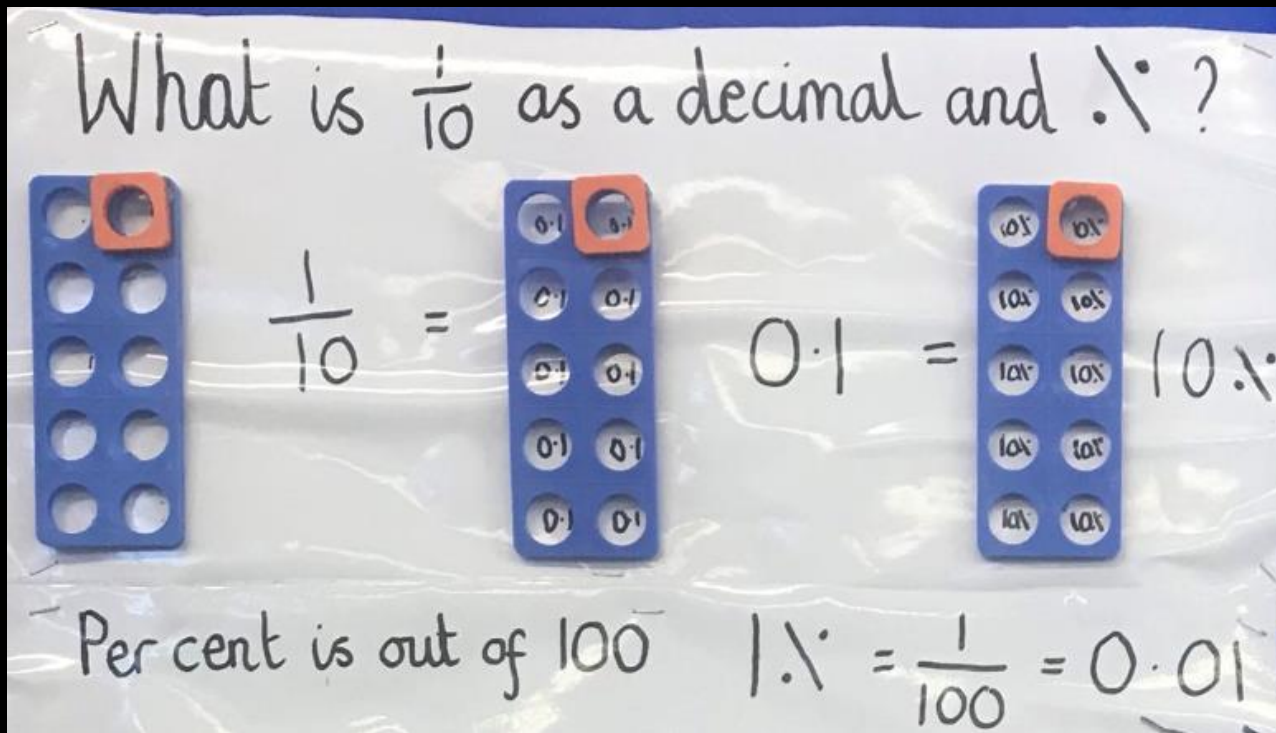
They can then simplify the fraction by applying understanding of mixed numbers and improper fractions.

What is $\frac{1}{10}$ as a decimal and %?

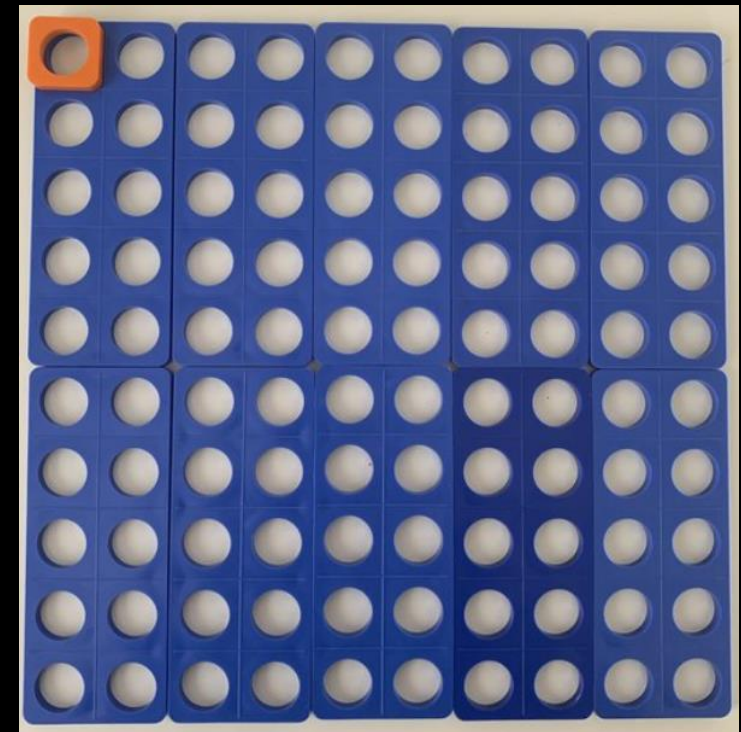
$\frac{1}{10} = 0.1 = 10\%$

Per cent is out of 100

$1\% = \frac{1}{100} = 0.01$



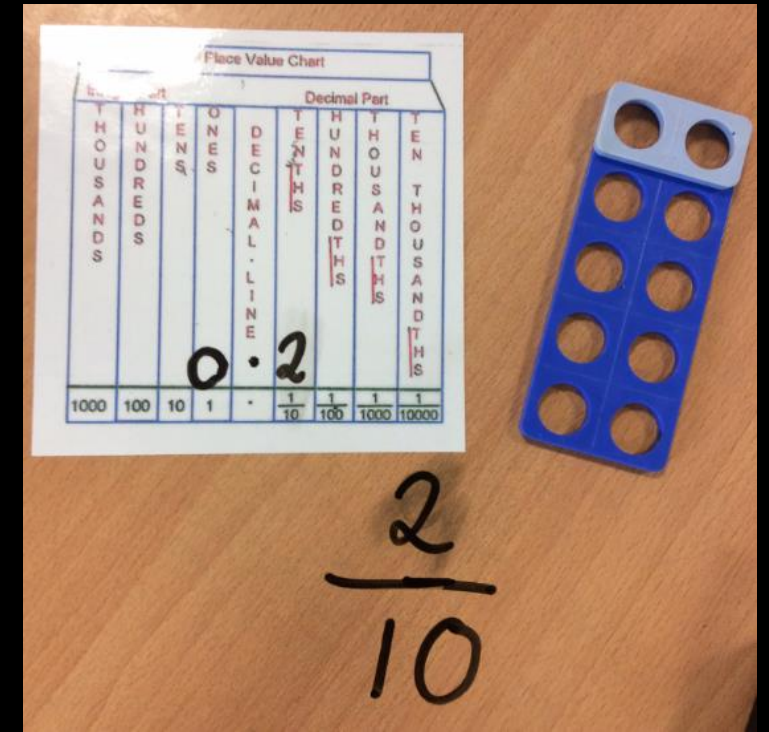
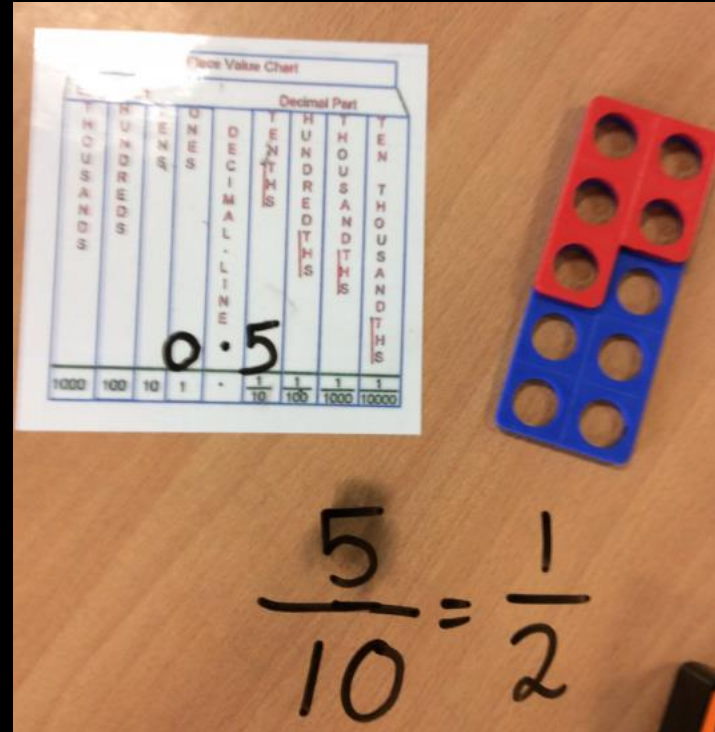
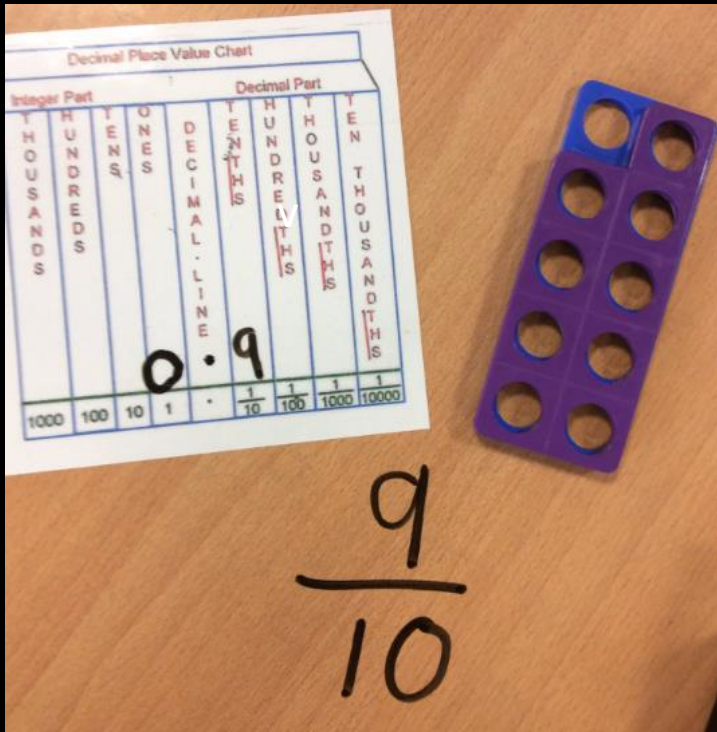
The image shows a piece of white paper with handwritten text and three blue Numicon rods. The first rod has one red ring at the top, representing 1/10. The second rod has one red ring at the top and one white ring at the bottom, representing 0.1. The third rod has one red ring at the top and ten white rings, representing 10%. The text 'What is 1/10 as a decimal and %?' is written at the top. Below the rods, the equations '1/10 = 0.1 = 10%' and 'Per cent is out of 100' are written. At the bottom, the equation '1% = 1/100 = 0.01' is written.



