Numbers and the Number System

Comparing and ordering fractions





Educational context

This activity group develops work from Numbers and the Number System 2 and from the *Number, Pattern and Calculating 4 Teaching Resource Handbook.* Essentially, the activities are about using fractions to describe proportions, recognizing that a whole range of equivalent fractions can be used to describe the same proportion, and that proportions expressed as fractions can be ordered. Children are thus using fractions in these activities to develop their communicating about proportions, both within specific contexts and more generally. Later in Calculating 10, children will explore similar activities to help them make connections between proportion and ratio.

At the heart of this work, children are developing ways of recognizing and producing equivalent fractions. They then use this ability to compare fractions (or proportions) and order them in terms of size (magnitude).

These ideas are both important and challenging so children will need plenty of time, discussion and illustration to develop their communicating about fractions and proportions in these ways.

Learning opportunities

- To compare and order fractions whose denominators are all multiples of the same number.
- To use < and > signs to record the ordering of fractions.
- To simplify fractions to their lowest forms by finding common factors.
- To use equivalent fractions in context to scale up or down.

Words and terms for use in conversation

part-whole relationships, comparing, equivalent fractions, denominator, numerator, proportion, 'in every', unit fraction, proper fraction, improper fraction,

mixed fraction, factors, common factor, divisible by, multiple, times, divide, equivalence, scale up, reduce, simplest forms, common denominator, proper fraction, improper fraction, greater than, less than

Assessment opportunities

Look and listen for children who:

- Use the words and terms for use in conversation effectively.
- Explain comparisons between fractions whose denominators are multiples of the same number and notice the effect of odd and even numerators.
- Use knowledge of multiples to convert fractions into equivalent fractions and illustrate this with structured apparatus.
- Compare fractions and order them using < > symbols.
- Make connections between scaling up and multiplying with the inverse of scaling down and dividing.
- Use knowledge of multiples and factors to simplify fractions to their lowest forms.

Explorer Progress Book 5b, pp. 4–5

After completing work on this activity group, give small focus groups of children their Explorer Progress Books and ask them to work through the challenges on the pages. As children complete the pages, assess what progress they are making with the central ideas from the activity group. Refer to the assessment opportunities for assistance.

Explore More Copymaster 13: Who Gets More?

After completing work on Activity 3, give children Explore More Copymaster 13: Who Gets More? to take home.

Number, Pattern and Calculating 5 – Teaching Resource Handbook – Comparing and ordering fractions

Numbers and the Number System

Focus activities

Activity 1: Comparing and ordering proper fractions whose denominators are multiples of the same number

Have ready: Numicon Shapes, Numicon Coloured Counters, number rods, interlocking cubes

Step 1

Remind children about the lemonade activity from Numbers and the Number System 2, Activity 6. Ask them if they made two batches of lemonade, one with 2 cups of lemon juice in every 5 cups and the other with 3 cups of lemon juice in every 5 cups, which lemonade would taste stronger. Look and listen for children who can reason that having more lemon juice in the lemonade would make a stronger lemony taste. Help children to compare the different recipes with fractions and agree that we could write that one recipe has $\frac{2}{5}$ of lemon juice and the other has $\frac{3}{5}$.

Step 2

Give children some time to compare these fractions and allow plenty of time for them to make connections between the fractions and the strength of the drink. As they work, encourage them to illustrate these relationships on a number line (see Fig. 1).

Agree that to find the strongest lemonade they need to decide which fraction is closest on the number line to pure lemon juice. Help them to understand that 5 cups of lemon juice in every 5 cups makes a drink that is purely lemon juice and locate this on the number line as $\frac{5}{5}$ or 1 whole. Agree that $\frac{2}{5}$ is closer to pure lemon juice than $\frac{2}{5}$ so the recipe made with 3 cups of lemon juice in every 5 cups would be stronger than one made with 2 cups in every 5 cups.

