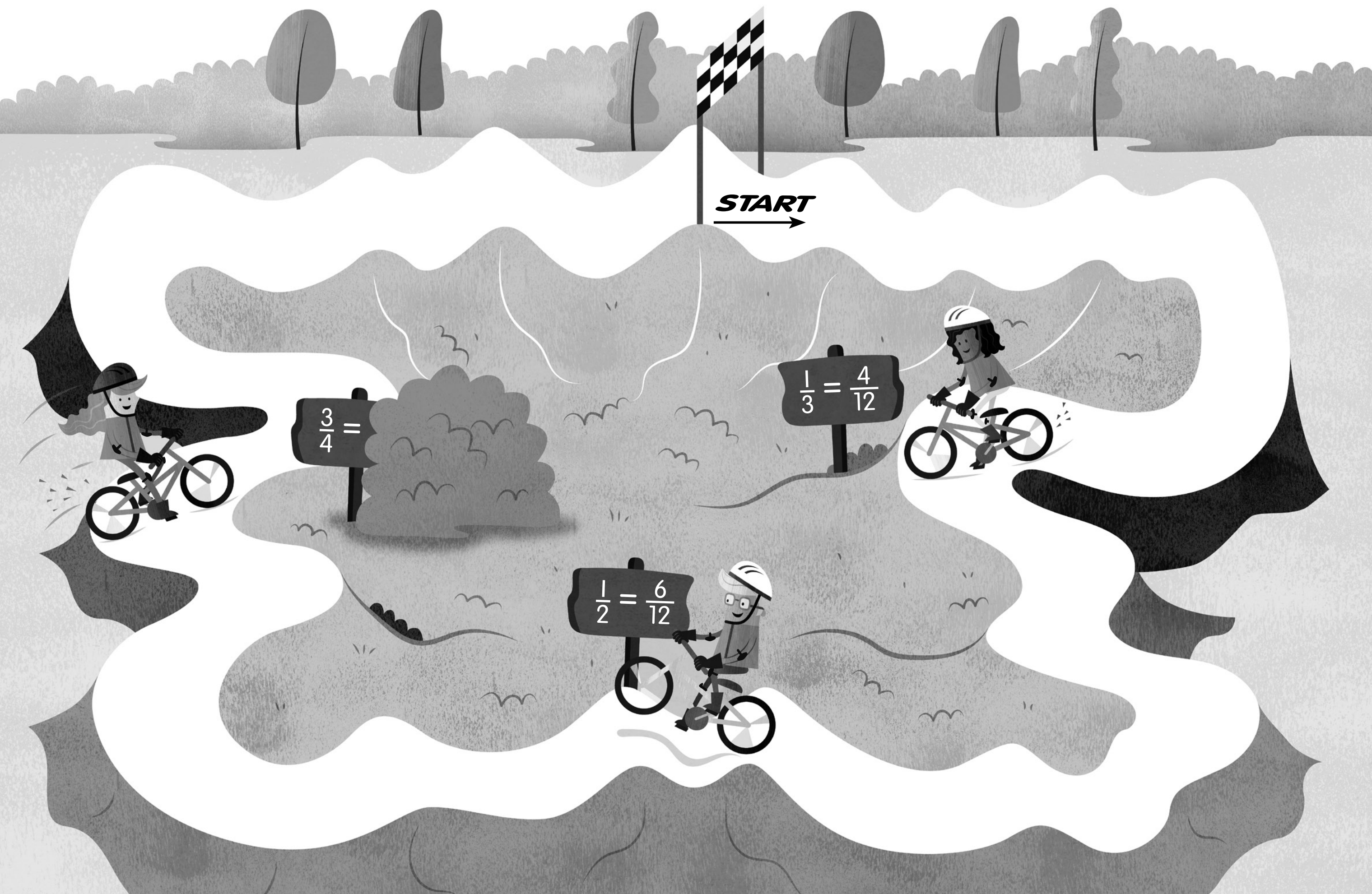
numicon

Pupil Book 6 Answers



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www.oxfordprimary.co.uk/numicon

About Numicon

Numicon is a distinctive multi-sensory approach to children's mathematical learning that emphasizes three key aspects of doing mathematics: communicating mathematically, exploring relationships and generalizing.

Numicon was founded in the daily experience of intelligent children having real difficulty with maths, the frequent underestimation of the complexity of the ideas that young children are asked to face when doing maths and recognition of the importance of maths to them and to society as a whole.

Numicon aims to facilitate children's understanding and enjoyment of maths by using structured imagery that plays to children's strong sense of pattern. This is done through research-based, multi-sensory teaching activities.

Numicon takes into account the complexity of abstract number ideas and seeks to foster the self-belief necessary to achieve in the face of challenge or difficulty.

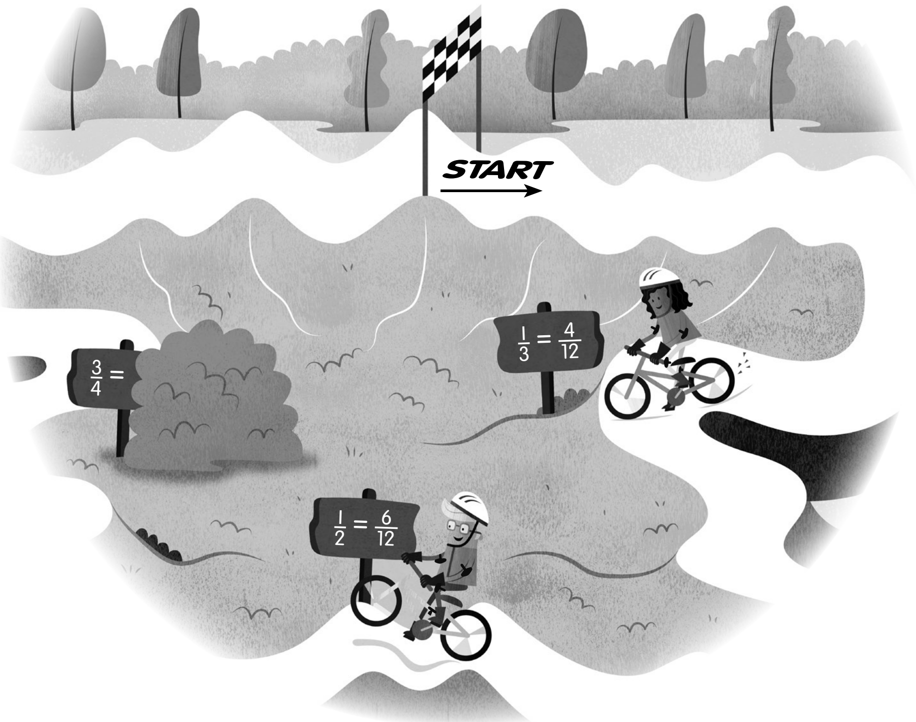
Through the combination of communicating mathematically (being active, talking and illustrating), exploring relationships and generalizing, children are given the support to structure their experiences: a vital skill for both their mathematical and their overall development.

A multi-sensory approach, particularly one that makes use of structured imagery, provides learners with the opportunity to play to their strengths, thereby releasing their potential to enjoy, understand and achieve in maths. By watching and listening to what children do and say, this enjoyment in achievement is also shared by teachers and parents.

Numicon strives to support teachers' subject knowledge and pedagogy by providing teaching materials, Professional Development and on-going support that will help develop a better understanding of how to encourage all learners in the vital early stages of their own mathematical journey.

Pupil Book 6 Answers

Written by Jayne Campling, Andrew Jeffrey,
Adella Osborne and Dr Tony Wing



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Contents

Using Numicon Pupil Books

Introduction

The Numicon Pupil Books have been created to help children develop mastery of the mathematics set out in Numicon Teaching Resource Handbook (TRH) activities. The questions in the Pupil Books extend children's experiences of live TRH activities, giving them the opportunity to reason and apply what they have learned, deepen their understanding, take on challenges and develop greater fluency.

Just like the teaching activities, all Pupil Book pages are designed to stimulate discussion, reasoning, and rich mathematical communicating. The Numicon approach to teaching mathematics is about dialogue. It is about encouraging children to communicate mathematically using the full range of mathematical imagery, terminology, conventions and symbols.

All questions in the Pupil Books relate to specific Numicon TRH activities. At the top of each Pupil Book page you can find details of the Activity Group the page relates to (for example Calculating 1). The number after the decimal point tells you which focus activities the page accompanies (so Calculating 1·2 goes with focus activity 2). It is crucial to teach the relevant focus activities before children work on the questions. The Pupil Book questions are designed for children who are succeeding with specific TRH activities, and will invite them to think more deeply about a topic. If you find that children are struggling with a focus activity, details are given in the Teaching Resource Handbooks of other live activities, provided earlier in the progression, which you can work through together to support them until they are ready to move on.

There's a recommended order to teach the Activity Groups in and the Pupil Book materials follow this order of progression too, as you'll see from the contents page. You can use this order to help children see how their ideas and understanding builds upon what they have learned before.

These Pupil Book questions have been developed as a large bank that you can select from to best meet the changing needs of the children in your class. You can decide which questions are suitable for which children at which time, and no child is expected to find every question useful. How you choose to use the questions might also vary, for example you may find that particular questions are useful to discuss and work through together as a class.

Intelligent practice

The 'Practice' sections target two areas. Routine practice is used to promote fluency with particular aspects or techniques. Non-routine practice questions offer challenges in varied ways designed both to improve fluency and to deepen and extend understanding. Practice for simple fluency usually comes first and the questions on each page become progressively more challenging.

Going deeper

'Going deeper' questions are designed to develop children's growing mastery of an area, challenging their understanding beyond routine exercises. In these sections children are commonly asked to check, explain and justify their strategies and thinking. Trying to explain something clearly helps promote, and is a key indicator of, developing mastery.

Using the Pupil Books

Doing mathematics involves much more than logic, and children's emotions are crucially important. Thoughtful progress is more likely to happen through encouraging curiosity and good humour, and engaging with children in a polite and calm way. This is why the phrasing and tone of Numicon Pupil Book questions is deliberately different to many mathematical textbooks. For example, we often begin questions for children with "Can you…?" If any child says simply "yes" or "no" in response, we'd suggest replying with "Can you show me how …?" or "That's interesting, can you say anything about why not?" These invitations are effective beginnings to the kinds of open conversation and discussions that are at the heart of the Numicon approach.

Some Pupil Book questions have a pair work symbol to signal that these require specific work with a partner, and help with classroom management. These are not the only questions where working with a partner is likely to be beneficial however. All Pupil Book questions should be seen as opportunities for rich mathematical communicating between anyone and everyone in the classroom at all times, and this should be actively encouraged wherever you think appropriate. The Numicon approach is crucially about dialogue – action, imagery and conversation.

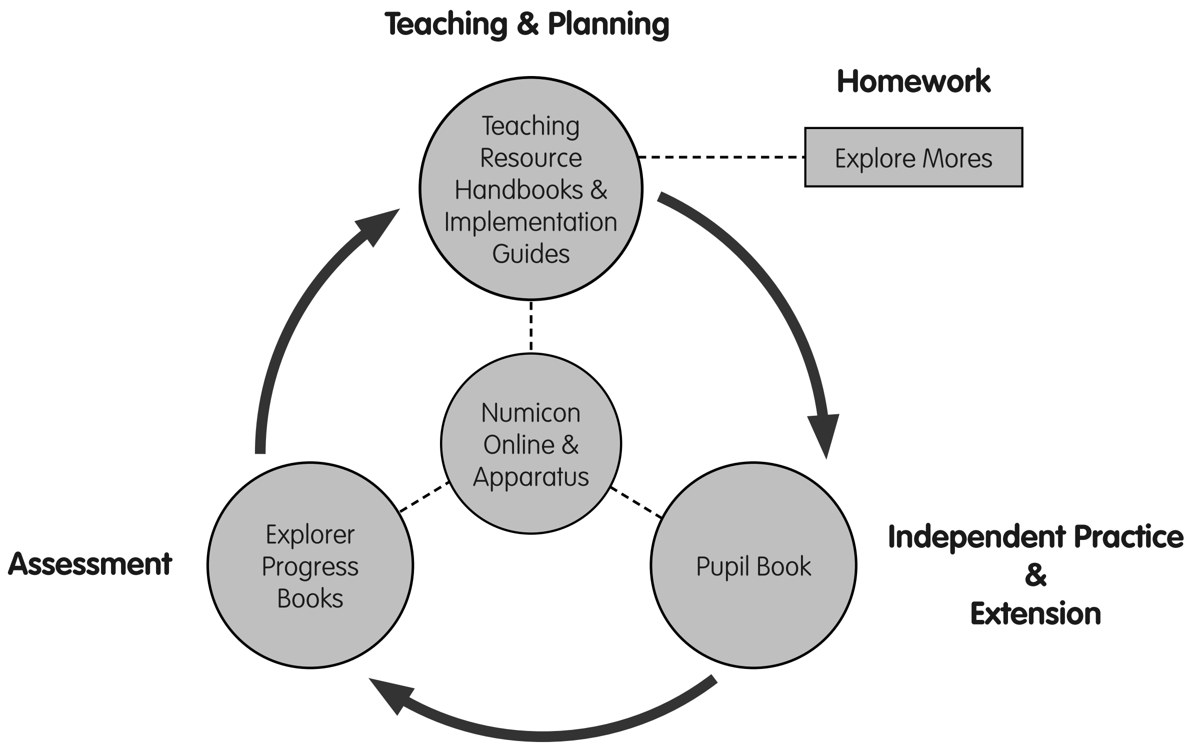
Finally, the Pupil Book questions are there to be enjoyed. Children who are supported, and who are succeeding, generally relish challenge and further difficulty. We hope you as teachers will also enjoy the journeys and pathways that these books will take children and their teachers jointly along.

Dr Tony Wing

A guide to the Numicon teaching resources

Numicon Pupil Books fit with the other resources shown here to fully support your teaching. You can also find additional resources, including an electronic copy of this answer book, on Numicon Online. This is available on the Oxford Owl website (www.oxfordowl.co.uk).

Numicon resources



Teaching Resource Handbooks

There is a Number, Pattern and Calculating and a Geometry, Measurement and Statistics Teaching Resource Handbook for each year group. The teaching in these handbooks is carried out through activities. You will find detailed support for planning and assessment here, along with vocabulary lists, the key mathematical ideas covered and photocopy masters.

Implementation Guides

Each Teaching Resource Handbook comes with an Implementation Guide. These provide guidance on the Numicon approach, how to implement this in the classroom and valuable information to support subject knowledge, including explanations of the key mathematical ideas covered and a glossary of mathematical terms used.

Explore More Copymasters

The Explore More Copymasters provide homework that enables children to practise what they are learning in school. For Geometry, Measurement and Statistics these are given in the back of the Teaching Resource Handbook. For Number, Pattern and Calculating these are provided in a separate book.

A homework activity is included for every Activity Group. Each one includes information for the parent or carer on the mathematics that has been learned in class beforehand and how to use the work together. These activities can also be used in school to provide extra practice.

Explorer Progress Books

There are four Explorer Progress books for each year group (one for Geometry, Measurement and Statistics and three for Number, Pattern and Calculating). There are two pages in the Explorer Progress Books for each Activity Group which can be used to assess children's progress, either immediately after the Pupil Book questions or at a later point to find out what learning has been retained. These progress books give children the opportunity to apply what they have learned to a new situation.

Apparatus

Physical apparatus	Apparatus on the Interactive Whiteboard Software

A wide range of apparatus and structured imagery is used in Numicon to enable children to explore abstract mathematical ideas. You can find digital versions of this apparatus in the Interactive Whiteboard Software available through Numicon Online. Here you can manipulate the apparatus from the front of the class and save anything you have set up for future use.