Fractions, decimals and percentages

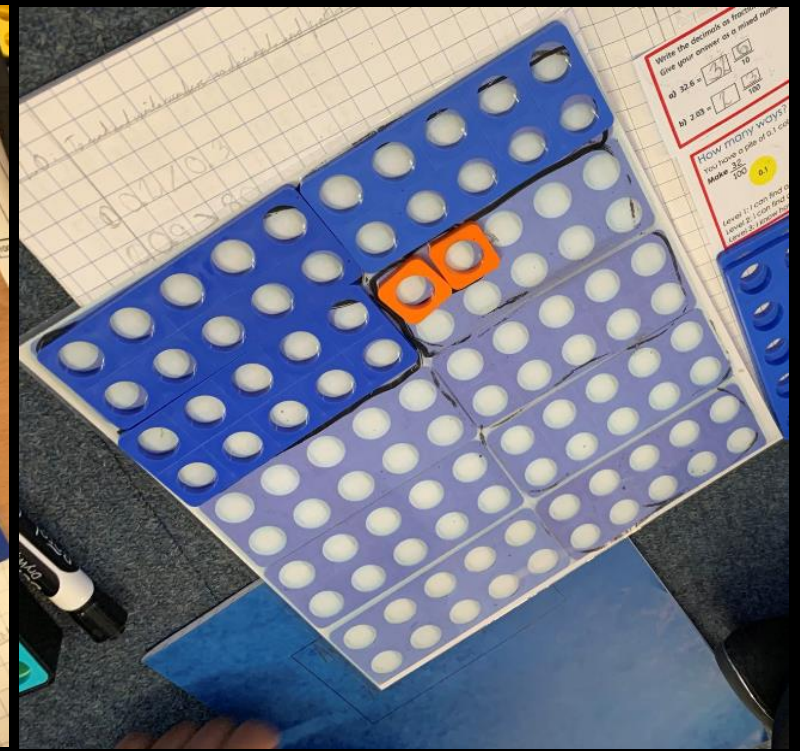
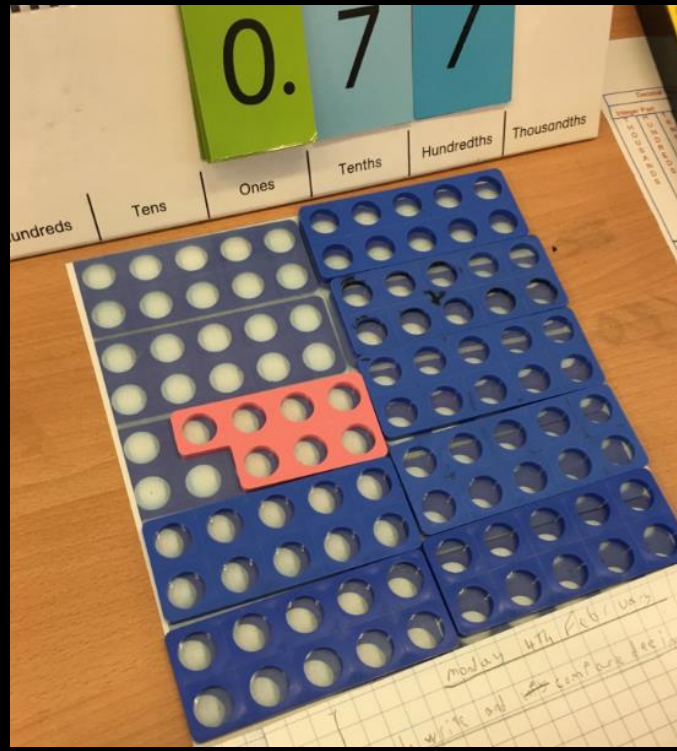
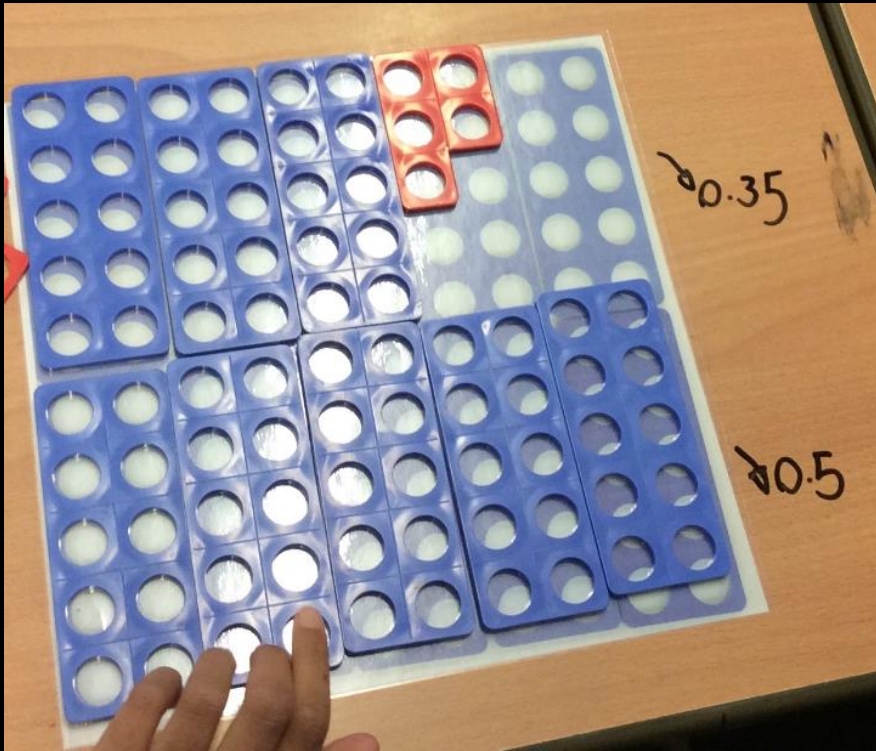
Children recognize that $\frac{1}{10} = 0.1 = 10\%$, as the whole has been divided by 10.

As children become confident with tenths, they move on to looking at 100ths using the Numicon 100 square.