Step 3

Next, ask children if we used 4 cups of lemon juice in every 5 cups of lemonade and 6 cups of lemon juice in every 10 cups to make two batches of lemonade whether they could write fractions to compare them. Help children to recognize that we could write $\frac{4}{5}$ and $\frac{6}{10}$ to represent the amount of lemonade and give them time to talk about these recipes and fractions. Look and listen for children who can explain that these fractions are both more than a half but it is not so easy to compare these fractions directly because they have different denominators.



Step 4

Tell children to use, draw or write anything that might help them to compare these fractions. Look and listen for children trying out their own ideas, before asking them if finding equivalent fractions or using apparatus and number lines might be useful.

Step 5

Encourage children to explore how they could represent $\frac{4}{5}$ and $\frac{6}{10}$ on number lines or with apparatus. Look and listen for children who draw number lines which are the same length, dividing one line into fifths and the other into tenths (see **Fig. 2**).

Step 6

Show children how a double number line (see Fig. 3) with fifths at the top and tenths at the bottom can be useful when comparing these fractions and agree that we can see that $\frac{4}{5}$ is equivalent to $\frac{8}{10}$ and that this is closer to 1 than $\frac{6}{10}$.

Step 7

Help children also to use Shapes and Counters or rods to illustrate that $\frac{4}{5}$ is equivalent to $\frac{8}{10}$, e.g. Fig.4.

Look and listen for children who can now explain that if a recipe has $\frac{6}{10}$ or $\frac{8}{10}$ of lemon juice the one with $\frac{8}{10}$ or 8 cups in every 10 cups will have a stronger taste of lemon.

Step 8

Repeat the activity with other pairs of fractions whose denominators are multiples of the same number, e.g. $\frac{2}{3}$ and $\frac{5}{6}$. Look and listen for children who realize that they can compare the fractions as the denominators are multiples of the same number.

Step 9

Allow plenty of time for children to explore these ideas with pairs of fractions with common denominators. Include examples like $\frac{4}{6}$ and $\frac{4}{12}$ or $\frac{3}{4}$ and $\frac{6}{8}$ and encourage children to explain that they can sometimes compare fractions with common denominators in two ways, e.g. for $\frac{4}{6}$ and $\frac{4}{12}$ we can compare these as sixths ($\frac{4}{6}$ and $\frac{2}{6}$) and twelfths ($\frac{8}{12}$ and $\frac{4}{12}$) but this would not be the case for $\frac{4}{6}$ and $\frac{5}{12}$. Help children explain this with reference to odd and even numerators.



6

Activity 2: Comparing and ordering proper fractions in a different context

Have ready: Numicon Shapes, Numicon Coloured Counters, number rods

Step 1

Explain to children that during a music lesson with percussion instruments, a group of children have been composing some rhythms. One child plays their instrument for the first 2 beats out of every 10, another plays the first 3 beats out of every 4 and a third plays the first 4 beats out of every 5. The children all count at the same speed and start at the same time.

Step 2

Practise counting and clapping the beats that each child has chosen to play and then ask children if they can think of a way to illustrate these on paper to see which children are playing their instruments at the same time. Explore children's ideas and help them to draw number lines to show the rhythms, using letters to represent the different children, e.g. Fig. 5 or different coloured music notes or Counters, e.g. Fig. 6.

Step 3

Ask children which instrument is played the most often and look and listen for children who can reason that the child who plays their instrument 4 beats in every 5 plays the most often. Next ask children if they can write the number of beats played by each child as a fraction of beats each child counts. Look and listen for children who can write the fractions $\frac{2}{10}$, $\frac{3}{4}$ and $\frac{4}{5}$.

Step 4

Ask children to talk about which of these fractions is the largest or smallest and how this relates to the frequency of beats played in the composition. Look and listen for children who can remember that a good way to compare fractions is to convert them into equivalent fractions with a common denominator and to illustrate these on a number line.