Numicon Online for planning and assessment support

Many other resources are provided on Numicon Online to support your planning, teaching and assessment. There are editable planning documents, photocopy masters and videos to support teaching. Assessment resources here include assessment grids for the Explorer Progress Books and milestone tracking charts to monitor children's progress throughout the year. You can access all these resources, along with the Interactive Whiteboard Software, through the Oxford Owl website (www.oxfordowl.co.uk).

Planning chart

The chart below shows you how the Activity Groups in the Teaching Resource Handbooks and the Pupil Book pages fit together and the key learning that is covered. The order follows the recommended teaching progression.

Key to abbreviations used on the chart
NPC: *Number, Pattern and Calculating Teaching Resource Handbook*
GMS: *Geometry, Measurement and Statistics Teaching Resource Handbook*

NNS: Numbers and the Number System
Geo: Geometry
Calc: Calculating
PA: Pattern and Algebra
Mea: Measurement

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
NNS 1: Working with numbers beyond a million and decimals (Number, Pattern and Calculating 6, pages 72–77)	**pp. 2–5**	**NPC Milestone 1** • Understand the value of each digit in large numbers up to ten million and numbers with up to 3 decimal places • Order numbers with up to 8 digits and position them on a number line
Calc 1: Adding and subtracting negative numbers in context, and large numbers (Number, Pattern and Calculating 6, pages 88–95)	**pp. 6–9**	**NPC Milestone 1** • Use different approaches to add and subtract negative numbers in context
Calc 2: Multiplying and dividing (Number, Pattern and Calculating 6, pages 96–105)	**pp. 10–13**	**NPC Milestone 1** • Use appropriate mental methods to add, subtract, multiply and divide increasingly large numbers
Mea 1: Statistics, charts and graphs (Geometry, Measurement and Statistics 6, pages 53–63)	**pp. 14–17**	**GMS Milestone 1** • Calculate the mean average of a set of data • Create, use and interpret conversion graphs • Convert between metric and imperial speeds • Construct and interpret pie charts to solve problems
PA 1: Multiples, factors and primes (Number, Pattern and Calculating 6, pages 34–42)	**pp. 18–21**	**NPC Milestone 2** • Identify common factors, common multiples and prime numbers
NNS 2: Fractions (Number, Pattern and Calculating 6, pages 78–85)	**pp. 22–25**	**NPC Milestone 2** • Compare and order fractions by expressing them as equivalent fractions with a common denominator
Calc 3: Estimating, rounding and equivalence (Number, Pattern and Calculating 6, pages 106–114)	**pp. 26–29**	**NPC Milestone 2** • Use estimation to check answers to calculations

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
Calc 4: Column methods for adding and subtracting (Number, Pattern and Calculating 6, pages 115–123)	**pp. 30–33**	**NPC Milestone 2** • Use column methods of adding and subtracting for larger numbers and decimals
Calc 5: Percentages (Number, Pattern and Calculating 6, pages 124–132)	**pp. 34–37**	**NPC Milestone 2** • Understand, recall and use equivalences between simple fractions, decimals and percentages
Geo 1: 2D shapes and angles (Geometry, Measurement and Statistics 6, pages 25–34)	**pp. 38–41**	**GMS Milestone 1** • Use formal notation to denote parallel, perpendicular and equal length lines in geometric diagrams • Recognize and classify a wide range of 2D shapes based on their properties • Calculate missing angles in polygons, along straight lines, around a point and that are vertically opposite • Construct triangles and other polygons from given properties
Calc 6: Exploring calculations: multi-step, non-routine problems and order of operations (Number, Pattern and Calculating 6, pages 133–140)	**pp. 42–45**	**NPC Milestone 3** • Use the BODMAS convention for order of operations to solve problems
Calc 7: Ratio and proportion (Number, Pattern and Calculating 6, pages 141–149)	**pp. 46–49**	**NPC Milestone 3** • Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples • Recognize proportionality in contexts when the relations between quantities are in the same ratio
Mea 2: Areas of 2D shapes (Geometry, Measurement and Statistics 6, pages 64–72)	**pp. 50–53**	**GMS Milestone 2** • Use formulae to find the area of triangles (area = $\frac{1}{2} \times b \times h$) and parallelograms (area = $b \times h$) and understand why they work • Find the area of composite shapes by partitioning into triangles and/or rectangles
Calc 8: Converting fractions and decimals (Number, Pattern and Calculating 6, pages 150–156)	**pp. 54–57**	**NPC Milestone 3** • Convert simple fractions to decimal fractions by dividing
PA 2: Exploring number sequences and relationships (Number, Pattern and Calculating 6, pages 43–51)	**pp. 58–61**	**NPC Milestone 3** • Generate and describe linear number sequences including expressing term to term and general rules of number patterns
Mea 3: 3D shapes – nets and surface area (Geometry, Measurement and Statistics 6, pages 73–78)	**pp. 62–65**	**GMS Milestone 2** • Recognize and create nets of cubes • Create nets of cuboids and prisms • Use nets to calculate surface area

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
Calc 9: Written column methods of multiplying (Number, Pattern and Calculating 6, pages 157–164)	**pp. 66–69**	**NPC Milestone 4** • Use short and long multiplying and dividing to solve problems, including those involving decimals
Calc 10: Introducing long written methods of dividing (Number, Pattern and Calculating 6, pages 165–175)	**pp. 70–73**	**NPC Milestone 4** • Use short and long multiplying and dividing to solve problems, including those involving decimals
Mea 4: Volume and scaling (Geometry, Measurement and Statistics 6, pages 79–89)	**pp. 74–77**	**GMS Milestone 3** • Carry out calculations involving lengths and volumes of cubes and other cuboids, using formulae where appropriate • Convert between different metric units of volume • Use and understand the effects of scaling on area and volume
Calc 11: Adding and subtracting with fractions (Number, Pattern and Calculating 6, pages 176–182)	**pp. 78–81**	**NPC Milestone 4** • Add and subtract fractions and mixed numbers
Calc 12: Multiplying and dividing fractions (Number, Pattern and Calculating 6, pages 183–187)	**pp. 82–85**	**NPC Milestone 4** • Multiply simple pairs of proper fractions • Divide proper fractions by whole numbers
PA 3: Using algebra to solve problems (Number, Pattern and Calculating 6, pages 52–60)	**pp. 86–89**	**NPC Milestone 4** • Express missing number problems algebraically • Enumerate possibilities of combinations of two unknowns
Geo 2: Circles (Geometry, Measurement and Statistics 6, pages 35–42)	**pp. 90–93**	**GMS Milestone 3** • Recognize and name the radius, diameter and circumference of any circle • Recognize that the diameter of any circle is twice the radius
Calc 13: Solving non-routine problems using all four operations (Number, Pattern and Calculating 6, pages 188–194)	**pp. 94–97**	**NPC Milestone 5** • Solve non-routine problems using all four operations
Geo 3: Transformations in the four quadrants (Geometry, Measurement and Statistics 6, pages 43–51)	**pp. 98–101**	**GMS Milestone 3** • Read and plot points using coordinates in all four quadrants • Describe, draw and translate 2D shapes using the coordinates of their vertices • Reflect points and shapes in both x and y axes using coordinates • Describe the movements of shapes accurately using the language of transformations
PA 4: Using symbols and letters for variables and unknowns (Number, Pattern and Calculating 6, pages 61–70)	**pp. 102–105**	**NPC Milestone 5** • Use symbols and letters to represent variables and unknowns in mathematical situations

Cover

Practice

$\frac{9}{12}$ The other fractions shown on the track are all shown with their equivalents in twelfths. The equivalent of $\frac{3}{4}$ in twelfths is $\frac{9}{12}$.

Going deeper

To find out how far the children have travelled altogether you need to add all the fractions.

$\frac{4}{12} + \frac{6}{12} + \frac{9}{12} = \frac{19}{12}$ or $1\frac{7}{12}$ laps of the track.

Children may also say that, if you combine all their distances, the children have been around the track just over one and a half times.

Encourage children to look for the lowest common denominator when adding fractions. Here this is twelfths, so they will find that there isn't a lower common denominator for all three fractions. Children may find other ways to add the fractions however, including adding the fractions on a number line or adding $\frac{1}{2}$ ($\frac{2}{4}$) and $\frac{3}{4}$ to get $\frac{5}{4}$ and then adding on $\frac{1}{3}$. The fractions can also be added together in different orders.

Page 2: Exploring large numbers (NNS 1·1)

Practice

1 **a** Molly's: Five million, two hundred and twenty-three thousand, two hundred and fifty-six. Ben's: Eight million, three hundred and forty-one thousand, six hundred.

b Yes, because 8 million is greater than 5 million.

c 300 000 (Switzerland) and 3000 (Norway)

2 **a** Hungary

b Hungary, Serbia, Hong Kong, Sierra Leone, Luxembourg, Seychelles

3 Seychelles – 7, Hong Kong – 7 million, Luxembourg – 70 000, Sierra Leone – 7 million, Hungary – 70, Serbia - 700 000 and 70 000.

The position of the 7 determines its value.

Going deeper

1 101 400

2 9 531 712, 9 541 712, 9 551 712, 9 561 712, 9 571 712, 9 581 712, 9 591 712, 9 601 712, 9 611 712

Page 3: Looking at the value of digits in large numbers (NNS 1·2)

Practice

1 2 105 121

2 She has not put zero as a place holder in the hundreds column. It should be 1 131 042.

3 **a** 3 blue, 0 purple, 4 green, 5 black, 6 yellow, 0 orange, 2 red

b Combinations will vary. (Look for totals of 20 with at least one of each colour and at least 3 blues), e.g. 3 blue, 1 purple, 4 green, 4 black, 5 yellow, 1 orange, 2 red.

Going deeper

1 **a** 10 000 000 (10 blue marbles)

b 39 (3 orange marbles and 9 red marbles)

2 Answers will vary. Look for a total of 10 in the answers with a 0 in thousands and hundreds column, e.g. 3 212 011.

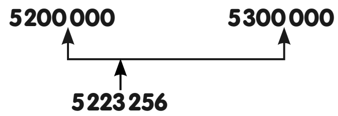
3 3 020 410

Page 4: Rounding large numbers (NNS 1·3)

Practice

1 5 200 000

2



3 **a** 5 220 000 **b** 5 223 000

c 5 223 300 **d** 5 223 260

4 **a** 7 000 000

b 7 100 000

c 7 090 000

d 7 086 000

e 7 085 600

f 7 085 630

Going deeper

1 Answers will vary. Example responses may include: 3 335 000, 3 337 000.

2 Answers will vary but should be between 1 061 500 and 1 062 499.

3 1 275 000 and 1 284 999

Page 5: Exploring the values of digits in decimal numbers (NNS 1·4)

Practice

1 **a** Because 0·3 contains 3 tenths and 0·25 contains 2 tenths and 5 hundredths. 3 tenths is bigger.

b $3 = \frac{3}{10}$ or 0·3, $2 = \frac{2}{10}$ or 0·2, $5 = \frac{5}{100}$ or 0·05

2 **a** 1·8 > 1·65 **b** 0·503 < 0·65 **c** 2·067 < 2·12

3 Answers will vary. Reponses should be between 0·501 and 0·749.

Going deeper

1 Answers will vary but might include 1·451, 1·455, 1·459.

2 0·123 and 0·246

3 Answers will vary.

4 Answers will vary.

NPC Milestone 1

- Understand the value of each digit in large numbers up to ten million and numbers with up to 3 decimal places
- Order numbers with up to 8 digits and position them on a number line

Page 6: Using negative numbers (Calc 1·1)

Practice

1 **a** January, February, November, December

b January

c June, July, August

2 February and December, June and August

Going deeper

1 **a** colder

b warmer

c warmer

2 **a** $^{-}7$ or $^{-}8$

b Answers may vary but could include: $^{-}5$, $^{-}4$, $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, or 3.

3 Answers will vary but the first should be colder than $^{-}50$ e.g. $^{-}55$, the second between $^{-}50$ and $^{-}15$, e.g. $^{-}20$ and the third between $^{-}15$ and 1, e.g. 0.

Page 7: Finding differences (Calc 1·2)

Practice

Guidance Children could label their own blank number line and use the counter to help them.

1 $^{-}5$°C

2 $^{-}7$°C

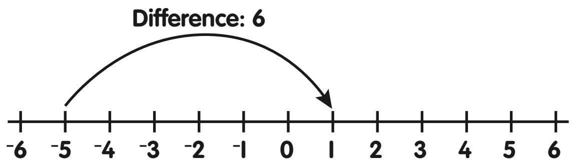
3 **a** 9°C **b** 10°C **c** 11°C

Going deeper

1 **a** $^{-}1$, $^{-}2$, $^{-}3$ **b** Answers will vary.

2 Answers will vary but might include: $^{-}2$ and 10, $^{-}8$ and 4, $^{-}6$ and 6.

3 Children should show that if they move the jump, the difference stays the same but the start and end point of the jump alters.



Page 8: Profit and loss (Calc 1·3)

Practice

1 Monday, Tuesday, Wednesday and Friday

2 Monday £54 – £80 = $^{-}$£26 (loss)

Tuesday £18 – £60 = $^{-}$£42 (loss)

Wed £90 – £100 = $^{-}$£10 (loss)

Thurs £126 – £120 = £6 (profit)

Friday £108 – £110 = $^{-}$£2 (loss)

Saturday £216 – £170 = £46 (profit)

Sunday £252 – £190 = £62 (profit)

3 £15 000

Going deeper

1 Monday £27 – £65 = $^{-}$£38 (loss)

Tuesday £9 – £55 = $^{-}$£46 (loss)

Wed £45 – £75 = $^{-}$£30 (loss)

Thurs £63 – £85 = $^{-}$£22 (loss)

Fri £54 – £80 = $^{-}$£26 (loss)

Saturday $^{-}$£108 – £110 = $^{-}$£2 (loss)

Sunday $^{-}$£126 – £120 = £6 (profit)

2 Calculations will vary. An example could be: £240 – £890 = $^{-}$£650.

3 Word problems will vary. An example could be: 'A car wash makes £240 one week but pays its staff a total of £890. Does it make a profit or loss? By how much?

Page 9: Adding and subtracting large numbers (Calc 1·4 & 1·5)

Practice

1 plane a: 2238 kg plane b: 4450 kg

2 4169, children should recognise that it is easier to subtract 8400 and then adjust.

3 236 094, children may use the inverse or count up to reach the target number.

Going deeper

1 **a** 14 262

b 14 450

c Look for children who show the difference on a number line and slide the jump along.

2

+	16 472	18 448
39 951	56 423	58 399
46 284	62 756	64 732

3 Answers will vary.

NPC Milestone 1

- Use different approaches to add and subtract negative numbers in context

Page 10: Multiplying using factors (Calc 2·1)

Practice

The way children factorize may be different but an example is given for each below.

1 24 × 25

6 × (4 × 25) = **600**

2 **a** 15 × 36

(15 × 6) × 6

90 × 6 = **540**

b 18 × 25

9 × (2 × 25)

9 × 50 = **450**

c 120 × 42

40 × (3 × 7) × 6

40 × 21 = 840

840 × 6 = **5040**

Going deeper

1 46 × 16

23 × 2 × 8 × 2

2 24 × 25 = 12 × 50

If you halve one amount and double the other amount, the answer is the same. So, e.g. if you had 2 × 3 then 1 × 6 is the same, if you had 4 × 5 then 2 × 10 would be the same so you can generalise. Children may also draw a diagram to show how if you halve one and double the other, it is the same.