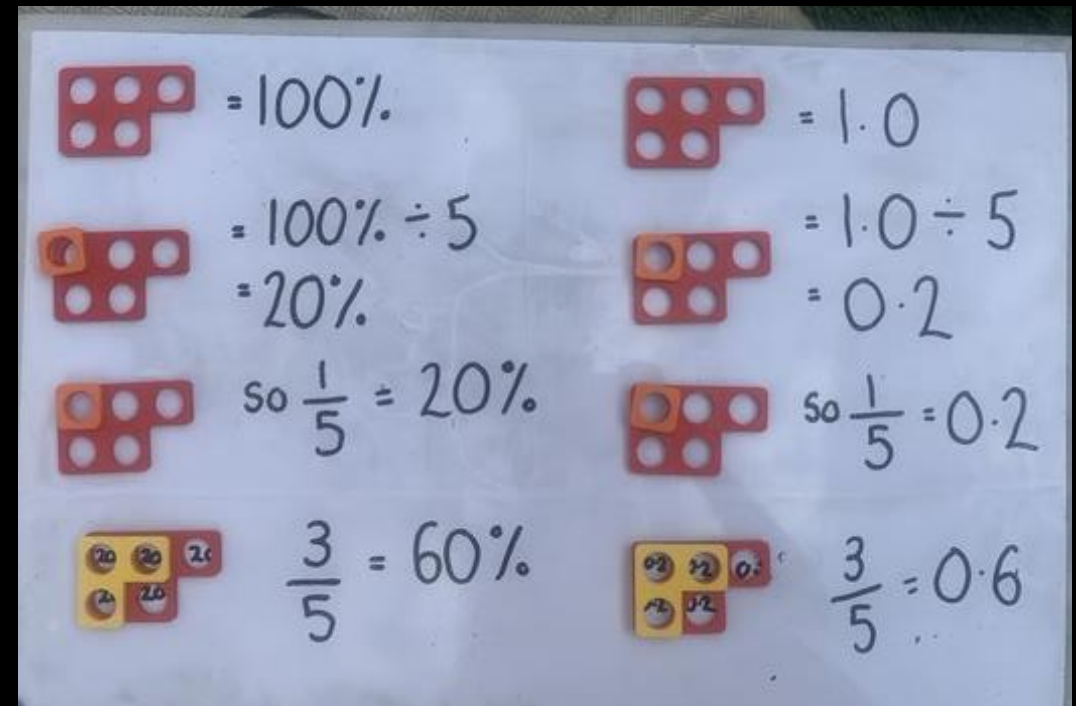
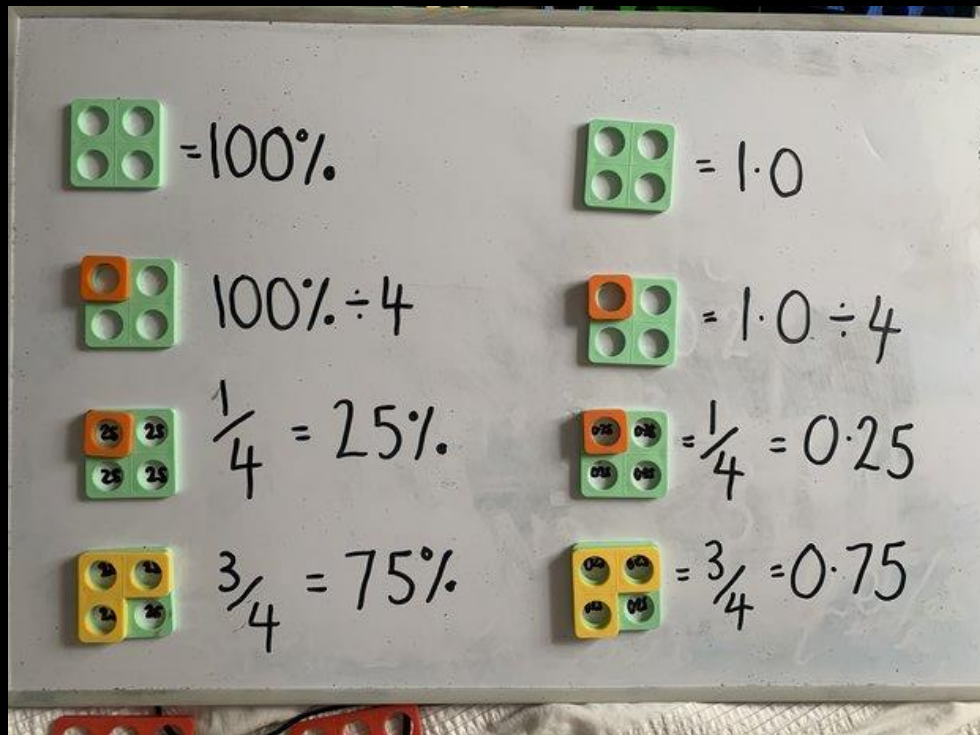
Fraction and
decimal
equivalence

Children apply their understanding to write different amounts of tenths as decimals.



Moving from
tenths to
hundredths

Children build on prior learning to write hundredths in different forms eg. 0.35 , 35% and $\frac{35}{100}$.



Converting fractions, decimals and percentages.

Children learn how to share the whole (100%) by the denominator. They then multiply that amount by the numerator to find the percentage, decimal and fraction equivalence.

Calculate 10% of £20



$$£20 \div 10 = £2$$

Calculate 30% of £20



$$£20 \div 10 = £2$$

$$£2 \times 3 = £6$$

Find

10% of 320

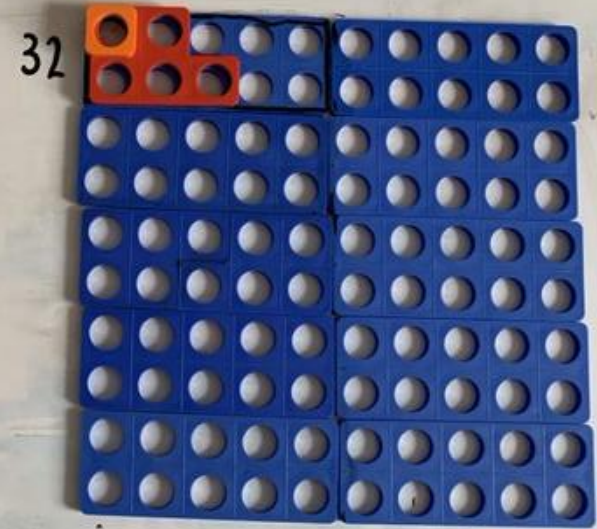
$$320 \div 10 = 32$$

5% of 320

$$32 \div 2 = 16$$

1% of 320

$$320 \div 100 \text{ or } 32 \div 10 = 3.2$$



Finding % of amounts

Children begin to calculate percentages by exploring a 10 plate as representative of 100%. From this, they can find % when multiples of 10 (10%, 20% etc.)

Children then move to the Numicon 100 square to explore 5%, 1% and so on. Children should recall prior learning to recognize the equivalence between finding 10% and finding one tenth, 1% and 1 hundredth.

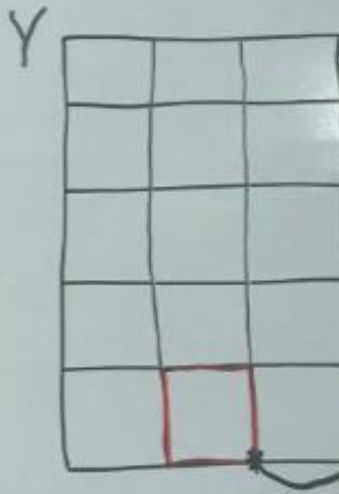
Geometry, Position and Direction



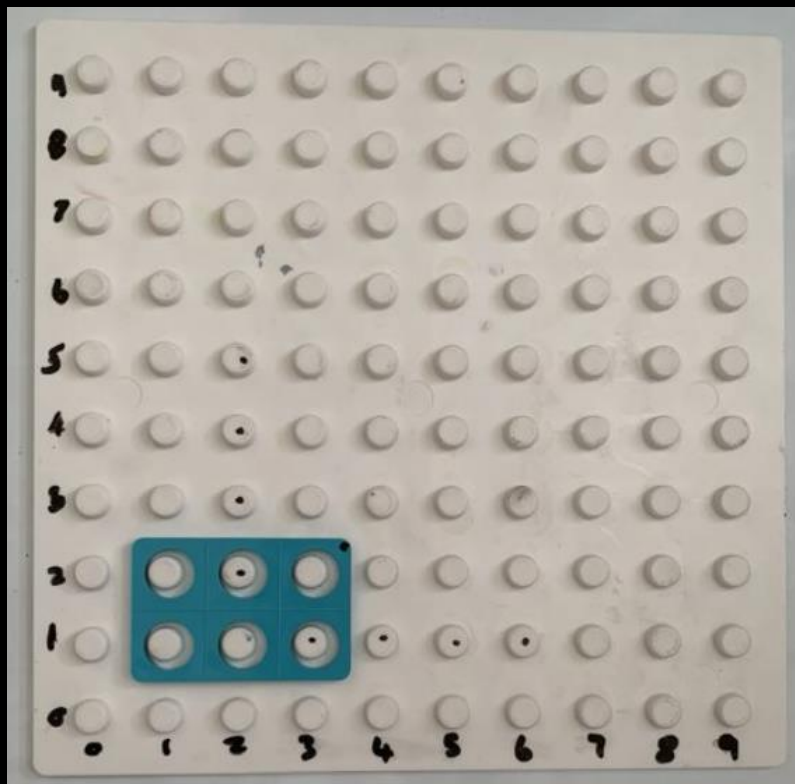
something is moved
without changing size or ori



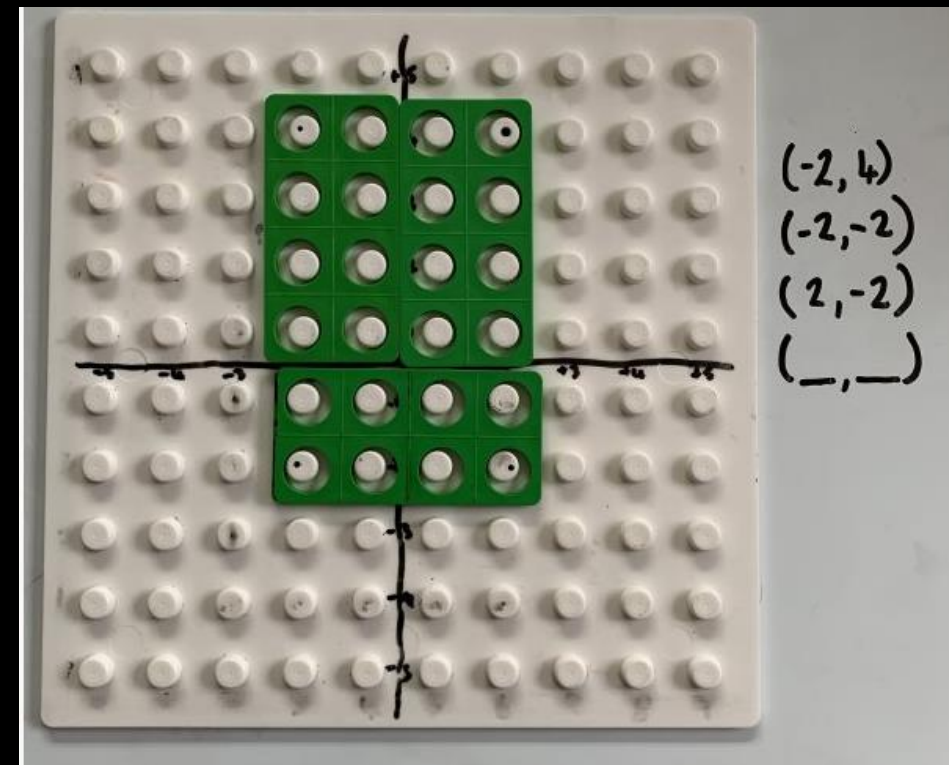
to the left



2 across to
3 up.