Step 5

Ask children to look closely at the fractions $\frac{2}{10}$, $\frac{3}{4}$ and $\frac{4}{5}$ and to talk about how they can convert these fractions into equivalent fractions with a common denominator. Give children the chance to explore these fractions with rods or Shapes and to write equivalent fraction families, e.g. Fig. 7. Look and listen for children explaining for themselves that all these fractions can be written as a fraction of 20, because 20 is a common multiple of 10, 4 and 5.

Step 6

Some children may choose to use Shapes to help them write equivalent fraction families, e.g. Fig. 8. Use Shapes and work with children to illustrate the equivalence by placing three Counters over every 4-shape and encourage children to talk about the proportion of Counters by helping them use the language 'in every' as they make further links between quarters, tens and fifths, e.g. 'There are three Counters over every four holes.'

Step 7

Using the illustrations for support, help children to agree that we can write our fractions as $\frac{4}{20}$, $\frac{15}{20}$ and $\frac{16}{20}$ and that $\frac{16}{20}$ would be the furthest along the number line and closest to one whole, so this fraction represents the player who is playing the most beats. Ask children to order all the fractions.

Step 8

Repeat the discussion from steps 1–7 with various combinations from the following list: $\frac{1}{2}$, $\frac{4}{10}$, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{3}{10}$, $\frac{4}{5}$, $\frac{3}{4}$.



Focus activities

Activity 3: Using greater than and less than signs to record comparisons

Have ready: Numicon 0–100 Numeral Cards,

Numicon Shapes, Explore More Copymaster 13: Who Gets More? Step 1

Give children 6–24 Numeral Cards and ask them to find two Numeral Cards that show multiples of the same number.

Look and listen for children selecting, e.g.

6 and 12 and explaining that the numbers are both multiples of 2, 3 and 6.

Step 2

Record 6 and 12 as denominators and choose two smaller numbers as the numerators, to show two proper fractions, e.g. 2/6 and 5/12

Step 3

Ask children which fraction is greater. Agree that to check this we could convert these fractions to equivalent fractions with common denominators.

Look and listen for children who suggest

Converting 2/6 to 4/12 by multiplying both the denominator and the numerator by 2. See image.



Agree that 5/12 is greater than 4/12.

Work with children to record this using greater than and less than symbols:

5/12 > 4/12 or 4/12 < 5/12. Agree that this also means 5/12 > 2/6 and 2/6 is < 5/12

<u>Step 4</u>

Ask children whether they could also convert 2/6 and 5/12 to equivalents with common denominators by dividing. Look and listen for children who can explain that both denominators could be converted to 2, 3 or 6 by dividing (e.g. $6 \div 2 = 3$ and $12 \div 4 = 3$); agree that these are all common factors of both denominators. Help children to explain that the conversion is not possible, however, since in each case the numerators are not also divisible by the same number (e.g. $2 \div 2 = 1$, so 2/6 = 1/3, but 5 is not divisible by 4, so it is not possible to convert 5/12 to thirds).

<u>Step 5</u>

Ask children if there are any other common denominators that would work.

Agree that both fractions can be converted to twenty-fourths by doubling both parts of 5/2 and multiplying both parts by 4. Some children may recognize that there is an infinite number of possibilities, using multiplying (thirty-sixths, forty-eighths, ...).

<u>Step 6</u>

Repeat Steps 1–5 with other pairs of fractions whose denominators are multiples of the same number, e.g. 3/5 and 4/10, 4/5 and 12/20. If appropriate, extend to other pairs of fractions,

Eg. ¾ and 4/5. (which children encountered in Activity 2).

Look and listen for children recognizing that both quarters and fifths can be converted to twentieths.

Ask them to explain, illustrating with Shapes.

Extend the activity further by encouraging children to suggest a method for finding the lowest common denominator (LCD; suggestions might include listing the multiples of each denominator, then looking for the lowest number appearing in both lists).

After completing work on this activity, give children the opportunity to take home and complete *Explore More Copymaster 13: Who Gets More* ? This will help children to understand the sizes of different fractions and the relationships between them.

Paired work Have ready: a list of fractions with denominators that are multiples of the same number, e.g.