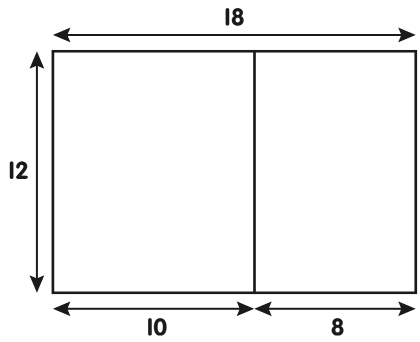
3 175 × 28 = 25 × 7 × 4 × 7 = (25 × 4) × 7 × 7 = 4900

Page 11: Multiplying using partitioning (Calc 2·2)

Practice

1 12 × 18

(12 × 10) + (12 × 8) = 120 + 96 = **216**



2 **a** 15 × 32

15 × 3 × 10 = 450

15 × 2 = 30

450 + 30 = **480**

b 52 × 45

50 × 40 = 2000, 50 × 5 = 250

2 × 40 = 80, 2 × 5 = 10

2000 + 250 + 80 + 10 = **2340**

c 38 × 19

30 × 10 = 300, 8 × 10 = 80

30 × 9 = 270, 8 × 9 = 72

300 + 80 + 270 + 72 =

500 + 220 + 2 = **722**

3 Children may partition in different ways.

Going deeper

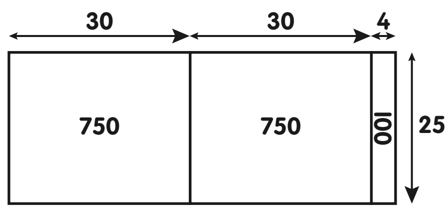
1 180 × 12

80 × 12 = 960, 100 × 12 = 1200

1200 + 960 = **2160**

2 25 × 64 = **1600**

Answers will vary but could include:



3 Answers will vary.

Page 12: Partitioning and dividing (Calc 2·3 & 2·5)

Practice

Guidance Encourage children to simplify the divisions by dividing or multiplying each number by the same – make links to simplifying fractions.

1 Look for children who can explain that by keeping the ratio the same, the answer is not affected, e.g. £3 shared between 6 will give everyone 50p and £1·50 shared between 3 will also give 50p each.

2 **a** 300 ÷ 6 = 150 ÷ 3 = 50

b 848 ÷ 16 = 212 ÷ 4 = 53

c 540 ÷ 9 = 60 ÷ 1 = 60

Children's answers may vary.

3 160 ÷ 32 = 40 ÷ 8 = 5

20 ÷ 4 = 5

Going deeper

1 75 ÷ 3 = 25. The only factors of 6 are 2 and 3 (other than 6 and 1) and 2 was already used in the example at the top of the page.

2 192 ÷ 12 = 16

96 ÷ 6 = 16

48 ÷ 3 = 16

64 ÷ 4 = 16

3 342 ÷ 18 = 19

18 × 20 = 360

360 – 342 = 18

Page 13: Multiplying and dividing by 10, 100 and 1000 (Calc 2·4)

Practice

1 Divide by 100 as all the digits have moved two places to the right.

2 Answers will vary but examples include:

16 becomes 0·16

1·5 becomes 0·015

56 becomes 0·56

3 **a** 6400 **b** 10 750 **c** 140 800

4 1000

5 100

Going deeper

1 Answers will vary. Make sure children are moving the digits the correct number of places.

3 Answers will vary.

An example could be 150.

NPC Milestone 1

- Use appropriate mental methods to add, subtract, multiply and divide increasingly large numbers

Page 14: Introducing the mean (average) (Mea 1·1)

Practice

1 Add the four scores together and divide by 4.

2 Molly and Ravi

3 Ben

Going deeper

1 If the mean of three runs is 11 seconds, the total must total less than 33 seconds. Therefore Jonah must run his final race in less than 11·4 seconds. Any answer less than this is correct.

2 74 skips

Page 15: Using the mean (Mea 1·2)

Practice

1 **a** Mean is 500·64 g, 1% of this is 5·0064 g so she should reject any bags weighing less than 495·6 g: in this case, just the bag that is 493·5 g.

b Either answer is possible depending on justification: as the mean is above half a kilogram (500 g) this seems reasonable, but an answer of no is also acceptable as there are several bags below 500 g.

Going deeper

1 332·3 ml

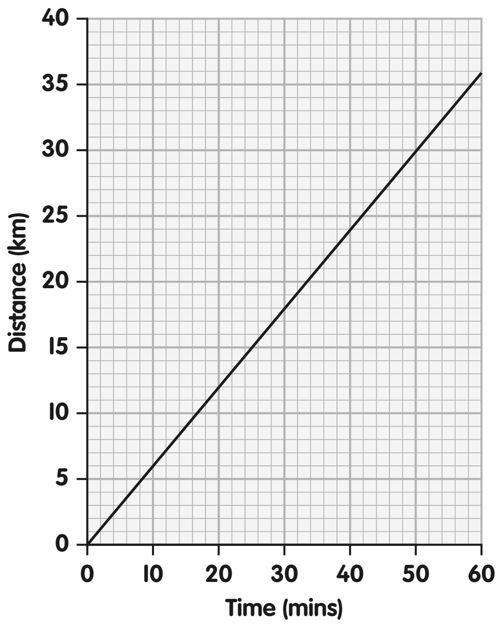
2 35 years

Page 16: Average speeds and units of speed (Mea 1·3 & 1·4)

Practice

1 36 km/h

2



3 Draw vertical line up from 20 minutes then across to 12 km; answer: 12 km.

4 75 laps = 75 × 200 m = 15 000 m, or 15 km. Draw a line across from 15 km to the graph, then down to 25 mins; answer: 25 minutes.

Going deeper

1 She has driven 60 km and travelled for 45 minutes.

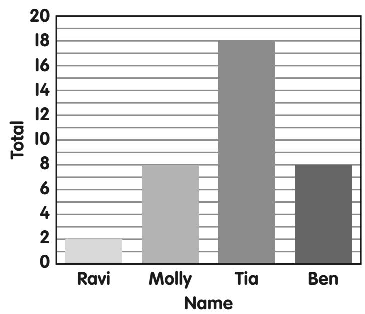
2 If children need support with this question, remind them that 1 mile ≈ 1·6 km. 5 mph ≈ 8 km/h, so Zane runs faster than Nadia.

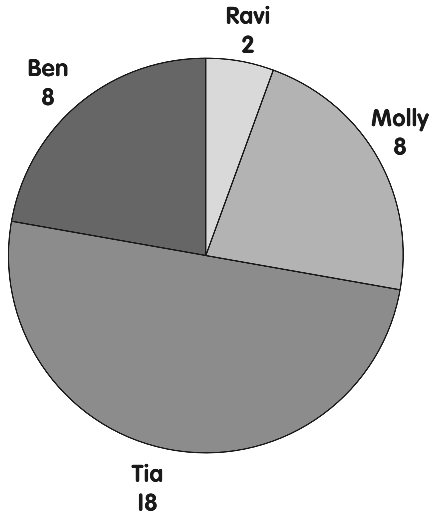
The 10 mile race ≈ 16 km, so at 9 km/h Zane would take $\frac{16}{9}$ hours or $\frac{16}{9} \times 60$ minutes = $\frac{960}{9}$ minutes. Converting this fraction to a mixed number gives us $106\frac{2}{3}$ minutes. At 5 mph, Nadia would take exactly 2 hours or 120 minutes, so she was just over 13 minutes slower than Zane.

Page 17: Constructing and interpreting pie charts (Mea 1·5)

Practice

1 a





b For quantities, a bar chart is more useful. For proportions, a pie chart is more useful.

2 a $\frac{1}{2}$; pie chart is best here.

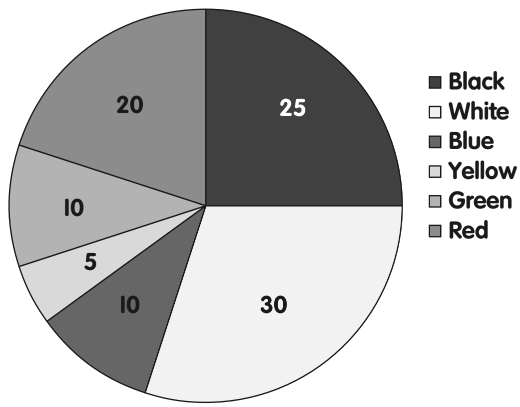
b 16; bar chart is best here.

Going deeper

1 a

Black	White	Blue	Yellow	Green	Red
25%	30%	10%	5%	10%	20%

b Pie chart would be most useful as it shows proportions of total amount best.



c 10% is green; 10% of 200 is 20 green dresses.

GMS Milestone 1

- Calculate the mean average of a set of data
- Create, use and interpret conversion graphs
- Convert between metric and imperial speeds
- Construct and interpret pie charts to solve problems

Page 18: Exploring factors and multiples (P&A 1·1 & 1·2)

Practice

1 The circular number chain starts with the number at the top, which in the example on page 18 is 6. This number is then followed by a multiple of the starting number followed by a factor of the second number. The sequence of multiple, factor then repeats until the circle reaches 6 again.

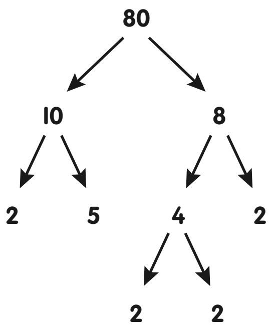
2 An example of a similar circular chain is 5, 15, 3, 24, 4, 20, 5.

3 Answers will vary.

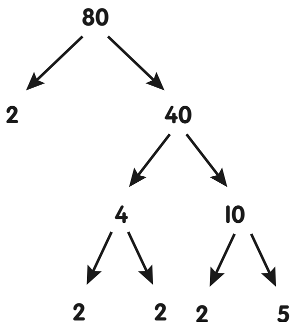
This example includes 15 numbers but other examples could be longer: 4, 40, 10, 30, 3, 24, 8, 16, 2, 50, 5, 60, 6, 36, 4.

Strategies include: avoiding primes and choosing multiples that have several factors and have not already been listed.

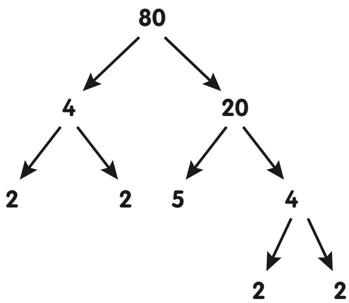
4 Answers may vary. Some examples might include:



$80 = 2 \times 5 \times 2 \times 2 \times 2$



$80 = 2 \times 2 \times 2 \times 2 \times 5$



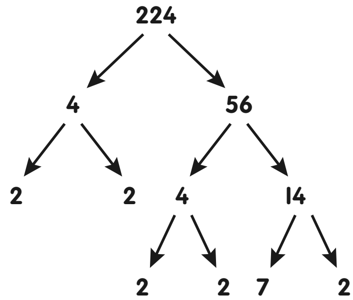
$80 = 2 \times 2 \times 5 \times 2 \times 2$

Going deeper

1 Starting with 5 at the top, you could have: 5, 45, 9, 54, 6, 30, 5 or 5, 30, 6, 54, 9, 45.

2 Answers will vary. One example might be because they start and end with the same number.

3 Answers will vary: suggestions might include using primes and other numbers which are the smallest possible factors, others may say they start with half or quarter facts.

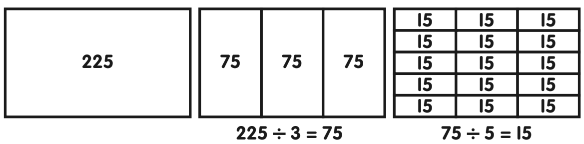


$224 = 2 \times 2 \times 2 \times 2 \times 7 \times 2$

Page 19: Using factors when multiplying and dividing (P&A 1·3)

Practice

1 $225 \div 15 = 15$



$225 \div 3 = 75$ $\quad 75 \div 5 = 15$

2 Answers will vary. Examples for **a** might include:

a 16×24

$4 \times 4 \times 24$

$8 \times 2 \times 2 \times 12$

$4 \times 4 \times 3 \times 8$

$8 \times 2 \times 4 \times 6$

$2 \times 2 \times 2 \times 2 \times 2 \times 12$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 6$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Some children might suggest that they can use prime factors to make the calculation easier to do mentally.

b An example for **b** might be:

$504 \div 6 = 84$, $84 \div 4 = 21$

c An example for **c** might be:

198 ÷ 3 = 66, 66 ÷ 6 = 11

d An example for **d** might be:

5 × 5 × 5 × 3 = 375 or 5 × 5 × 3 × 5 = 375

3 **a** $\frac{300}{18}$ $\frac{50}{3}$ **b** $\frac{252}{21}$ $\frac{36}{3} = 12$

Going deeper

1 Answers will vary. Examples might include:

a 23 × 15 23 × 3 × 5 5 × 23 × 3

b 16 × 36 6 × 6 × 16 6 × 6 × 4 × 4 6 × 6 × 8 × 2

36 × 8 × 2

c 28 × 13 14 × 2 × 13 7 × 2 × 2 × 13 4 × 7 × 13

d 19 × 17 Both these numbers are prime so we can't use factors to find different ways to calculate this example.

2 Answers will vary but most children will probably say **d** as this cannot be simplified.

3 The prime factors of 720 are 2 × 2 × 2 × 2 × 3 × 3 × 5. If you are dividing 720 by 16 you can cancel out 16 or 2 × 2 × 2 × 2 and you are left with 3 × 3 × 5 which is 45. 720 ÷ 16 = 45

Page 20: Using the lowest common multiple (LCM) (P&A 1·4)

Practice

1 The first time the lights flash together will be after 350 seconds. Some children will realise that the lights flash at the lowest common multiple of 14, 10 and 25 and explain that 50 is the first multiple of 10 and 25 but this is not a multiple of 14. 70 is the first multiple of 14 and 10 but it is not a multiple of 25. To find a multiple of 14 and 25, these numbers can be multiplied together. 14 × 25 = 350. 350 is the lowest common multiple of 14, 25 and 10.

2 Answers will vary. An example might be that the common multiples of 5 and 8 between 50 and 150 are 80 and 120.

3 Answers will vary, examples might include:

24, 60, 54, 84 are all common multiples of 6 and 2

24, 60, 84 are common multiples of 4 and 6

24, 60, 54, 30 are all multiples of 3 and 2

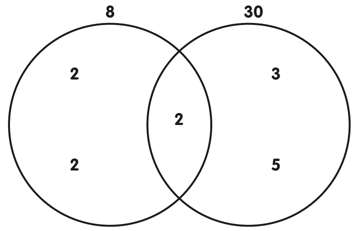
60, 30 are multiples of 10, 2, 3 and 5

24, 60, 84, 48 are all multiples of 4 and 2

24, 48 are multiples of 8, 4 and 2

Going deeper

1

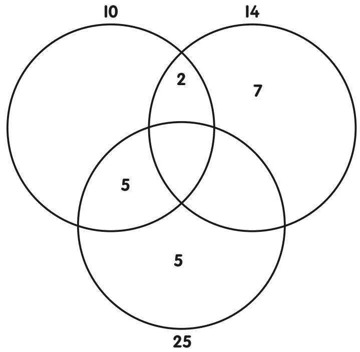


2 × 2 × 2 × 3 × 5 = 120

The LCM of 8 and 30 is 120.

2 The Venn diagram shows the prime factors of 12 (2 × 2 × 3 = 12) and 18 (2 × 3 × 3 = 18). The LCM of these numbers is 36 (2 × 2 × 3 × 3 = 36).

3

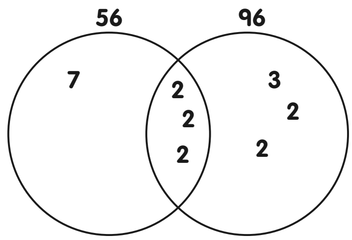


5 × 5 × 2 × 7 = 350

Page 21: Exploring highest common factors (HCF) (P&A 1·5)

Practice

1 **a**



The HCF of 56 and 96 is 8. This can be seen in the intersection of the diagram (2 × 2 × 2 = 8).

b The HCF of 72 and 90 is 18.

c The HCF of 81 and 27 is 27.