$(1, 1)$ $(3, 1)$
 $(1, 2)$ $(_, _)$

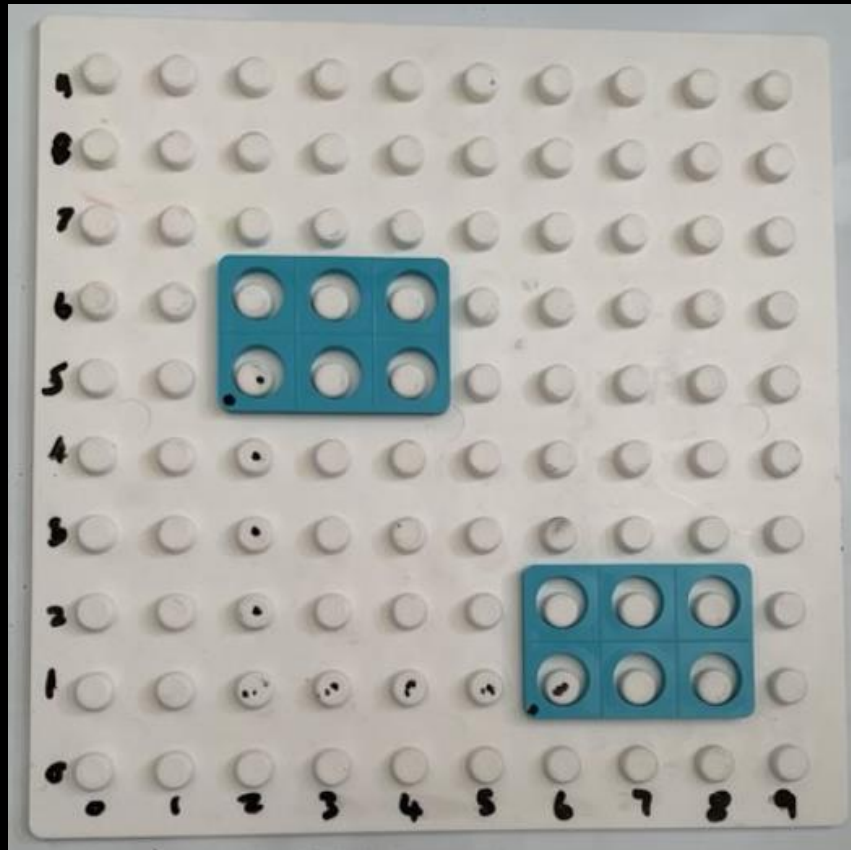


$(-2, 4)$
 $(-2, -2)$
 $(2, -2)$
 $(_, _)$

Co-ordinates

Children to use the base board and place tiles onto the grid. Children can then mark out the co-ordinates of each corner of the shape.

You could get creative with this and turn it into a four-quadrant grid, as this will allow children to explore negative numbered co-ordinates.



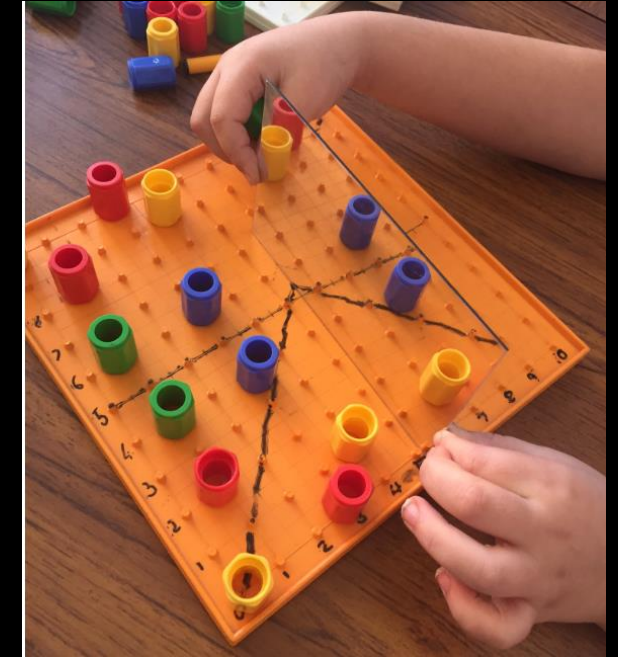
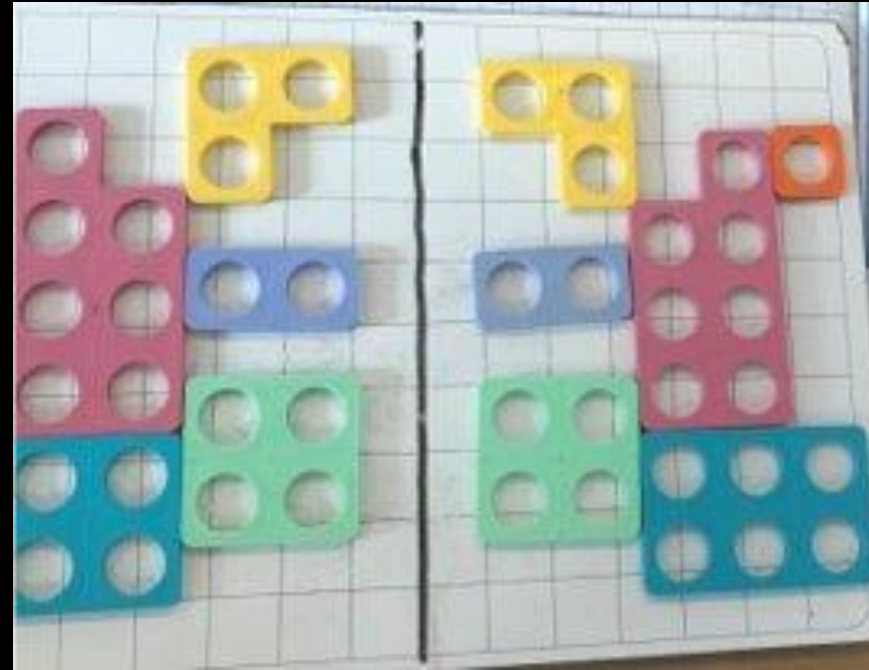
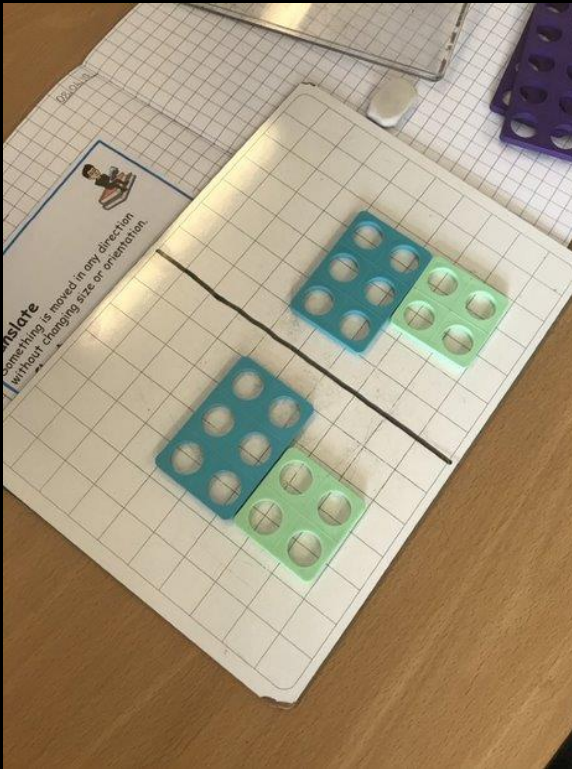
Shape B has been translated _____

4 left
4 up

Translation

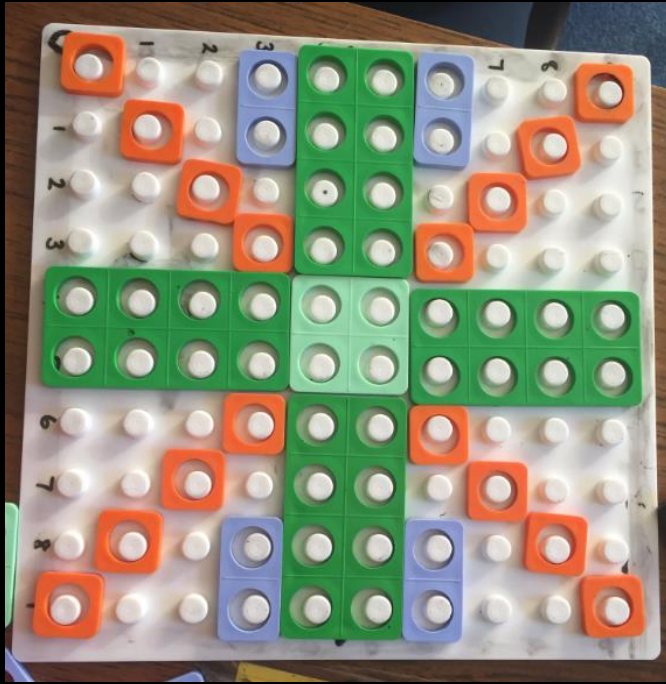
Children can use the base board to direct other children to move (or translate) the shape in a horizontal and then vertical direction.