Children take turns to pick two fractions for their partner to write a 'greater than' or 'less than' statement. The child who chose the fractions checks the statement. Numicon Student Book 5 pp. 50–53 Numicon Student Book 5 Answer Book p. 32

Activity 4: Simplifying fractions by finding common factors

Have ready: Numicon Shapes or number rods Numicon 10s Number Line Laminate labelled with multiples of 10 or Numicon 1–100 cm Number Rod Track

<u>Step 1</u>

Remind children that in Numbers and the Number System 2, Activity 6 they used equivalent fractions to scale up a recipe for lemonade, e.g. if 1 cup of lemon juice and 4 cups of water make one 5-cup jug of lemonade, then 2 cups of lemon juice and 8 cups of water make two 5-cup jugs.

Work with children to write the proportions of lemon juice and water as fractions of a whole and record them as adding calculations:

$$\frac{1}{5} + \frac{4}{5} = I \text{ and } \frac{2}{10} + \frac{8}{10} = I.$$

<u>Step 2</u>

Ask children how many cups of lemonade a recipe would make if it asked for 9 cups of lemon juice and 36 cups of water. Agree 45 cups, because 9 + 36 = 45. Work with children to write the adding calculation with fractions: 9/45 + 36/45 = 1.

<u>Step 3</u>

Ask children whether we could make less lemonade, but with the same strength.

Give children plenty of time to think about this for themselves. Look and listen for those who use their knowledge of inverses to suggest dividing to scale the recipe down instead of multiplying to scale up.

Step 4

Look at the fractions for the amounts of lemon juice and water: 9/45 + 36/45 = 1.

Encourage children to talk about the relationships between the numerators and denominators.

Help children reason that because the numerators and the denominators are multiples of 9 they are all divisible by 9. Confirm by illustrating with Shapes or rods on the 10s Number Line Laminate or the Number Rod Track (see image).

9 × 4 = 36 36 ÷ 9 = 4

Numbers and the Number System

6

Step 5

Agree that the recipe made with $\frac{9}{45}$ of lemon juice can be reduced to $\frac{1}{5}$ of lemon juice by dividing the numerator and denominator by 9 and in the same way $\frac{36}{45}$ of water can be reduced to $\frac{4}{5}$ of water by dividing by 9. These reduced recipes will have the same lemon strength. Help children to realize that the fractions of lemon juice and water in this recipe can be written in many different ways but $\frac{1}{5}$ of lemon juice and $\frac{4}{5}$ of water are the fractions that represent this recipe in its simplest form. This means it cannot be reduced any further.

Step 6

Repeat this discussion with a new recipe using $\frac{14}{49}$ of lemon juice and $\frac{35}{49}$ of water.

Activity 5: Simplifying fractions to their lowest terms

Have ready: number rods

Step 1

Talk to children about fractions out of context, e.g. $\frac{12}{20}$ and ask them to write some equivalent fractions. Look and listen for children who can write a list of equivalent fractions for $\frac{12}{20}$ by scaling up e.g. $\frac{24}{40}$, $\frac{36}{60}$ and also those who can scale down by halving, e.g. $\frac{6}{10}$ and $\frac{3}{5}$, or by looking for a common factor.

Step 2

Help children to look for a common factor in 12 and 20. Use rods to build and compare fraction walls for 12 and 20, e.g. Fig. 11.

From looking at the fraction walls agree that 1, 2 and 4 are common factors of 12 and 20. Help children to work with both 2 and 4 to simplify the fraction and to realize that if they choose the largest common factor 4, they will find the simplest form of the fraction.

Step 3

Help children to divide $\frac{12}{20}$ by 4 to get $\frac{3}{5}$ and to illustrate this with rods, e.g. Fig. 12.

Step 4

Repeat with fractions like $\frac{4}{10}$, $\frac{6}{15}$, $\frac{9}{27}$, $\frac{8}{12}$, $\frac{10}{14}$, $\frac{12}{26}$, $\frac{10}{15}$. Work with children to use rods to find the largest common factors and to reduce these fractions to their simplest forms.