2 Answers will vary, examples might include 14 and 21, 21 and 28, 28 and 35, 21 and 35, 14 and 35. These numbers are both multiples of 7 but are not also multiples of 14 or 21, etc.

3 Pairs of numbers between 50 and 100 that have a HCF of 6 are:

54 and 60, 54 and 66, 54 and 78, 54 and 90, 54 and 84

60 and 66, 60 and 78, 60 and 90

66 and 90, 66 and 78, 66 and 90, 66 and 72, 66 and 84

72 and 78, 72 and 90

84 and 90

90 and 96

Going deeper

1 The HCF of two whole numbers up to and including 100 is 50 – which is the HCF of 50 and 100.

2 The HCF of two whole numbers up to and including 100 where the smaller number is not a factor of the larger is 33 – which is the HCF of 66 and 99.

3 Pairs of numbers between 50 and 100 whose only common factor is 7 are:

56 and 63, 56 and 77, 56 and 91

63 and 70, 63 and 77, 63 and 91, 63 and 98

70 and 77, 70 and 91

77 and 84, 77 and 91, 77 and 98

These have been found by working systematically, pairing multiples of 7 in the range of 50 to 100 but avoiding pairs of multiples that are also both multiples of 14, 21 and 28.

NPC Milestone 2

- Identify common factors, common multiples and prime numbers

Page 22: Exploring fractions (NNS 2·1)

Practice

1 No, Ben is not correct. $\frac{3}{8}$ is less than $\frac{1}{2}$.

2 $\frac{3}{8}, \frac{9}{16}, \frac{7}{10}, \frac{3}{4}, \frac{4}{5}$

3 Answers will vary but examples include: $\frac{7}{8}, \frac{2}{3}, \frac{3}{5}, \frac{9}{10}$.

4 a $\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{7}{9}$

b $\frac{1}{8}, \frac{5}{24}, \frac{1}{3}, \frac{5}{12}, \frac{5}{6}$

c $\frac{3}{14}, \frac{1}{4}, \frac{3}{7}, \frac{1}{2}, \frac{5}{7}$

Going deeper

1 $\frac{4}{5}$ because this is $\frac{8}{10}$ which is 0·8 and $\frac{3}{4}$ is 0·75.

2 Answers will vary. Example response: $\frac{1}{15}, \frac{3}{6}, \frac{5}{9}, \frac{6}{8}, \frac{9}{12}$.

A proper fraction is one where the numerator is smaller than the denominator.

3 Answers will vary.

Page 23: Converting and comparing fractions (NNS 2·2)

Practice

1 a Ravi got the higher mark as he got 17 out of 20, whereas Molly got $\frac{3}{4}$ correct, which is 15 out of 20.

b Tia got the higher mark as she got $\frac{2}{3}$ of the questions correct which is 20 out of 30, whereas Ben got 18 out of 30.

2 $\frac{16}{20}$

3 $\frac{25}{30}$

4 a $\frac{9}{24}, \frac{4}{24}, \frac{10}{24}, \frac{8}{24}$

b $\frac{28}{40}, \frac{25}{40}, \frac{30}{40}, \frac{24}{40}$

c $\frac{28}{36}, \frac{30}{36}, \frac{24}{36}, \frac{21}{36}$

Going deeper

1 $\frac{18}{30}, \frac{2}{3}, \frac{3}{4}, \frac{17}{20}$

2 Answers will vary, e.g.

a $\frac{1}{3}, \frac{3}{10}$

b $\frac{8}{21}, \frac{3}{8}$

3 $\frac{9}{10}, \frac{11}{12}, \frac{10}{11}, \frac{12}{13}, \frac{13}{14}$

Children should recognise that they need a fraction greater than 0·8 and to get that the numerator needs to be as close as possible to the denominator.

Page 24: Simplifying fractions (NNS 2·3)

Practice

1 $\frac{18}{80} = \frac{9}{40}$

Explanations will vary but should discuss finding a common factor that both the numerator and denominator divide by.

2 $\frac{2}{80} = \frac{1}{40}$

$\frac{4}{80} = \frac{1}{20}$

$\frac{6}{80} = \frac{3}{40}$

$\frac{8}{80} = \frac{1}{10}$
$\frac{11}{80}$
$\frac{12}{80} = \frac{3}{20}$
$\frac{10}{80} = \frac{1}{8}$
$\frac{5}{80} = \frac{1}{16}$
$\frac{3}{80}$
$\frac{1}{80}$

The ones that cannot be simplified do not share any common factors.

3 **a** $\frac{5}{40} = \frac{1}{8}$
b $\frac{3}{27} = \frac{1}{9}$
c $\frac{16}{56} = \frac{2}{7}$
d $\frac{48}{72} = \frac{2}{3}$

4 $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}$
Multiply both the numerator and denominator by the same amount.

Going deeper

1 Answers will vary. Look for children who recognise that fractions that do not have obvious common factors are harder to simplify.

2 'Sometimes true' – It works with even numerators and denominators.

Page 25: Ordering mixed numbers (NNS 2·4)

Practice

1 Vanessa because $4\frac{5}{6}$ is greater than $4\frac{3}{4}$.

2 $3\frac{1}{2}, 3\frac{2}{3}, 4\frac{3}{4}, 4\frac{5}{6}$

3 **a** $1\frac{3}{8}, 1\frac{2}{5}, 2\frac{1}{4}, 2\frac{3}{10}$
b $1\frac{1}{6}, 1\frac{2}{9}, 2\frac{1}{4}, 2\frac{5}{12}$

4 Answers will vary but might include $1\frac{3}{10}$ because $1\frac{1}{4}$ is 1·25 and $1\frac{1}{3}$ is 1·333 and $1\frac{3}{10}$ is 1·3.

Going deeper

1 Answers will vary. Example response could be: $3\frac{3}{5}, 4\frac{1}{2}, 4\frac{4}{5}$.

2 $2\frac{3}{5}, 2\frac{7}{10}, 2\frac{2}{3}$

3 Answers will vary. Look for children using appropriate language.

NPC Milestone 2

- Compare and order fractions by expressing them as equivalent fractions with a common denominator

Page 26: Estimating and rounding (Calc 3·1)

Practice

1 **a** Germany: 6 030 000, Japan: 9 280 000, UK: 1 680 000, France: 1 970 000, Spain: 2 730 000, Italy: 1 010 000, India: 4 130 000

b The given number line is graduated in steps of 1 000 000 and, in order, the countries (from smallest) are: Italy, UK, France, Spain, India, Germany, Japan.

2

Country	a	b	c
Germany	6 000 000	6 033 000	6 033 200
Japan	9 300 000	9 278 000	9 278 200
UK	1 700 000	1 682 000	1 682 200
France	2 000 000	1 970 000	1 970 000
Spain	2 700 000	2 733 000	2 733 200
Italy	1 000 000	1 014 000	1 014 200
India	4 100 000	4 126 000	4 125 700

Going deeper

1 India

2 101 000

3 92 800

Page 27: Estimating quantities and costs (Calc 3·2)

Practice

1 It is worth noting that in rounding to the nearest 100, Ravi has rounded one figure down (133 to 100; 25% lower), and the other figure up (365 to 400; 10% higher). So, rounding 133 to 130 would definitely give a closer estimate; 130 × 400 = 52 000. (See also answer to GD1 below.)

2 Molly has rounded the actual product of 133 and 365 (48 545) to the nearest 10 000, and 2·3 to the nearest whole number (2). A closer estimate would be 2·3 × 50 000 = 115 000 litres.

3 Molly has rounded £1·948 to £2. Then £2 × 100 cubic metres = £200. The actual cost can be calculated by multiplying annual family usage in cubic metres by £1·948. [(133 × 365 × 2·3) ÷ 1000] × £1·948 = 111·654 × £1·948 = £217·50. Molly's estimate is low because she rounded down annual family usage to 100 000 litres from the actual amount of 111 654 litres.

Going deeper

1 The difference between the original figures and the rounded figures is much greater when rounding to the nearest 100 than when rounding to the nearest 10. In particular, rounding 133 to 100 is a 25% reduction, whereas rounding 133 to 130 is only a 2% reduction, thus making the resulting estimate much more accurate.

2 (Example) Try rounding 230 831 up to 231 000 households. Average use per household per day is 133 × 2·3 = 305·9 litres. If we round up 305·9 to 310 then the Edinburgh daily usage in cubic metres is approximately (231 000 × 310) ÷ 1 000 = 231 × 310 = 71 610 cubic metres. This estimate is going to be high because we rounded both the number of households and average daily household usage up.

Page 28: Rounding (Calc 3·4)

Practice

1 **a** Since the given recipe is to make 15 cheese straws, to make 300 straws multiply the given amounts by 20. 350 g × 20 = 7000 g = 7 kg. 7 ÷ 1·4 = 5, so five 1·4 kg bags of flour will be needed.

b One 1·4 kg bag of flour is enough to make 60 cheese straws (1400 g ÷ 350 = 4; 4 × 15 = 60). 800 ÷ 60 = 13·333 … = $13\frac{1}{3}$, so 14 bags of flour will be needed to make one day's production.

c At the end of a day $\frac{2}{3}$ of a bag of flour will be left over from making 800 straws; $\frac{2}{3}$ of a bag of flour is enough to make $\frac{2}{3}$ of 60 straws = 40 straws. Since there are 8 straws in a pack, 40 ÷ 8 = 5 so there will be 5 extra packs.

Going deeper

1 Answers will vary. (Example) 7 804 999 rounds to 7 800 000 both to the nearest 10 000 and the nearest 100 000. In general, the first two and the last three digits of the 7-digit number can be anything; the digit in the 10 000s column must be zero, and the digit in the 1000s column must be < 5.

2 Molly: 399 874

Ravi: 420 356

Ben: 425 129

Tia: 399 996

Page 29: Estimating answers to calculations (Calc 3·6)

Practice

1 **a** (25 × 10) + (6 × 4) = 274

b (25 × 10) + 6 = 256

c 10(50 + 25) – (6 × 3) = 732

d (25 × 6 × 4) – ((50 ÷ 10) – 3)) = 598

e 25(10 + 6) + (4 × 3) = 412

2 **a** 660 + 330 = 990 (higher) **b** 125 × 5 = 625 (higher)

c 1500 – 750 = 750 (higher) **d** 270 ÷ 9 = 30 (lower)

e 325 + 575 = 900 (higher) **f** 9 × 20 = 180 (lower)

g 3350 – 400 = 2950 (higher) **h** 63 ÷ 7 = 9 (lower)

Going deeper

1 Round 138·8 to 139. Rounding 9·3 to 9 would be a larger adjustment than rounding 138·8 to 139.

2 Sanjay may not be correct. The highest possible figure for 2014 is 5 149 999, whereas the highest possible figure for 2013 is 5 499 999.

3 **a** 7(75 – 25) – 8 + 4 + 2 = 348

b ((7 × 75) – 25)) + ((8 × 2) ÷ 4)) = 504

c (8 × 75) + (7 × 25) – 4 – 2 = 769

d 4(75 – 25) + 2(8 + 7) = 230

NPC Milestone 2

- Use estimation to check answers to calculations

Page 30: Exploring column methods for adding (Calc 4·1)

Practice

1 Estimates will vary.

a 0·225 kg

b 0·26 kg

c 0·351 kg

2 A – 0·165 kg

B – 0·074 kg

C – 0·2 kg

Going deeper

1 Estimates will vary.

a 1·097 kg

b 0·801 kg

c 1·231 kg

2 A (0·158 kg) and B (0·067) = 0·225 kg

B and C = 0·26 kg

B and D = 0·342 kg

A (0·158 kg) and C (0·193) = 0·351 kg

A, B and C = 0·418 kg

A (0·158 kg) and D (0·275) = 0·433 kg

C and D = 0·468 kg

3 Answers will vary. Example response:

0·225 kg – 0·067 kg = 0·158 kg

Pages 31 to 34

Page 31: Working out change by subtracting (Calc 4·2)

Practice

1 **a** £3·84

b £5·31

2 **a** 7·29

b 2·91

c 2·45

d 2·37

Example response for using inverse to check: 7·29 + 2·38 = 9·67

Going deeper

1 1kg of each = £9·54

Ben is £1·09 short.

Ravi would have 70p left.

2 In discussions look for children discussing place value errors, commutativity (i.e. just swapping the digits), exchange.

a 4·594

b 3·876

c 1·878

3 Answers will vary but look for children using the inverse to come up with calculations, e.g. 2·547 + ? = ? then ? – ? = 2·547.

Page 32: Subtracting using equal adding (Calc 4·3)

Practice

1 Lily had added ten to each number and then one hundred to each number.

2 **a** 626

b 357

c 5·92

d 4·58

3 **a** 136 **b** 1000 **c** 4·85 **d** 10·2

Going deeper

1 Answers will vary.

a Answers will vary. Only the ones digit will be larger in the number that is being subtracted, e.g. 452 – 127.

b Answers will vary. Only the ones digit and the tens digit will be larger in the number that is being subtracted, e.g. 615 – 278.

2 Answers will vary.

Page 33: Missing number problems (Calc 4·4)

Practice

1

$$\begin{array}{r} 5{\cdot}67 \\ +\ 8{\cdot}45 \\ \hline 14{\cdot}12 \end{array} \qquad \begin{array}{r} 12{\cdot}48 \\ +\ 5{\cdot}84 \\ \hline 18{\cdot}32 \end{array} \qquad \begin{array}{r} 14{\cdot}26 \\ -\ 6{\cdot}18 \\ \hline 8{\cdot}08 \end{array} \qquad \begin{array}{r} 18{\cdot}39 \\ -\ 12{\cdot}84 \\ \hline 5{\cdot}55 \end{array}$$

2 a, b Answers and calculations will vary.

Going deeper

1 **a**

$$\begin{array}{r} 3579 \\ +\ 5356 \\ \hline 8935 \end{array} \quad \text{or} \quad \begin{array}{r} 3576 \\ +\ 5359 \\ \hline 8935 \end{array}$$

b Answers will vary.

NPC Milestone 2

- Use column methods of adding and subtracting for larger numbers and decimals

Page 34: Exploring percentages, fractions and decimals (Calc 5·1)

Practice

1 Ravi could divide £100 by 3 to calculate how much the '$\frac{1}{3}$ off' offer would save, and also recognize in the other offer that 30% of £100 is $\frac{30}{100}$ of £100, which is £30. So $\frac{1}{3}$ of £100 = £33·33 off, is the better offer.

2 **a** Since $\frac{2}{3}$ is 0·666 … and rounds to 0·67, $\frac{2}{3} > 0{\cdot}66$ (just).

b $\frac{13}{50} = \frac{26}{100}$ = 26%, so 28% > $\frac{13}{50}$

c 0·75 = $\frac{75}{100}$, so 80% > 0·75

3 Answers will vary.

Going deeper

1 The short answer is that the second bike was more expensive. If £140 is 70% of the original price of the first bike (i.e. the price after 30% has been taken off), then the original price of the first bike was £200 (if £140 is 70%, then 10% is £20, so the original 100% = £200). Similarly for the second bike if £160 is 50% of its original price, then the original price was £320.

2 Simple fractions tend to be used in everyday measuring contexts, e.g. 'half each', or 'three-quarters of the way', or 'two-thirds full'. Also for some special offers, e.g. '$\frac{1}{3}$ off!'

3 Food packaging (e.g. 0·13 g salt), or currency exchange rates (e.g. 1·22 USD). Decimals also tend to be used when there is any electronic calculating to be done.

4 Food packaging (e.g. ingredients: Cereals (39%), discounts e.g. '20% 0ff', VAT, and in news items interest rates, inflation, and pay rises are generally all announced in percentages. Percentages are generally used in situations where ease of comparison is important.