As they do this, they begin to explore and notice that the shape does not change in its appearance, it only moves position.



Reflection

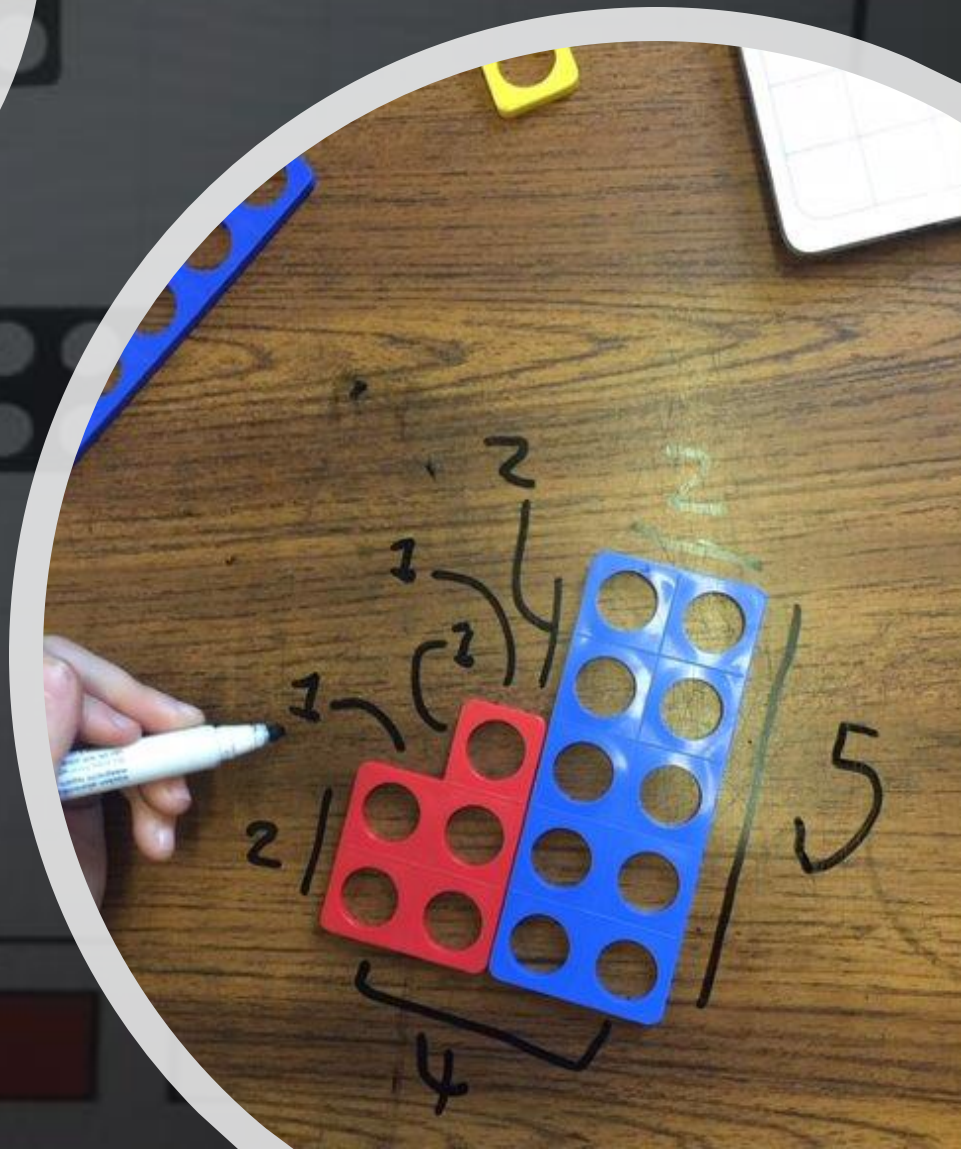
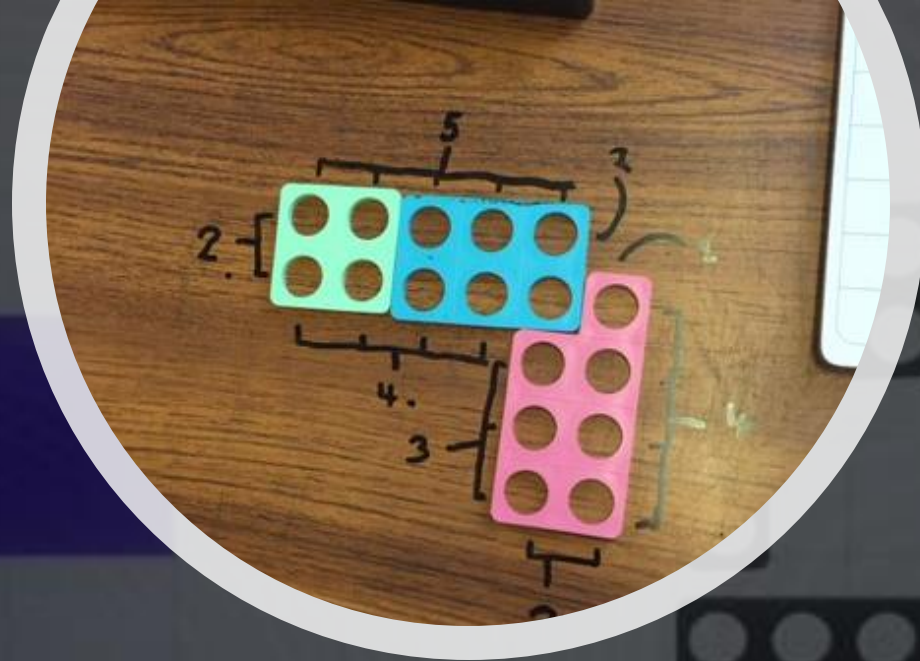
Children learn to reflect an image across a mirror line. They explore horizontal, vertical and diagonal lines of reflection.

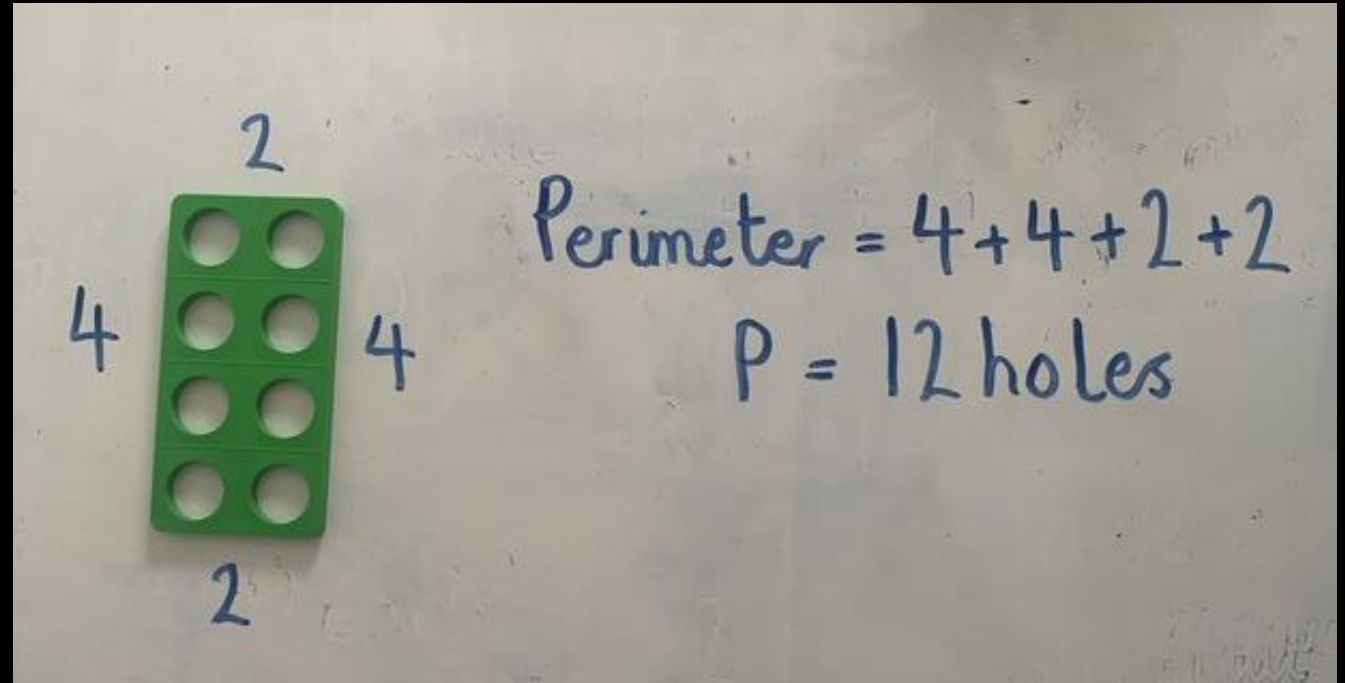
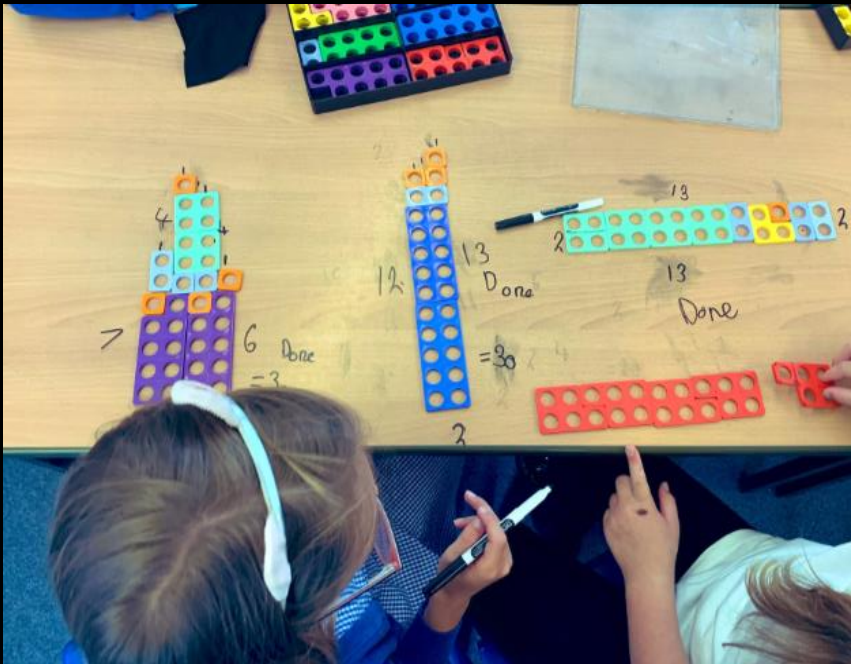