Practice and discussion

Whole-class

- Discuss with children how and when the mathematics they have been learning could help them in solving problems.
- Ask children to write scores using fraction notation and compare them, e.g. which is the better score, ⁸/₁₀ or ¹⁷/₂₀? Talk about sports results, e.g. which team is doing better, Team A or Team B, if Team A has won ¹²/₁₅ games and Team B has won ⁹/₁₂ games?
- Ask children to use the > and < symbols to show the relationships between pairs of fractions with denominators that are multiples of the same number (e.g. $\frac{2}{8}$ and $\frac{4}{12}$).
- Select a set of fraction cards where the denominators are multiples of the same number and play higher than/lower than games.
- Ask children to find equivalent fractions from a list where the denominators are all multiples of the same number.
- Give fraction statements with missing numerators or denominators for children to complete, e.g. ⁵/₈ > [□]/₂₄ or ³/₉ > ⁴/_□.
- Remind children about multiples and factors. Ask them to count in multiples and to find all the factors of any given number.
- Show children two numbers and talk with them about common factors, e.g. 24 and 32.
- Continue a list or family of equivalent fractions following a pattern, e.g. ³/₅, ⁶/₁₀, ⁹/₁₅ and simplify fractions, e.g. ¹⁵/₃₅.

Independent

Paired work for Activities 1 and 2

Have ready: Numicon 0-100 Numeral cards

Children take turns to make two proper fractions using 2–24 Numeral Cards, making sure the denominators are multiples of the same number, e.g. $\frac{3}{4}$ and $\frac{7}{12}$. Compare these using a double number line.

Individual work for Activities 1 and 2

Have ready: Pairs of fractions with denominators that are multiples of the same number, e.g. $\frac{4}{6}$ and $\frac{9}{18}$

Ask children to select a pair of fractions and compare these fractions on a double number line.

Explore More – extension and practice for school and home

30

Numicon – Number, Pattern and Calculating 5

Numbers and the Number System 6, Comparing and ordering fractions

.....

Who Gets More?

How this will help your child

- This activity will allow your child to understand the size of different fractions and the relationship between them.
- It will also help them to describe parts of a whole as fractions.

Words and phrases to use

equal, greater than (>), less than (<), more, bigger, less, smaller, fraction, half, third, quarter, sixth, eighth, twelfth, larger than (>), smaller than (<)

You will need

Scissors

During the activity, look at what your child can do

.....

- Find, write and say fractions that are parts of a whole.
- Compare fraction amounts.
- Recognize equivalent fractions.

What to do

- Cut out the fraction strips, fraction and symbol cards and game board from the Who Gets More? sheet. The fraction strips will all be the same length when cut out.
- Shuffle the fraction cards and share them out between you and your child. Put a pile, face down, in front of each player.
- Explain to your child that the cards show different fractions.
- Ask your child to take the first card from the top of their pile, e.g. $\frac{1}{3}$, and to say the name of the fraction on the card, e.g. 'one-third'. Ask them to place the card on the shaded box on the left-hand side of the game board.
- You then select your first fraction card, e.g. $\frac{4}{8}$. Read the name of the fraction out to your child, e.g. 'four-eighths', and place the card on the right-hand side of the game board.
- Ask your child to say which fraction is the largest or smallest, or if they are equal. Ask them to place the correct symbol on the game board between the fraction cards to show this. Remind them that the wide part of the symbol points at the larger amount.
- Check that the correct symbol has been used by folding the fraction strips to show each amount and then comparing them.
- The player whose fraction represents the most, wins the two cards. If the fractions are equal, these cards are put aside and are out of the game.
- Keep taking turns to choose a card first until all the cards have been used up. The player with the most fraction cards is the winner.

Next steps ...

2

- Try playing the game without using the fraction strips.
- Create your own fraction strip and cards of different fractions ($\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{10}$, etc.) to extend the game.
- Talk about and compare fractions in everyday life, e.g. when cutting up fruit, or sharing out food between people.