Page 35: Comparing percentages (Calc 5·2)

Practice

1 **a** Novak family (30%) **b** Murray family (55%)

c Murray family (5%)

2 Novak family (5%)

3 £24 out of £50 was spent on dairy; 48%.

Going deeper

1 Altogether they spent £9 out of £50 (18%), so more than 10%.

2 7 out of 35 total absences is $\frac{1}{5}$ or 20%. 3 absences out of 30 is 10%, so Ash Class had the better record.

Page 36: Percentage increases and decreases (Calc 5·3 & 5·4)

Practice

1 The first offer gives 120 tissues for £3, which is equivalent to £2·50 for 100 tissues (100 is $\frac{5}{6}$ of 120, and $\frac{5}{6}$ of £3 is £2·50). The second offer gives 100 tissues for £2·40, so the second offer is better.

2 and 3 The important thing is to look for which calculations children choose to do, and also their methods for calculating these percentages. Although all children should understand that $x\% = \frac{x}{100}$ of something, and be able to calculate percentages this way, encourage children to look for calculations that they can do mentally, e.g. 1% of 358 seconds is 3·58 seconds, or 10% of 3820 ml is 382 ml. Also look for children working out 5% by halving 10%, working out 25% by halving 50%, 2·5% by halving 5%, and 70% by using 7 × 10%. Which children know that 33% is very nearly $\frac{1}{3}$?

Going deeper

1 10% of £50 is £5, so 5% is £2·50 and 2·5% is £1·25. £5 + £2·50 + £1·25 = £8·75.

2 Encourage children to test this by using a convenient amount, e.g. £1.

£1 + 20% = £1·20; £1·20 – 10% of £1·20 = £1·20 – 12p = £1·08

£1 – 10% = 90p; 90p + 20% of 90p = 90p + 18p = £1·08

Discuss the fact that if this arithmetic works with £1 it will work with any number of pounds, or in fact any amount of anything, not just money. Understanding why this always works involves realising that we can use the expression '100%' to stand for 'all' of anything, i.e. a 'whole amount' of anything, and that the arithmetic will always be the same.

Page 37: Using percentages with data (Calc 5·5)

Practice

1 **a** Answers will vary, e.g. children could *either* calculate 18% of 63 000 000 by multiplying 63 million by $\frac{18}{100}$, or could reason: 10% of 63 000 000 is 6 300 000, so 20% is 12 600 000 and 2% is 1 200 000. Subtracting 2% from 20% gives 12 600 000 subtract 1 260 000, which is 11 340 000, or 11 300 000 to the nearest 100 000.

b 5% of 63 000 000 is $\frac{1}{20}$ × 63 000 000 = 3 150 000

c 16% of 63 000 000 is (3 × 5%) + 1% = (3 × 3 150 000) + 630 000 = 10 080 000

d If 15 marbles are 20% (or $\frac{1}{5}$) of the total, then the total number of marbles is 5 × 15 = 75.

Going deeper

1 Increasing by 25% is equivalent to adding $\frac{1}{4}$, so each amount can be multiplied by 125% or $\frac{5}{4}$. This gives:

240 g flour × $\frac{5}{4}$ = 300 g flour

30 g butter × $\frac{5}{4}$ = 37·5 g butter

50 ml water × $\frac{5}{4}$ = 62·5 ml water

30 g sugar × $\frac{5}{4}$ = 37·5 g sugar

120 g starter × $\frac{5}{4}$ = 150 g starter

50 g beaten egg × $\frac{5}{4}$ = 62·5 g beaten egg

1 pinch of salt × $\frac{5}{4}$ = 1 pinch and 'a bit'

This recipe will make 8 × $\frac{5}{4}$ = 10 sourdough rolls.

2 If 147 000 000 is 60% (or $\frac{3}{5}$) of the number of sparrows in 1984 then the number of sparrows in 1984 must have been: 147 000 000 × 5 ÷ 3 = 245 000 000 sparrows.

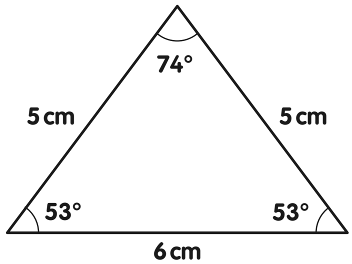
NPC Milestone $\boxed{2}$

- Understand, recall and use equivalences between simple fractions, decimals and percentages

Page 38: Exploring 2D shapes and angles (Geo 1·1 & 1·2)

Practice

1 Acute isosceles triangle

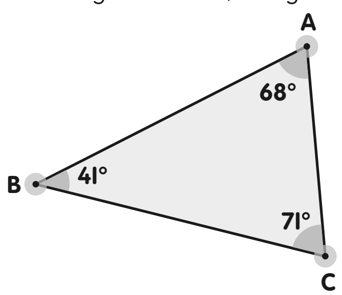


2 Drawings of triangles will vary but the side lengths should all total 16 cm.

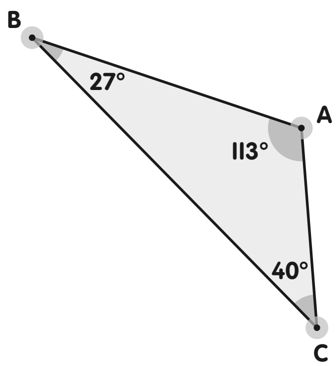
3 Answers will vary.

4 Triangles will vary.

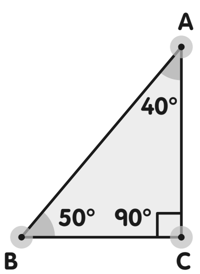
a Acute angled scalene (all angles less than 90°), e.g.



b Obtuse angled scalene (one angle greater than 90°), e.g.



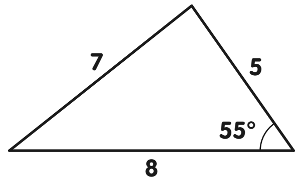
c Right-angled scalene



Going deeper

1 Look for children who can generalize that to make a triangle with perimeter 16 cm, the sum of the two shortest lengths must be greater than the longest.

2 **a** Missing side length: 7 cm; Perimeter: 20 cm



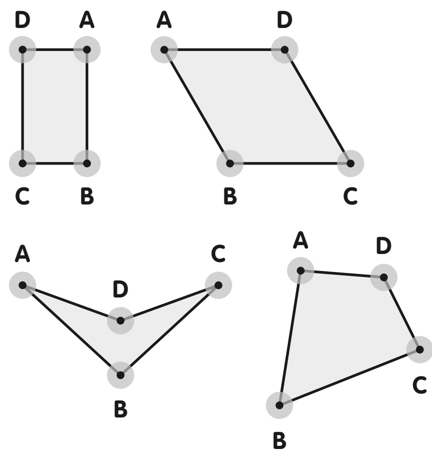
b An equilateral triangle will not be possible with whole numbers of cm because 20 is not a multiple of 3. Remind children of the earlier generalization from Q1.

c Answers will vary.

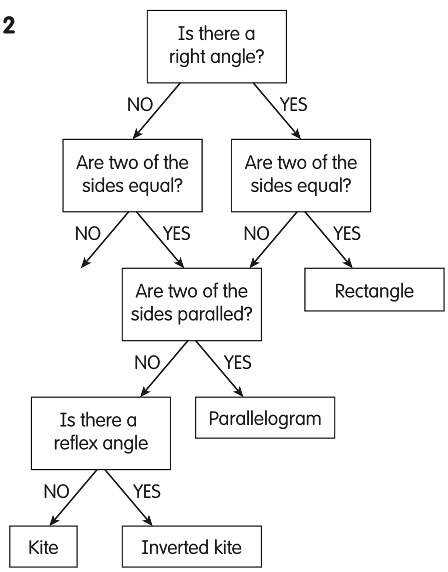
Page 39: Exploring quadrilaterals (Geo 1·3)

Practice

1 Diagrams will vary.

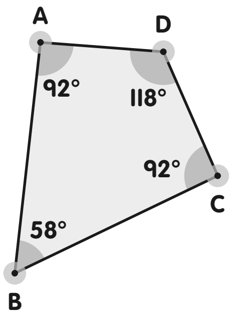


2

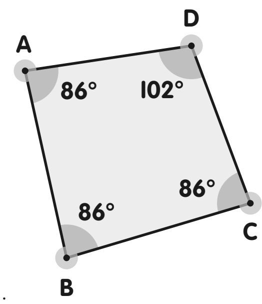


Going deeper

1 **a** Yes, it is possible.



b Yes, it is possible



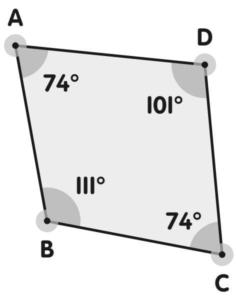
c Not possible because if three are right angles, the fourth must also be a right angle.

2 A square is a rhombus – ALWAYS.

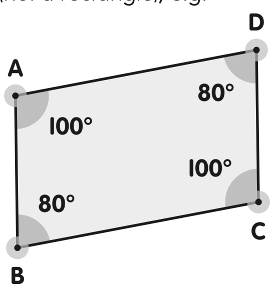
(A square is the only type of rhombus where all four angles are equal.)

A trapezium is a rectangle – SOMETIMES
(A trapezium has at least one pair of parallel sides.)

A quadrilateral has two obtuse and two acute angles – SOMETIMES, e.g.



A parallelogram has one pair of obtuse angles – SOMETIMES (not a rectangle), e.g.



Page 40: Exploring angles in regular polygons (Geo 1·4)

Practice

1 **a** Look for children who explain that for any polygon, by using a pencil to make the exterior angle journey around it, the pencil completes a full turn (360°).

b Look for children who can draw a picture to show this.

2 1080° because the external angles are 360 ÷ 8 = 45 so the internal angles are

180 – 45 = 135°

8 × 135 = 1080°

3 Exterior = 30° (360° in total)

Interior = 150° (1800° in total)

Going deeper

1 900°

2 $n(180 - (360 \div n))$

3 Look for children who can generalize that the number of triangles that a given polygon can be divided into by drawing diagonals is always the same.

Page 41: Finding missing angles (Geo 1·5)

Practice

1 36°

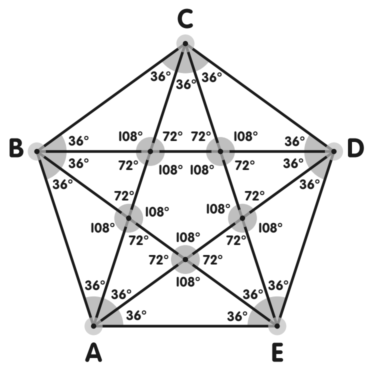
2 108° – 36° = 72°

3 **a** b = 118°, a and c = 62°

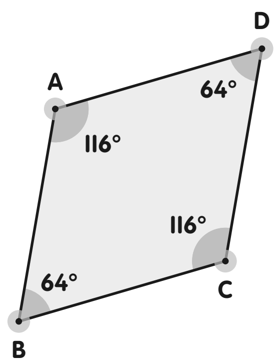
b e , h and j = 32°, f and d, i and g = 148°

Going deeper

1



2



3 Answers will vary.

GMS Milestone 1

- Use formal notation to denote parallel, perpendicular and equal length lines in geometric diagrams
- Recognize and classify a wide range of 2D shapes based on their properties
- Calculate missing angles in polygons, along straight lines, around a point and that are vertically opposite
- Construct triangles and other polygons from given properties

Page 42: Exploring multi-step problems (Calc 6·1)

Practice

1 Option A: The cafe would need to buy 7 tables at a total cost of £1400 and 40 chairs at a cost of £400 = £1800.

Option B: The cafe would need to buy 10 tables at a total cost of £1500 and 40 chairs at a cost of £400 = £1900.

Option A is the most cost effective.

2 **a** and **b** Total income per hour would be 60 people × £4·50. 60 × £4·50 = £270 per hour.

9 a.m. to 4 p.m. is 7 hours, so total income would be £270 × 7 = £1890 per day.

Going deeper

1 **a** They would need to find out how much the children spend each day (€30 × 3) and the adults (€75 × 2), and add these together. They would then need to multiply this by 7 to find the spending for a whole week: €1680.

b They could do the calculations in a different order, e.g. each child spends €30 × 7 in the week, and each adult spends €75 × 7 so they could work out what this came to for 3 children and 2 adults and should get the same figure.

2 He might multiply 3 kg by 14 to find he will use 42 kg of salt. He could then reason that he needs to buy 5 bags (4 bags would only give him 40 kg which isn't enough). He could then say that a bag of 10 kg costs 23p × 10 or £2·30, then multiply £2·30 by 5 to get £11·50.

Page 43: More multi-step problems (Calc 6·2)

Practice

1 The total area of the wall is 30 m^2.

The combined area of the posters is 4 m^2 + 1 m^2 + 1 m^2 = 6 m^2.

6 is one fifth or 20% of 30.

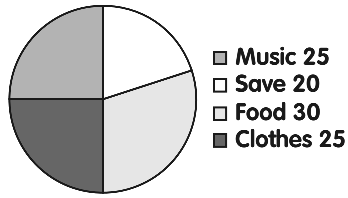
2 Answers will vary, e.g. 1 + 2 × 3 – 4 = 3.

3 Answers will vary, e.g. (1 + 2) × 3 – 4 = 5.

Going deeper

1 £480

Aria's spending



2 18% are squirrels, and there are 72 of them.

Page 44: BODMAS and the order of operations (Calc 6·3 & 6·4)

Practice

1 3 cans of soup and 4 cucumber portions, paid £3·74 with a £20 and received £16·26 change.

2 All give the answer 28 except **b** which gives 24.

3 Brackets are unnecessary in **a** and **e**.

a 11 **b** $^{-}3$ **c** 2 **d** 32 **e** 2

Going deeper

1 Use either **b** or **c**. Brackets in **c** make us find the daily amount before multiplying.

2 Answers will vary.

Page 45: Using BODMAS to solve problems (Calc 6·5)

Practice

1 12 × (28 + 1) or (28 + 1) × 12. There are 348 people in the school.

2 **a** 8 **b** 22 **c** $\frac{1}{2}$ **d** 8·75 **e** $\frac{1}{8}$ **f** 24 **g** 0 **h** 7 **i** 27

Going deeper

1 **a** 4 + 2 × 6 = 16 **b** 4 + 2 + 6 = 12 **c** 4 – (2 + 6) = $^{-}4$

d 4 ÷ (2 × 6) = $\frac{1}{3}$ **e** 4 × 2 + 6 = 14 **f** (4 + 2) ÷ 6 = 1

2 Answers will vary, e.g. 1 + 2 × 3 – 4 = 3.

3 Answers will vary, e.g. (1 + 2) × 3 – 4 = 5.

NPC Milestone 3

- Use the BODMAS convention for order of operations to solve problems

Page 46: Exploring ratio and proportion (Calc 7·1)

Practice

1 **a** 2:3 **b** 1:2 **c** 3:2

2 **a** 25 red cubes **b** 20 blue cubes

3 Answers will vary, but pairs of number rods and Numicon Shapes are extremely useful for illustrating such direct comparisons, e.g. a pink rod and a white rod next to each other illustrate a 4:1 comparison directly.

Going deeper

1 The important point to realise is that just doubling the number of cubes used in the original model will not allow you to make a scaled up version of the original (see *Number, Pattern and Calculating 6 Teaching Resource Handbook*, page 142, Activity 1). As all three dimensions (length, breadth, and height) of the original need to be scaled up in the same proportion, i.e. doubled, the number of cubes will have to be multiplied by 2 × 2 × 2 = 8 times to produce a scale model of the original whose sides have been 'doubled'. So Tomo is wrong.

2 It would have 24 yellow cubes (i.e. 3 × 8). In its simplest form, the ratio of yellow to blue blocks should remain as 3:2, as it is in the original.