Symmetry

Children continue to explore reflection and symmetry by creating patterns that are symmetrical, following a horizontal, vertical or diagonal line of symmetry.

Measurement

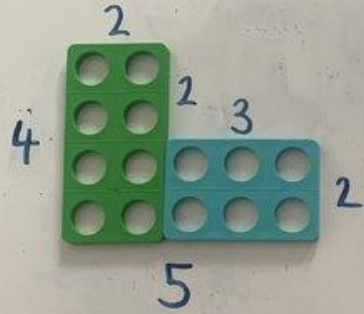




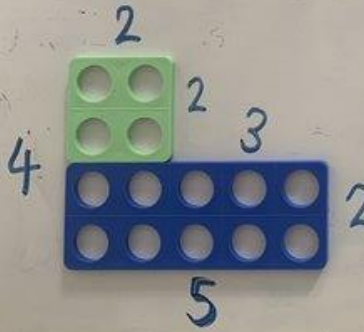
Perimeter of rectangles

Children begin to calculate the perimeter of rectangles.

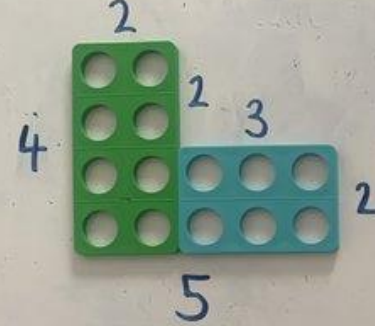
Children practise with different examples to generalize that to find the perimeter of a shape they must find the sum of all the sides.



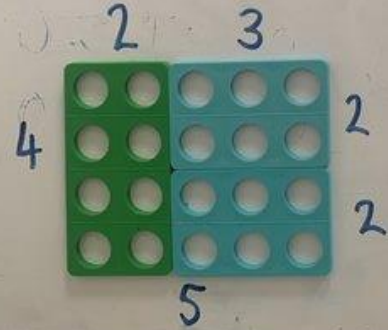
$$\begin{aligned} \text{Perimeter} &= 5 + 4 + 2 + 2 + 3 + 2 \\ &= 18 \text{ holes} \end{aligned}$$



$$\begin{aligned} \text{Perimeter} &= 5 + 4 + 2 + 2 + 3 + 2 \\ &= 18 \text{ holes} \end{aligned}$$



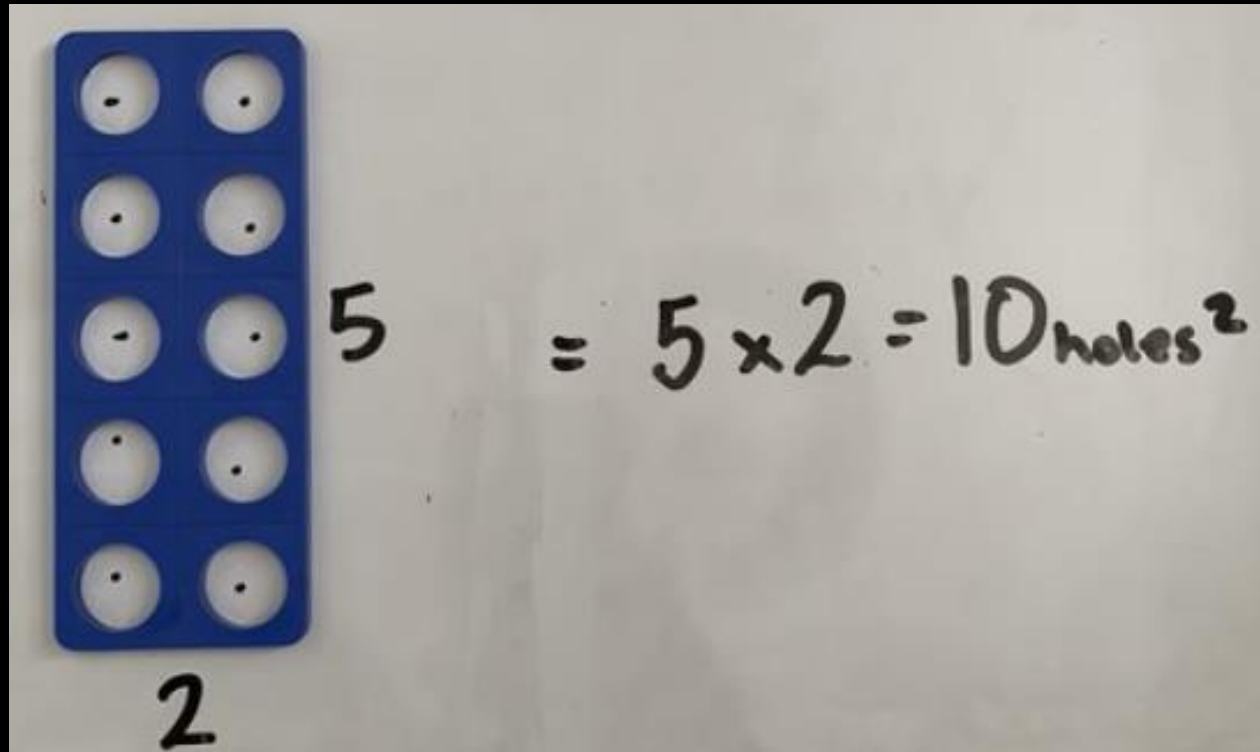
$$\begin{aligned} \text{Perimeter} &= 5 + 4 + 2 + 2 + 3 + 2 \\ &= 18 \text{ holes} \end{aligned}$$



$$\begin{aligned} \text{Perimeter} &= 5 + 4 + 2 + 3 + 2 + 2 \\ &= 18 \text{ holes} \end{aligned}$$

Perimeter (of rectilinear shapes)

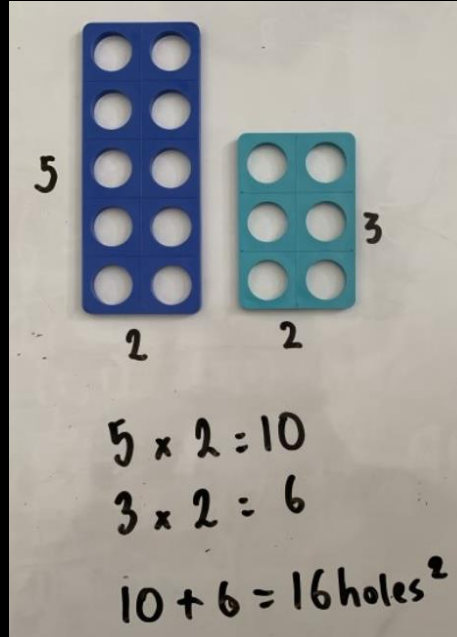
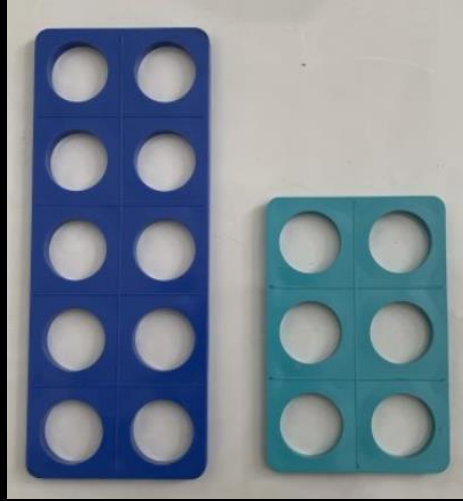
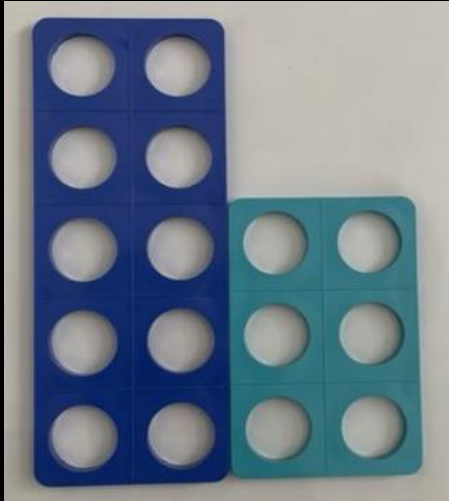
Children build on their understanding of the perimeter of rectangles by creating compound rectilinear shapes. There needs to be a clear understanding of counting the total distance around the outside of the shape. Children can explore building different shapes with the same perimeter.