© Oxford University Press 2015. This page can be copied for use in the purchasing school.



Who Gets More?



Number, Pattern and Calculating 5

Numbers and the Number System 6, Comparing and ordering fractions

Student Book 5

Numbers and the Number System 6·1



Numbers and the Number System 6.3

Using 'greater than' and 'less than' signs



Practice

- 1 Choose five pairs of fractions from the list and show which is the smaller fraction in each pair by using the '<' sign.
- 2 Choose five different pairs of fractions from the list, and show which fraction is greater in each pair by using the '>' sign.
- 3 Can you put all the fractions in the list in order, beginning with the smallest? Can you explain to your partner how you did this?

.....

Going deeper

52

- 1 Using the numbers 5 and 9 only once, can you make this statement true?
- 2 Using the numbers 5, 6 and 9 only once, can you make this statement true? Is there another way to do this?



Common denominators



Practice

1 If the piece of music is 36 beats long, who will have played their instrument the most times? Who will have played the fewest times? Can you explain the quickest way to work out these answers?

2 Choose three of the fractions below and put them in order of their size, beginning with the smallest.

| 2 | 3 | 3 | 5 | 7 | 8 |
|---|---|---|---|----|----|
| 3 | 8 | 4 | 9 | 12 | 15 |

3 Choose a different set of three fractions from the same list and put them in order of size, beginning with the largest. Can you show these in position on a number line?

Going deeper

1 Which of the fractions in the list above is closest to 1?

- 2 Look at the list of fractions above. a How many different sets of three fractions can you make from
 - the list? How do you know you have found all the possible sets?
 - **b** Which set of three fractions gives the highest total when you add them together?
- 3 Dev has dug $\frac{1}{4}$ of a vegetable patch, Aroha has dug $\frac{1}{3}$ and Nina has dug $\frac{3}{8}$ of the patch. Who has dug the most? Who has dug the least? How much of the vegetable patch still has to be dug?

Numbers and the Number System 6.4

51

Simplifying proper fractions



Practice

- 1 The lemonade above was made from 15 cups of lemon juice and 25 cups of water. What proportion of the lemonade is lemon juice and what proportion is water? Can you simplify your fractions to their lowest terms?
- 2 Can you write six different fractions that are all equivalent to $\frac{3}{5}$?
- 3 Simplify each of the fractions below to its lowest terms.



.....

Going deeper

- 1 Can you simplify $\frac{48}{53}$? Can you explain your answer?
- $2\,$ What is the closest fraction to $\frac{48}{53}$ that you could simplify? Can you explain why you think this is the closest fraction?
- 3 What advice would you give to someone who wants to know how to simplify fractions in the quickest way?

Explorer Progress Book 5b, pages 4–5

53

Numbers and the Number System 6-2

Assessment Cards for students to reflect on their learning



| NPC Milestone 3 | Use knowledge of factors and multiples to find equivalent fractions and to simplify fractions to their lowest terms | | |
|-----------------|---|---------|--|
| | Compare and order fractions with denominators which are multiples of the same number | NPC5:3b | |
| | Use the inverse relationships between adding and subtracting, and multiplying and dividing, to complete calculations with missing numbers | NPC5:3c | |
| | Use efficient written column methods for adding and subtracting whole numbers up to 10 000 and decimals with up to 3 decimal places | NPC5:3d | |
| | Use known multiplying facts to multiply and divide whole numbers and decimals by 10, 100, and 1000 | NPC5:3e | |

Explorer Progress – an assessment tool

Numbers and the Number System 6: Comparing and ordering fractions



Football Matches

During a local football season: Team Appleton wins 2 out of every 3 matches. Team Marlow wins 3 out of every 6 matches. Team Tilbury wins 5 out of every 9 matches. Team Duckfield wins 7 out of every 12 matches. Can you put the results in order, according to which team is doing the best?



Teacher notes



Teacher notes