Page 47: Investigating ratio problems (Calc 7·2)

Practice

1 It would take 4 strawberries, 6 blueberries, and 8 raspberries to make 1 glass of Berry Delight, so the answers can be calculated directly from this.

	a	b	c
strawberries	(5 × 4) 20	(12 × 4) 48	(7 × 4) 28
blueberries	(5 × 6) 30	(12 × 6) 72	(7 × 6) 42
raspberries	(5 × 8) 40	(12 × 8) 96	(7 × 8) 56

2 **a** $\frac{2}{9}$ **b** $\frac{1}{3}$ **c** $\frac{4}{9}$

3 **a** 3:4 **b** 3:2 **c** 1:2

Going deeper

1 Since these berries are in the ratio 2:3:7, it is helpful to think of berries going into the mixture in batches of 12, i.e. 2 strawberries for every 3 blueberries, and 7 raspberries (2 + 3 + 7 = 12). If there are to be 84 berries in total, 12 goes into 84 exactly 7 times, so the final ratios of berries will be: (7 × 2) : (7 × 3) : 7 × 7) = 14:21:49.

So 14 strawberries, 21 blueberries, and 49 raspberries. We can check this by noting that 14:21:49 is indeed the same ratio as 2:3:7, and that 14 + 21 + 49 = 84.

2 The numbers of actual berries in their correct ratio would have been 21:28:35, so the answers are:

a $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{5}{12}$

Page 48: Examining ratio and proportion using pie charts (Calc 7·4)

Practice

1 Since there are 14 times as many children in the whole school as there are in Owl class, to answer the questions multiply each Owl class value by 14. So individual answers are:

a 14 × 2 = 28 **b** 14 × 11 = 154 **c** 14 × 4 = 56

d 14 × 9 = 126 **e** 14 × 4 = 56

2 **a** $\frac{20}{30} = \frac{2}{3}$ **b** $\frac{6}{30} = \frac{1}{5}$

3 27 bags of popcorn is 3 times the number of children who chose it, i.e. popcorn is being bought at the rate of 3 bags per child. So to be fair (or consistent) 3 × 11 = 33 packets of grapes should be bought.

4 Each class will need 9 packets of popcorn from the box of 12 packets, which is $\frac{3}{4}$ of the box. $\frac{3}{4}$ × 14 (the total number of classes) is 10·5 so the school will need to order 11 boxes. Each box costs £5·40 so they will spend £59·40 in total.

Going deeper

1 36% of 250 = 90 Autumn born children, and $\frac{1}{5}$ of 250 = 50 Spring born children. So 90 + 45 + 50 = 185 children were born during Autumn, Winter, and Spring. Therefore 250 – 185 = 65 children were born in the Summer, and this figure can be checked by re-calculating with percentages throughout. 36% were born in the Autumn, 45 children (Winter) is 18% of the total, and $\frac{1}{5}$ (Spring) is 20% of the total. 36% + 18% + 20% = 74%, so the remaining 26% must have been born in the Summer (100% – 74% = 26%). 26% of 250 = $\frac{26}{100}$ × 250 = 13 × 5 = 65 children.

Page 49: Unequal sharing problems (Calc 7·5)

Practice

1 £120 is 6 × £20, so the wildlife sanctuary (WS) should get 6 × £4 = £24. The local hospital (LH) should get 6 × £6 = £36. The library (L) should get 6 × £10 = £60. (To check this, £24 + £36 + £60 = £120.)

2 £150 is 7·5 × £20, so the WS gets 7·5 × £4 = £30, the LH gets 7·5 × £6 = £45, and the L gets 7·5 × £10 = £75. (To check, £30 + £45 + £75 = £150.)

3 WS gets $\frac{4}{20}$ = 20%, the LH gets $\frac{6}{20}$ = 30%, and the L gets $\frac{10}{20}$ = 50%.

Going deeper

1 For every 10 marbles shared Dario gets 4 and Tom gets 6, giving Tom 2 more than Dario each time. If Tom finally ends up with 16 more marbles than Dario, they must have shared out 10 marbles 8 times. 8 × 10 = 80 marbles have been shared in the given ratio. (To check, Dario gets 32 marbles and Tom 48; 32 + 48 = 80, and 48 is 16 more than 32.)

2 Children can vary the problem by changing both the sharing ratio and the difference in totals. Note that the final difference has to be a multiple of the difference between the ratio values, just as 16 is a multiple of the difference between 4 and 6.

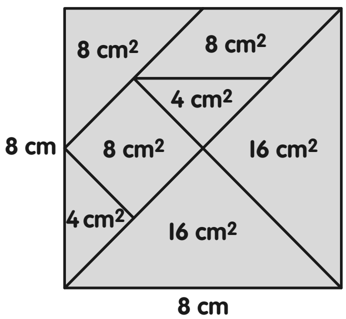
NPC Milestone 3

- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples
- Recognize proportionality in contexts when the relations between quantities are in the same ratio

Page 50: Exploring areas of 2D shapes (Mea 2·1)

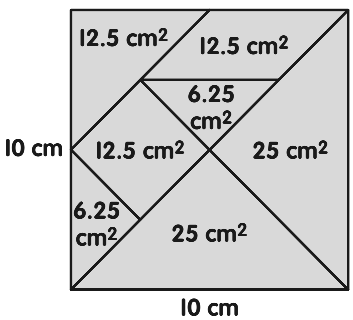
Practice

1



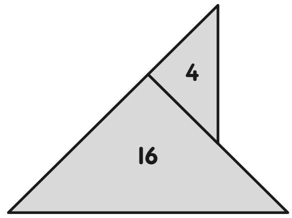
Find the area of the square (64 cm²) then see what fraction of the whole square each piece represents. The large triangles are each $\frac{1}{4}$ of the square, the small triangles are each $\frac{1}{16}$, and the medium-sized triangle, parallelogram and square are each $\frac{1}{8}$.

2



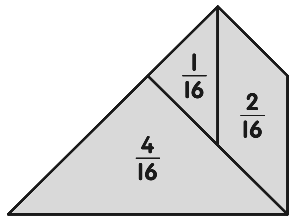
Going deeper

1 a Various. One possible solution:



b No, you can't. No two pieces add up to $\frac{7}{16}$ of the total figure.

c Yes – here is one possible solution:



2 These are both possible in various ways. Use three shapes that are $\frac{2}{16} + \frac{2}{16} + \frac{1}{16}$ for the first, and $\frac{4}{16} + \frac{2}{16} + \frac{1}{16}$ for the second.

Page 51: Finding the area of triangles (Mea 2·2 & 2·3)

Practice

1 9 cm² and 30 cm²

2 3 and 6 are perpendicular as base and height in the first triangle, and 5 and 12 are perpendicular in the second. A triangle's area is calculated by multiplying the base and the height and then halving the product.

Going deeper

1 a Neither wins – it is a draw. Player 1's triangles have areas of 11·25 and 13·75, a total of 25 cm². Player 2's triangles have areas of 7·5, 5, and 12·5, a total of 25 cm².

b It will always happen. The total of the bases of Player 1's triangles will always be 10, and the total of the bases of Player 2's triangles will always be 10, no matter how many triangles they are split into. The height of every triangle is 5 cm, so the total area will always be the same – half the total rectangle.

Page 52: Finding the area of parallelograms (Mea 2·4)

Practice

1 44 cm² and 19·2 cm²

2 A parallelogram's area is found by multiplying the base and height, which, in this case means calculating 11 × 4 and 3·2 × 6.

Going deeper

1 4·5 cm

2 The unknown number x, is 3 cm. The area can either be found by calculating 4 multiplied by 15 or 20 multiplied by x. Since both are the same (60 cm²), then x must be 60 ÷ 20 or 3 cm.

Page 53: Problem solving with composite shapes (Mea 2·5)

Practice

1 **a** Kitchen 9 m², Dining Room 9 m², Hall 2 m², Lounge 12·5 m², Bathroom 2·5 m²

b 50%

Going deeper

1 **a** Answers will vary.

b Answers will vary, e.g. assuming the triangle is half of the rectangle, the rectangle's area will be 20 cm², and the triangle's area will be 10 cm², so the rectangle and triangle sides can be worked out as 4 cm and 5 cm.

2 **a** Answers will vary.

b An infinite number of possibilities – it could be very long and not so high, or narrow and high or anywhere in between.

GMS Milestone 2

- Use formulae to find the area of triangles (area = $\frac{1}{2} \times b \times h$) and parallelograms (area = $b \times h$) and understand why they work
- Find the area of composite shapes by partitioning into triangles and/or rectangles

Page 54: Linking fractions with dividing (Calc 8·1)

Practice

1 $3 \div 4 = \frac{3}{4}$ of an apple

2 $3 \div 6 = \frac{3}{6} = \frac{1}{2}$ an apple

3 **a** $4 \div 5 = \frac{4}{5}$

b $6 \div 4 = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

c $8 \div 10 = \frac{8}{10} = \frac{4}{5}$

d $2 \div 6 = \frac{2}{6} = \frac{1}{3}$

4 Each person will receive a whole biscuit and a fraction of a biscuit.

(Mixed numbers/improper fraction)

Going deeper

1 Cut two apples in half and one apple into quarters giving each person $\frac{3}{4}$ in total.

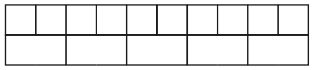
2 Answers will vary. Example response: 5 children were sharing 3 pizzas, equally. How much would they get each?

3 $\frac{3}{4} < \frac{5}{6}$ ($\frac{5}{6}$ is $\frac{1}{12}$ bigger so this should be the preference)

Page 55: Making connections between fractions and decimals (Calc 8·2 & 8·3)

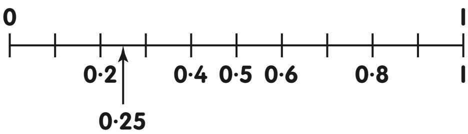
Practice

1 **a** 2

b 

2 0·2

3 **a** and **b**



4 0·75

Going deeper

1 $\frac{2}{5} = 0{\cdot}4$ so $\frac{4}{5}$ is double that which is 0·8.

2 1 ÷ 8 = 0·125, 125 g

3 Answers will vary. Look for children who can work out other decimal equivalents from those known already, e.g. 0·2 and $\frac{1}{5}$.

4 Answers will vary.

Page 56: Fractions and recurring decimals (Calc 8·4)

Practice

1 $\frac{1}{12}$ is 0·08333 because $\frac{1}{12}$ is smaller than $\frac{1}{11}$.

2 Yes – check children's dividing.

3 0·16666666

4 Look for children who compare each fraction with $\frac{1}{2}$ to make their estimate.

a 0·33333 **b** 0·555555 **c** 0·57143

Going deeper

1 $\frac{1}{15}$

2 Any decimal between 0·25 and 0·3333 e.g. 0·3.

3 **a** Answers will vary. Example response: $\frac{8}{15} = 0{\cdot}53333$.

Look for children who look for a numerator which is just more than half the denominator.

b Answers will vary. Example response: $\frac{7}{9} = 0{\cdot}77777777$. Look for children reasoning that the numerator needs to be close to the denominator (around $\frac{3}{4}$ of it).

Page 57: Adding fractions and decimals (Calc 8·5)

Practice

1 $0{\cdot}125, \frac{1}{6}, 0{\cdot}25, 0{\cdot}3, \frac{3}{8}, \frac{2}{5}, 0{\cdot}\dot{4}, 0{\cdot}\dot{6}, \frac{7}{10}, \frac{3}{4}$

2 0·65 because $\frac{2}{5} = \frac{4}{10}$ which is 0·4

3 0·15

4 $0{\cdot}3 + \frac{3}{4}$

5 Answers will vary, e.g. $0{\cdot}3 + 0{\cdot}25 = 0{\cdot}55$; $\frac{3}{4} - \frac{7}{10} = 0{\cdot}05$.

Going deeper

1 Answers will vary. $\frac{7}{10}$, $\frac{2}{5}$ and 0·3 may come up as these each have just one decimal place.

2 $\frac{1}{6}$ and 0·3

3 $\frac{3}{8} + 0{\cdot}\dot{6} = 1{\cdot}0416$

$0{\cdot}\dot{6} + \frac{2}{5} = 1{\cdot}0\dot{6}$

$0{\cdot}\dot{6} + \frac{7}{10} = 1{\cdot}36$

$0{\cdot}\dot{6} + \frac{3}{4} = 1{\cdot}41\dot{6}$

$\frac{7}{10} + \frac{3}{8} = 1{\cdot}075$

$\frac{7}{10} + \frac{2}{5} = 1{\cdot}1$

$\frac{7}{10} + \frac{3}{4} = 1{\cdot}45$

$\frac{3}{4} + 0{\cdot}3 = 1{\cdot}05$

$\frac{3}{4} + \frac{3}{8} = 1{\cdot}125$

$\frac{3}{4} + \frac{2}{5} = 1{\cdot}15$

4 Answers will vary.

Pairs with a difference of less than 0·25 are:

$0{\cdot}\dot{6} - 0{\cdot}\dot{4} = 0{\cdot}2$

$\frac{2}{5} - 0{\cdot}3 = 0{\cdot}1$

$\frac{3}{8} - 0{\cdot}25 = 0{\cdot}125$

$\frac{3}{4} - 0{\cdot}\dot{6} = 0{\cdot}08\dot{3}$

$0{\cdot}\dot{4} - 0{\cdot}3 = 0{\cdot}1\dot{4}$

NPC Milestone 3

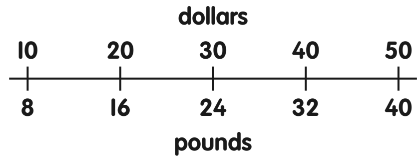
- Convert simple fractions to decimal fractions by dividing

Page 58: Linear sequences and graphs (P&A 2·1)

Practice

1 According to the graph £40 can be exchanged for $50.

2

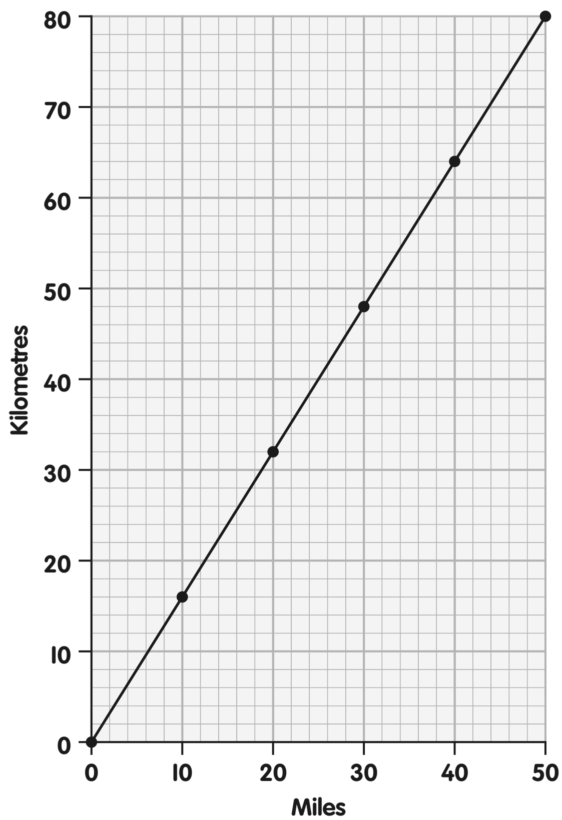


3 A general rule to convert pounds to dollars is to multiply the number of pounds by 1·25 and this will give you the converted dollars. This can also be written as 1·25 × pounds = dollars, or $1{\cdot}25\,x = y$ (where y stands for dollars and x stands for pounds).

Going deeper

1 To convert miles to kilometres you need to multiply the number of miles by 1·6. This can be written as miles × 1·6 = km or $1{\cdot}6\,x = y$ (where x is the number of miles and y is the number of kilometres).