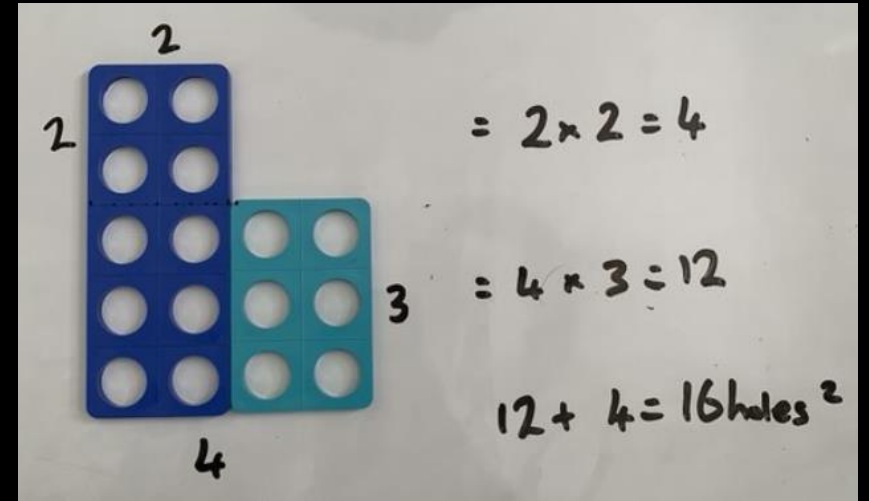
Area (of rectangles)

Children begin by exploring the area of rectangles. Children's first step is to "count the holes" to reinforce the concept that area is the space inside of a shape.

Children then apply their understanding of multiplication and arrays to generalize that the area of a rectangle $= L \times W$



Children may begin to explore the partitioning differently.



Area (of composite shapes)

Children build on their knowledge of area by combining rectangles to form composite shapes.

Children can partition these shapes back into rectangles in order to calculate the area of the entire shape.

$$\text{volume} = 1 \times 1 \times 1 = 1 \text{ cm}^3$$



$$\text{volume} = 3 \times 1 \times 1 = 3 \text{ cm}^3$$



$$\text{volume} = 3 \times 2 \times 1 = 6 \text{ cm}^3$$



$$V = 3 \times 2 \times 4 = 24 \text{ cm}^3$$



$$V = 2 \times 4 \times 3 = 24 \text{ cm}^3$$

$$2 \times 4 = 24 \text{ cm}^3$$



$$V = 3 \times 4 \times 2 = 24 \text{ cm}^3$$

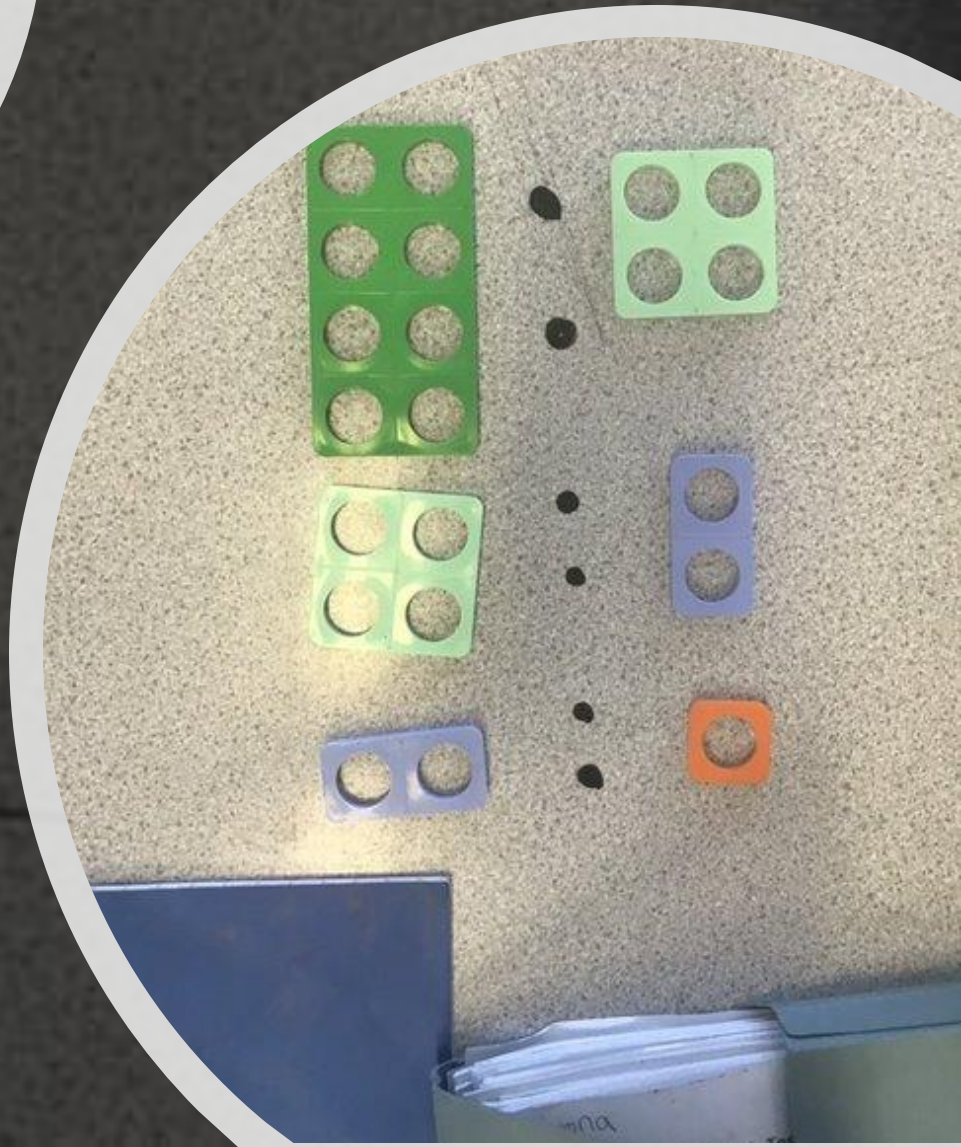
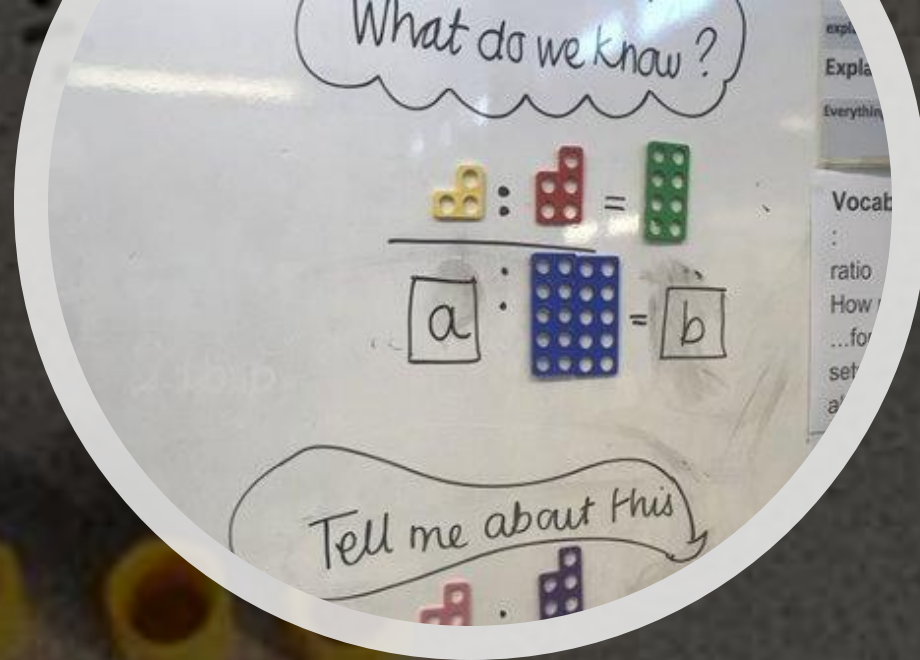
Volume

Children practise exploring how to calculate the volume of any cuboid.

Children build different cuboids to then generalize $V = W \times L \times H$

Children should articulate that orientation does not affect the volume of the shape.

Algebra, Ratio and Proportion



$$n^{\text{th}} \text{ term} = n + 3$$

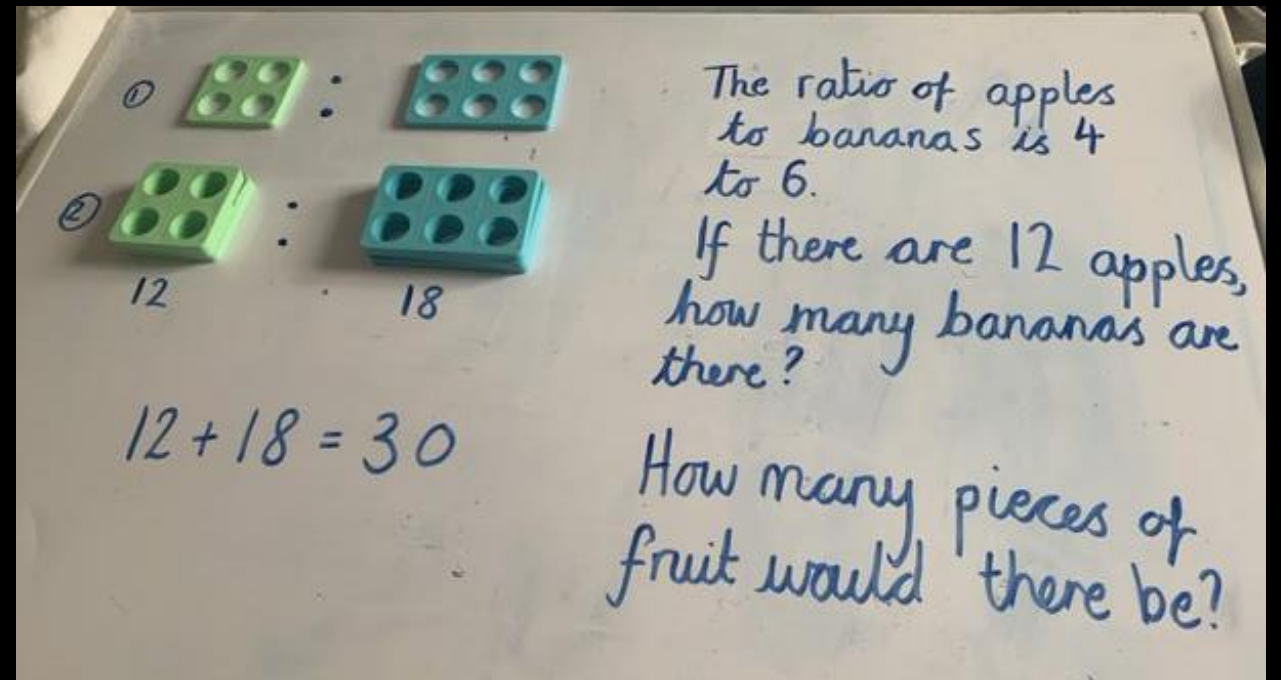
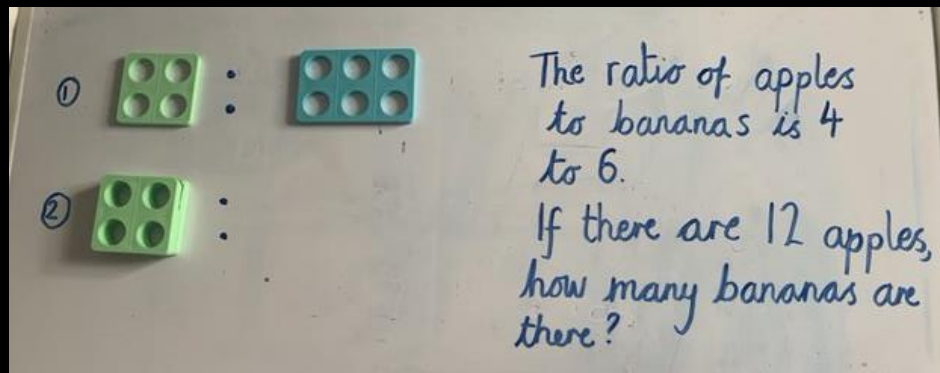


$$n^{\text{th}} \text{ term} = 2n + 3$$



Sequences

Children are given a sequence and explore what is constant with each term and what is changing. They can then create the next term in the sequence and finally determine the "nth term".



Ratio

Children use a stacking approach to calculate how many parts for a given ratio.



10 parts

$$10 \times \square = 50$$

$$10 \times 5 = 50$$



The ratio of apples to bananas is 4 to 6.

There are 50 pieces of fruit. How many bananas are there?

$$\text{Bananas} = 5 \times 6 = 30$$

$$\begin{aligned} \text{To check apples} &= 4 \times 5 = 20 \\ 20 + 30 &= 50 \end{aligned}$$



10 parts

$$10 \times \square = 50$$

$$10 \times 5 = 50$$



Ratio (part 2)

Children can then apply this method to more complex problems.

Children should articulate number of parts involved in a ratio (in the example $4 + 6 = 10$ parts) and use their knowledge of multiples to solve problems.

$$x + 2y = 20$$



x	y
8	6
2	9
4	8

17

$$x + 2y = 20$$

x and y are whole numbers **less than 10**

What could x and y be?

$x =$

$y =$


1 mark

Algebra

Children begin to explore alphabetical symbols and should start to articulate that multiples of the same letter (e.g. $3a$) is the same as $a + a + a$ where a is a constant.


$x + 2y = 20$ $x \text{ and } y < 10$

x	y	
18	1	x
9	$5\frac{1}{2}$	x



$x + 2y = 20$ $x \text{ and } y < 10$

x	y	
2	9	
10	5	?



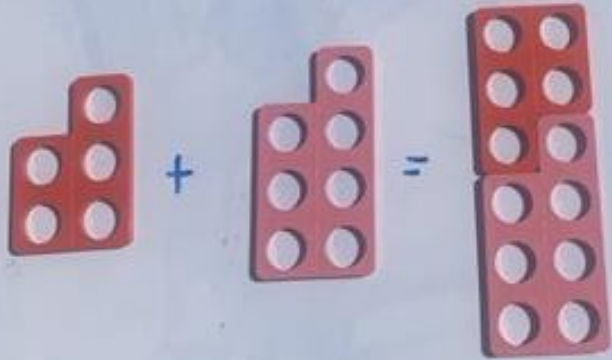
Algebra

Children should also have opportunity to explore non-examples as well as making conjectures, for example finding the smallest possible number for x , and answering probing questions such as: can x be odd?

Children are also encouraged to apply their place value knowledge in order to find examples which correspond with the conditions for x and y .

Always, sometimes, never?

odd + odd = even

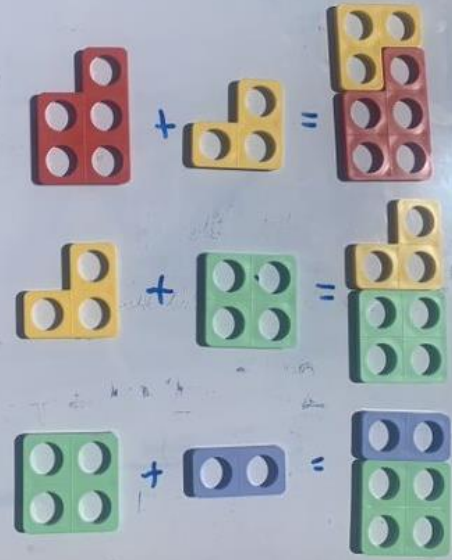


Always, sometimes, never?

odd + odd = even

odd + even = odd

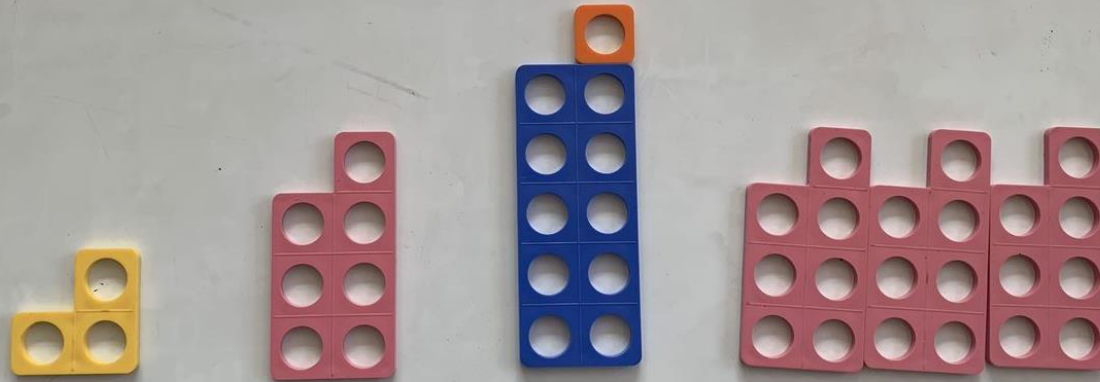
even + even = even



Investigations (LKS2)

Children can explore open ended tasks and begin to generalize and prove their thinking. Children can then take the learning further and ask further questions and specialize.

Always, sometimes, never



$$(n-4) + n + (n+4) = 3n$$

Investigations (UKS2)

With this example, as children test different examples, they should start to identify that the calculation $(-4 + 4)$ balances the equation. Children can then explore to create their own examples using this idea:
E.g. $(n-1) + n + (n+1) = 3n$