2



3 35 miles is 56 kilometres ($35 \times 1{\cdot}6 = 56$).

4 You need to divide the number of kilometres by 1·6 to convert kilometres to miles (km ÷ 1·6 = miles).

Page 59: Exploring number chains (P&A 2·3)

Practice

1 Halve an even number and add 5 to an odd number.

1, 6, 3, 8, 4, 2, 1

2, 1, 6, 3, 8, 4, 2

3, 8, 4, 2, 1, 6, 3

4, 2, 1, 6, 3, 8, 4

5, 10, 5

6, 3, 8, 4, 2, 1, 6

7, 12, 6, 3, 8, 4, 2, 1, 6

8, 4, 2, 1, 6, 3, 8

9, 14, 7, 12, 6, 3, 8, 4, 2, 1, 6

10, 5, 10

The number chains above are linked because they all settled to the same pattern of numbers in the same order apart from the chain for 5 and 10 which is a different chain.

2 **11**, 16, 8, 4, 2, 1, 6, 3, 8

12, 6, 3, 8, 4, 2, 1, 6

13, 18, 9, 14, 7, 12, 6, 3, 8, 4, 2, 1, 6

14, 7, 12, 6, 3, 8, 4, 2, 1, 6

15, 20, 10, 5, 10

16, 8, 4, 2, 1, 6, 3, 8

17, 22, 11, 16, 8, 4, 2, 1, 6, 3, 8

18, 9, 14, 7, 12, 6, 3, 8, 4, 2, 1, 6

19, 24, 12, 6, 3, 8, 4, 2, 1 , 6

20, 10, 5, 10

The number 13 has 13 numbers in the chain before it repeats. It has six numbers in the chain that lead up to the loop and then seven numbers in the loop.

3 **1**, 2, 6, 8, 9, 18, 14, 12, 11, 22, 16, 13, 26, 18

2, 6, 8, 9, 18, 14, 12, 11, 22, 16, 13, 26, 18

3, 6, 8, 9, 18, 14, 12, 11, 22, 16, 13, 26, 18

4, 7, 14, 12, 11, 22, 16, 13, 26, 18, 14

5, 10

6, 8, 9, 18, 14, 12, 11, 22, 16, 13, 26, 18

7, 14, 12, 11, 22, 16, 13, 26, 18, 14

8, 9, 18, 14, 12, 11, 22, 16, 13, 26, 18

9, 18, 14, 12, 11, 22, 16, 13, 26, 18

10, 5, 10

11, 22, 16, 13, 26, 18, 14, 12, 11

12, 11, 22, 16, 13, 26, 18, 14, 12

13, 26, 18, 14, 12, 11, 22, 16, 13

The number chains above are linked because they all settled to the same pattern of numbers in the same order apart from the chain for 5 and 10 which is different.

Going deeper

1 **1**, 4, 2, 1

2, 1, 4, 2

3, 12, 6, 3

4, 2, 1, 4

5, 20, 10, 5

6, 3, 12, 6

7, 28, 14, 7

8, 4, 2, 1, 4

9, 36, 18, 9

10, 5, 20, 10

The longest chain starts with 8.

2 If the first number is odd, then the chain rules requires this number to be multiplied by 4, so it is now a multiple of 4. As all multiples of 4 are even the next move is to halve it. If you halve a multiple of 4 you get another even number which needs to be halved. When a number is halved and halved it again this undoes the process of multiplying by 4. We can write this chain as: $x \longrightarrow 4x \longrightarrow 2x \longrightarrow x$.

If the first number is even then the first move in the chain will be to halve the number, if the number is then odd then next it will be multiplied by four and then because it's even it will be halved again. So multiplying by 4 and halving will result in the chain getting back to double the odd number, which is where it started. The even chain that leads to an odd number after one move can be written as: $x \div 2 \longrightarrow 4x \longrightarrow 2x \longrightarrow x$. In the case of the longest chain starting with 8, this chain requires two halving moves before an odd number is reached and it therefore has five not four moves.

3 The chains up to 20 also repeat on the fourth move apart from 16 which requires two halving moves before settling into the 4, 2, 1, 4 chain. This is because it is a multiple of 8.

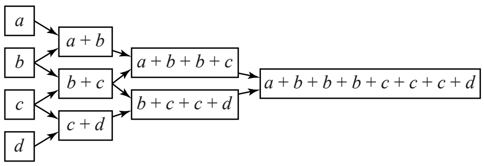
4 With other starting numbers up to 100, the multiples of 8 make the longest chains.

8, 4, 2, 1, 4
16, 8, 4, 2, 1, 4
24, 12, 6, 3, 12
32, 16, 8, 4, 2, 1, 4
40, 10, 5, 20, 10
48, 24, 12, 6, 3, 12
56, 28, 14, 7, 28
64, 32, 16, 8, 4, 2, 1, 4
72, 36, 18, 9, 36
80, 40, 20, 10, 5, 20
88, 44, 22, 11, 44
96, 48, 24, 12, 6, 3, 12

Page 60: Puzzles and generalizing (P&A 2·4)

Practice

1 If children have completed Activity 4 on page 47 of *Number, Pattern and Calculating 6 Teaching Resource Handbook* (see the figure 9 diagram from the handbook shown below) they will know that the final number will be A + 3B + 3C + D and they can work out that 6 + 33 + 42 + 5 = 86.



2 B and C need to be the smallest numbers: 5 and 6.

3 Answers will vary. Some might include reference to how to make the greatest or smallest total.

Going deeper

1 The last two numbers need to be as far apart as possible.

2 B and C need to be close together.

Page 61: Growing sequences (P&A 2·5)

Practice

1 Ten 5–rods plus a 10-rod.

2 $1 \times 2 + 2$, $2 \times 4 + 4$, $3 \times 6 + 6$, $4 \times 8 + 8$, $5 \times 10 + 10$

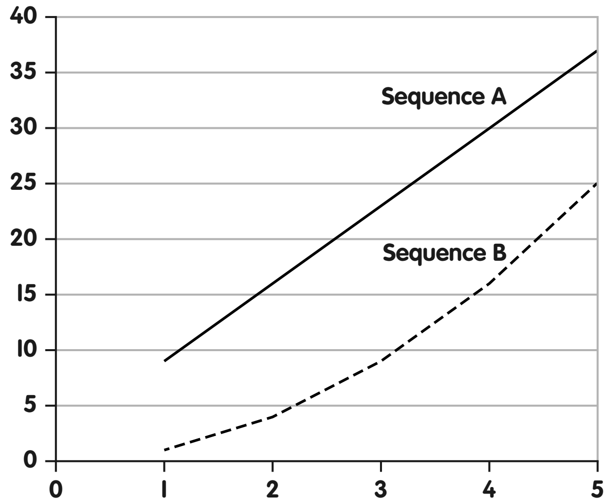
3 $20 \times 40 + 40$

4 $n \times 2n + 2n$ $\qquad$ $2n\,(n + 1)$

5 Answers will vary.

Going deeper

1



2 Answers will vary but might reference the shape of the graph with some explanation, e.g. Sequence A produces a straight line because the sequence increases regularly in steps of 7. Sequence B produces a curved line because the step size is growing by 2.

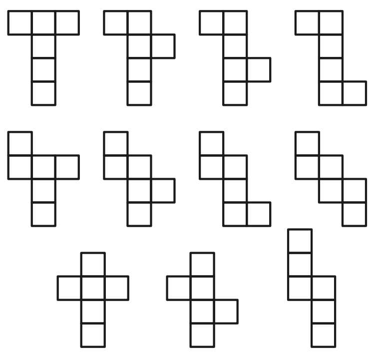
NPC Milestone 3

- Generate and describe linear number sequences including expressing term to term and general rules of number patterns

Page 62: Exploring nets of cubes and cuboids (Mea 3·1)

Practice

1 There are 11 possibilities in total.

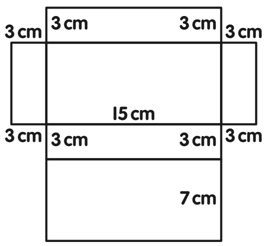


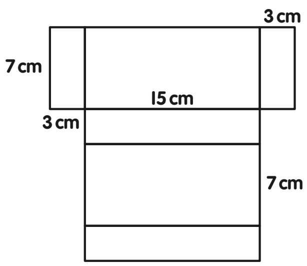
2 Polygons are 2D closed shapes made from straight sides. Polyhedrons are 3D shapes whose faces are all polygons.

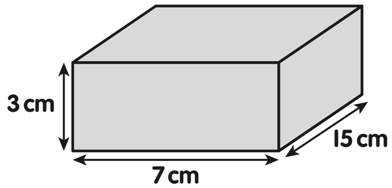
Going deeper

1 Some cannot be folded into cubes due to the faces overlapping when folded.

2 a







b 315 cm^3

Page 63: Investigating nets (Mea 3·2)

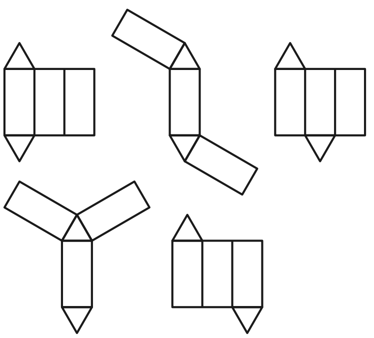
Practice

1 **a** and **b** Draw a net then make a box with dimensions 4 cm × 6 cm × 10 cm.

2 240 cm^3 (4 × 6 × 10 cm)

Going deeper

1 **a** and **b** Answers will vary, including

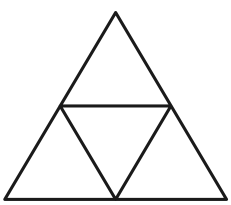


c 355·2 cm^2

Page 64: Platonic solids (Mea 3·2)

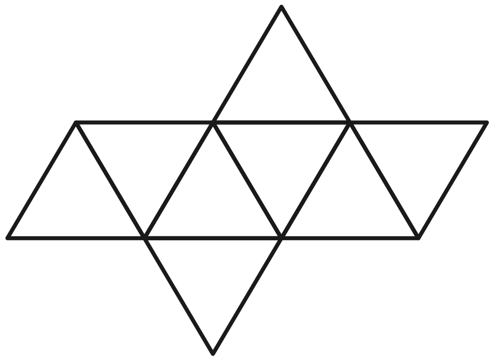
Practice

1 One possible way:



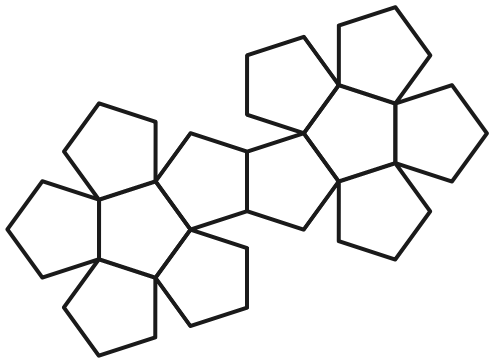
2 **Guidance:** Children could use the net of the tetrahedron they made for question 1 to help them create the net of the octahedron.

One possible way:



Going deeper

1 One possible way:



2

Shape	Faces	Vertices	Edges
Tetrahedron	4	4	6
Cube	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

Page 65: Solving problems involving surface area and nets (Mea 3·3)

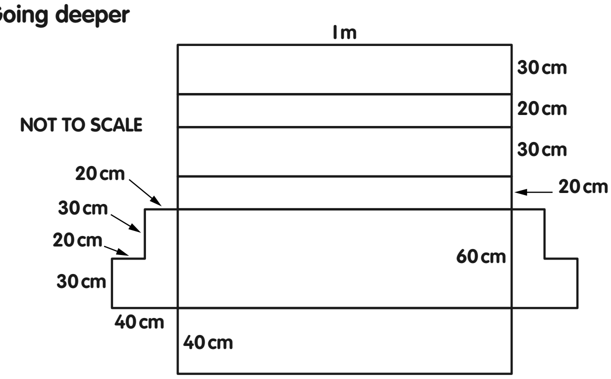
Practice

1 **a** 84·7 cm^2

b The total surface area to cover, in metres, is 84·7 m^2, so they would need to order 85 m^2 to cover all the cakes.

Going deeper

1



2 2 bottles – total area is 2·36 m². When children convert cm² into m² it may help them to visualize a small square in the corner of a larger square. 100 cm = 1 m but 10 000 cm² = 1 m².

GMS Milestone 2

- Recognize and create nets of cubes
- Create nets of cuboids and prisms
- Use nets to calculate surface area

Page 66: Multiplying using the short written method (Calc 9·1)

Practice

1 Look for children who use rounding to estimate.

a 24 600

b 92 292

2 Children should identify that **a** will be more than 10 000.

a 11 193 **b** 8524 **c** 8323

3 Look for children who identify that just 9 × 1000 is 9000 and there is also 600 to multiply by 9 so it is obvious the answer isn't big enough.

Going deeper

1 Answers will vary. Example response could be: Multiply 3075 × 10 and then take away 6150 (2 × 3075).

2 Answers will vary. Example response could be:

4175 × 3

2100 × 6

2995 × 4

Page 67: Multiplying decimals (Calc 9·2)

Practice

1 375 cm or 3·75 m

2 Estimates will vary.

a £25·68

b 50·4 cm

c 32·55 ℓ

Going deeper

1 She could do 9 × 62·5 cm or she could take her answer from question 1 and add on 3 × 62·5 cm.

2 By estimating children can place the decimal point in the appropriate place after calculating.

a £54·56

b 208·8 km

c 210·8 m

3 The first answer should be 1692 mm.

The second answer is correct.

The third answer should be 228·2 ℓ.

Page 68: Using long multiplication: whole numbers and decimals (Calc 9·3 & 9·4)

Practice

1 **a** Picture books – £54·75

Fantasy – £135

Fairy Tales – £73·50

Non-fiction – £239·52

Poetry – £189

Box-sets – £288

b £979·77

Going deeper

1 He has partitioned the number using place value and then multiplied each part. The sum of each row in the grid corresponds to a row in the long multiplication.

2 Divide it by 100 to convert pence to pounds.

3 Answers will vary. Example response: 6789 × 45 = 305 505.

Page 69: Problem solving using long multiplication (Calc 9·5)

Practice

1 The yellow star is worth 2, the blue rhombus is worth 6, the green cross is worth 3 and the red heart is worth 4.

a 338

b 7704

2 Answers will vary.

Going deeper

1 9

2 Answers will vary.

3 Answers will vary. Example response: 5642 × 12 = 67 704.

NPC Milestone 4

- Use short and long multiplying and dividing to solve problems, including those involving decimals

Page 70: Using long dividing to solve sharing problems (Calc 10·2)

Practice

1 **a** Answers will vary. Look for children using place value terminology correctly.

b 32 × 16 = 512

2 72

Going deeper

1 The word problems will all be different, just check they are sharing and not grouping scenarios.

a 216 leaflets shared between 12 classes, how many leaflets will each class get? (18)

b 780 newspapers shared between 15 newsagents, how many newspapers each? (52)

c £1508 lottery winnings shared between 26 people, how much so they all get each? (£58)

2 315 ÷ 21 = 15

1476 ÷ 18 = 82

888 ÷ 12 = 74

Page 71: Using long dividing to solve grouping problems (Calc 10·3 & 10·4)

Practice

1 **a** Children should be able to explain Molly's method by making links to multiplication facts.

b 27 × 35 = 945

c 43

2 17 £800 ÷ 16 is £50 and £52 ÷ 16 is £3, with 4 left over. If the remaining £4 is divided between the sixteen charities they would each get 25p and so a total of £52.2

3 32

Going deeper

1 **a** 2785 ÷ 22

b 3056 ÷ 12

2 **a** Will have a remainder of 22 because 72 is not a multiple of 25. 142 r22.

b Will divide exactly into 103 as 36 is a multiple of 12 as is 1200.

c Will have a remainder. 144 r 4.

d No remainder, will divide exactly into 16.

Page 72: Solving dividing problems with remainders (Calc 10·5 & 10·6)

Practice

1 £56·50 Children may reason by saying that £600 ÷ 12 is £50 and £72 ÷ 12 is £6, with £6 left over. If the remaining £6 is divided between the twelve charities they would each get 50p and so a total of £56·50.

2 Answers will vary.

£53·25 Children may reason by saying that £800 ÷ 16 is £50 and £52 ÷ 16 is £3, with £4 left over. If the remaining £4 is divided between the sixteen charities they would each get 25p and so a total of £52·25.

3 54·25 cm

4 98·2 ml

Going deeper

1 £5·75 more so £62·25

2 £18·75

3 **a** 774 cm

b 5047 ml or 5·047 ℓ

Multiply the two numbers together to find the missing number. And then do the dividing calculation to check you are correct.

Page 73: Dividing decimals equally using long or short dividing (Calc 10·7)

Practice

1 **a** Table 2 must be 20 people as it is the only amount that is a multiple of 10 and divides equally: £4·79.

Table 3 must be 15 people as it is the only amount that is a multiple of 5: £11·05.

Table 1 is therefore 12 people: £3·81.

b Possible response:

Table 1: £45·72 – 3, 4, 6

Table 2: £95·80 – 5, 10, 2

Table 3: £165·75 – 3, 5, 75

2 27·5 cm

Going deeper

1 Answers will vary. Example:

£155·50 ÷ 5 = £31·10

£260 ÷ 10 = £26

£135 ÷ 6 = £22·50

2 Answers will vary. Example:

a 2, 4, 5, 8, 10

b 3, 9, 15

3 5·6 m ÷ 2 = 280 cm

5·6 m ÷ 4 = 140 cm

5·6 m ÷ 5 = 112 cm

5·6 ÷ 7 = 80 cm

5·6 ÷ 8 = 70 cm

5·6 ÷ 10 = 56 cm

5·6 ÷ 14 = 40 cm

5·6 ÷ 16 = 35 cm

5·6 ÷ 20 = 28 cm

5·6 ÷ 28 = 20 cm

NPC Milestone 4

- Use short and long multiplying and dividing to solve problems, including those involving decimals

Page 74: Exploring volume and scaling (Mea 4·1)

Practice

1 **a** 72 cm³

b 3, 9, 15.

c 12 altogether. Dimensions are:

1 × 1 × 72

1 × 2 × 36

1 × 3 × 24

1 × 4 × 18

1 × 6 × 12

1 × 8 × 9

2 × 2 × 18

2 × 3 × 12

2 × 4 × 9

2 × 6 × 6

3 × 3 × 8

3 × 4 × 6

2 1 cm by 1 cm by 40 cm; 1 cm by 2 cm by 20 cm; 1 cm by 4 cm by 10 cm; 1 cm by 5 cm by 8 cm; 2 cm by 2 cm by 10 cm; 2 cm by 4 cm by 5 cm. You know you are correct because all the dimensions multiply to give an answer of 40.

Going deeper

1 36 cm³

2 Answers will vary, e.g. triangular face with base 2 cm, height 5 cm and prism length of 12 cm; triangular face with base 2 cm, height 6 cm and prism length of 10 cm; triangular face with base 4 cm, height 6 cm and prism length of 5 cm.

Page 75: Solving volume problems (Mea 4·2)

Practice

1 They must multiply to make 48, as this times 10 would give the volume of 480 cm³. So factor pairs of 48 are: 1 × 48, 2 × 24, 3 × 16, 4 × 12 and 6 × 8, all in cm.

2 6 cm by 6 cm by 6 cm

Going deeper

1 10 cm by 10 cm by 15 cm

2 Answers will vary, e.g. a parcel measuring 20 cm × 5 cm × 4 cm. The parcel will have a volume of 400 cm³.

Page 76: Units of volume (Mea 4·3)

Practice

1 **a** cm³ **b** mm³ **c** m³ **d** m³ **e** m³ **f** cm³

2 £3000

3 £3 000 000 (3 million)

Going deeper

1 **a** 45 m³

b £1125

c 22·5 m³

2 You would need to know the approximate equivalence relationship between cubic yards and cubic metres and then find the equivalent of 45 m³ in cubic yards.

Page 77: Scaling and scale factors (Mea 4·4 & 4·5)

Practice

1 **a** $9{\cdot}216\,m^3$ **b** 0·3 m × 0·4 m × 0·3 m **c** $0{\cdot}018\,m^3$

Going deeper

1 **a** $375\,m^2$ **b** $3750\,m^3$ **c** 2 m by 2 m

d $375\,cm^2$ **e** $3{\cdot}75\,m^3$

GMS Milestone 3

- Carry out calculations involving lengths and volumes of cubes and other cuboids, using formulae where appropriate
- Convert between different metric units of volume
- Use and understand the effects of scaling on area and volume

Page 78: Adding and subtracting with fractions (Calc 11·1)

Practice

1 **a** $\frac{2}{5}$

b 25: $\frac{2}{5}$ is 10, so $\frac{1}{5}$ is 5, so $\frac{5}{5}$ is 25

c $\frac{2}{5}$

2 **a** $\frac{4}{9}$

b Answers will vary.

Going deeper

1 **a** Kamal: £22·50, Theo £50

b £80

2 500 ml of purée, 1 litre of milk and 2 litres of water

Page 79: Adding and subtracting fractions whose denominators are multiples of the same number (Calc 11·2 & 11·3)

Practice

1 $\frac{3}{10}$. $\frac{2}{5}$ and $\frac{3}{10}$ is $\frac{7}{10}$, leaving $\frac{3}{10}$ remaining.

2 $\frac{1}{3}$

3 $\frac{7}{10}$

4 $\frac{1}{2}$

Going deeper

1 $\frac{2}{3}, \frac{13}{18}, \frac{7}{9}, \frac{5}{6}$. So $\frac{2}{3}$ is smallest, $\frac{5}{6}$ is largest.

3 **a** There are five possible pairs: 4 and 1, 3 and 3, 2 and 5, 1 and 7, 0 and 9.

b The missing numbers are 45 (denominator) and 4 (numerator).

Page 80: Adding and subtracting more fractions (Calc 11·4)

Practice

1 $\frac{7}{12}$

2 $\frac{7}{15}$

3 $\frac{7}{20}$

Going deeper

1 $\frac{7}{24}$

2 He put back $\frac{1}{12}$ of the original number of bricks.

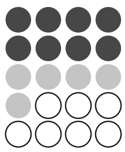
Page 81: Using arrays to support calculating with fractions (Calc 11·5)

Practice

1 **a** $\frac{10}{12}$ or $\frac{5}{6}$

b $\frac{2}{12}$ or $\frac{1}{6}$

2

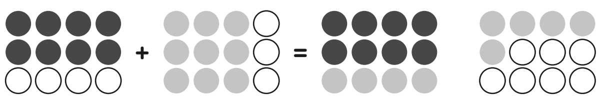


$\frac{7}{20}$ because $\frac{2}{5}$ is equivalent to $\frac{8}{20}$ and $\frac{1}{4}$ is equivalent to $\frac{5}{20}$. $\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$, which leaves $\frac{7}{20}$.

3 $\frac{7}{12}$

Going deeper

1 $\frac{2}{3}$ $\frac{3}{4}$ 1 $\frac{5}{12}$



2 Answers will vary, e.g. $\frac{3}{5} + \frac{4}{15}, \frac{2}{3} + \frac{3}{15}, \frac{6}{15} + \frac{7}{15}$.

NPC Milestone 4

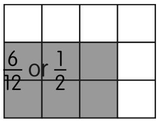
- Add and subtract fractions and mixed numbers

Page 82: Understanding multiplying fractions (Calc 12·1)

Practice

1 $\frac{3}{12}$ or $\frac{1}{4}$

2



Going deeper

1 Answers will vary, e.g.

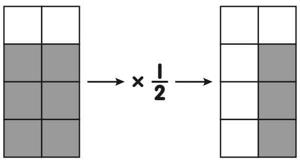
a $\frac{1}{3} \times \frac{1}{2}$ **b** $\frac{2}{3} \times \frac{3}{4}$ **c** $\frac{4}{5} \times \frac{1}{3}$

Page 83: Multiplying two fractions using a fraction machine (Calc 12·2)

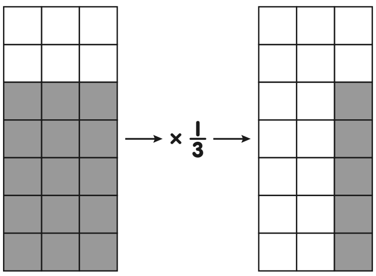
Practice

1 $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

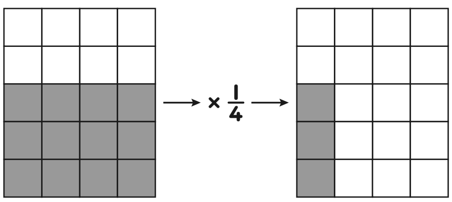
2 **a**



b



c



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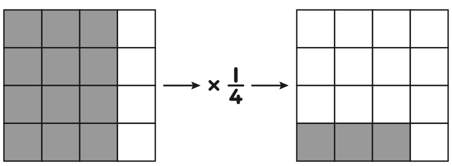
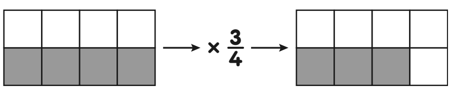


e



3 Answers may vary; importantly the answer is the product of the numerators over the product of the denominators.

Going deeper

1 She is working out $\frac{4}{10}$ or $\frac{2}{5} \times \frac{1}{2} = \frac{2}{10}$ or $\frac{1}{5}$. First she covers $\frac{2}{5}$ of the dark blue Shape, then half of the green Shape to find that she has covered $\frac{1}{5}$ of the dark blue Shape with the light blue Shape.

2 Answers will vary.

Page 84: Multiplying two fractions using a general rule (Calc 12·3)

Practice

1 $\frac{1}{2}$; various illustrations possible.

2 $\frac{1}{6}$

3 $\frac{1}{2}$; multiply numerators, multiply denominators, simplify

Going deeper

1 The friends receive $\frac{4}{9}$ of the original cake, the daughter has $\frac{2}{9}$.

2 Luke could have typed $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$ or any equivalent.

3 **a** $\frac{5}{6} \times \frac{2}{5}$ **b** $\frac{2}{3} \times \frac{2}{5}$ **c** $\frac{2}{5} \times \frac{1}{4}$ **d** $\frac{5}{6} \times \frac{1}{5}$

Page 85: Dividing a proper fraction by a whole number (Calc 12·6)

Practice

1 **a** $\frac{1}{6}$ each child

b 4 cakes each; various array diagrams possible.

2 **a** $\frac{1}{5}$ **b** $\frac{3}{20}$ **c** $\frac{3}{25}$

Going deeper

1 **a** $\frac{1}{5}$

b 100 ml

2 $\frac{1}{12}$ of her pocket money on each item.

NPC Milestone 4

- Multiply simple pairs of proper fractions
- Divide proper fractions by whole numbers

Page 86: Solving empty box problems (P&A 3·1)

Practice

1 For the first empty box problem there is only one possible answer, 26 + 34 = 60.

For the second empty box problem there are 59 possibilities, e.g. 59 + 1, … , 1 + 59.

For the third empty box problem there are an infinite number of possibilities because the total can be any whole number.

2 The prime factors of 462 are 2, 3, 7, 11.

$2 \times 3 \times 77$; $6 \times 11 \times 7$; $22 \times 3 \times 7$

3 Answers will vary. One example is $80 \div 4 = 20$. $160 \div 8 = 20$. $80 \div 2 \div 2$. $160 \div 4 \div 2$.

Going deeper

1 Using inverse: If $18 + a = 25$ then $25 - 18 = a$ so $a = 7$.

2 $6 \times a = 78$; $a \times 6 = 78$; $78 \div a = 6$; $78 \div 6 = a$

3 $\frac{3}{11} \times \frac{5}{2}$; $\frac{5}{2} \times \frac{3}{11}$; $\frac{1}{1} \times \frac{15}{22}$

Page 87: Symbolic notation (P&A 3·2)

Practice

1 $2a + 2b$

2 Answers will vary. Examples might include: 12, 7 = 61. 10, 8 = 74. 5, 9 = 86.

3 **a** $2b + a + c$ **b** $2c + b + d$

4 $3b + 3c + a + d$

Going deeper

1 The total of the circles is $a + b + c$. The total of the rectangles is $2a + 2b + 2c$. The total in rectangles is double the circle total.

2 $a^2 + b^2 + c^2$. There are a range of other possible rules for this diagram including $2a + 3b + 4c$ and $6a + b^2 + 2c$.

Page 88: Solving problems with algebra (P&A 3·3 & 3·4)

Practice

1 $a = 5$ $b = 7$ $c = 10$

2 **a** blue = 6, red = 9, yellow = 14

b yellow = 11, red = 16, blue = 14

c blue = 3, red = 7, yellow = 5

d blue = 6, red = 9, yellow = 4

Going deeper

1 $2b + a = 10$ and $2b + c = 14$, so c must be 4 more than a. This means that (from the middle row) $3a + 4 = 10$, so $3a = 6$, and $a = 2$. If $a = 2$, then $c = 6$. If $a = 2$, then (from the top row) $2b = 8$ and $b = 4$.

2 Answers will vary. One approach is to use the fact that $a \times b = 18$ and the fact that $a \times b \times c = 126$ to work out c. $c = 126$ divided by 18, which is 7. The other values can then be worked out using known facts.

Page 89: Finding all possibilities (P&A 3·5)

Practice

1 The following could be illustrated with number rods or in a table.

36: three 10-rods and a 6-rod

Followed by eighteen 2-rods

Then sixteen 2-rods and one 4-rod

Twelve 2-rods and three 4-rods

Ten 2-rods and four 4-rods

Eight 2-rods and five 4-rods

Six 2-rods and six 4-rods

Four 2-rods and seven 4-rods

Two 2-rods and eight 4-rods

2 Looking at the table or rods the one with 15 rods is 12 bikes 3 cars.

3 The number of wheels is 2 times the number of bikes (a) and 4 times the number of cars (b) which can be written as $W = 2a + 4b$.

Going deeper

1 $52 = 2a + 4b$ and $a + b = 21$. $a = 16$ and $b = 5$.

2 Answers will vary. Some children might say that they know that the number of cars cannot be more than 13 because $13 \times 4 = 52$. So this eliminates some of the possible pairs of numbers.

Others may explain that they started with multiples of 4 and worked out pairs of multiples of 4 and 2 that make 52.

Listing multiples of 4:

We can work out that 52 wheels is 13 cars and 0 bikes so 13 vehicles.

48 is 12 cars and 2 bikes which is 14 vehicles.

44 is 11 cars and 4 bikes which is 15 vehicles.

40 is 10 cars and 6 bikes which is 16 vehicles.

Following this pattern:

36 would be 17 vehicles

32: 18 vehicles.

28: 19 vehicles

24: 20 vehicles

20: 21 vehicles

20 is 5 cars and 16 bikes.

NPC Milestone 4

- Express missing number problems algebraically
- Enumerate possibilities of combinations of two unknowns

Page 90: Exploring circles (Geo 2·1 & 2·2)

All answers are given to 2 decimal places using Pi as 3·14 unless otherwise specified.

Practice

1 Diameter

2 a Approximately 24 cm using Pi as 3·0.

b Actual circumference is 25·12 cm using Pi as 3·14.

3 a Radius – 2·4 cm

Circumference – 15·07 cm

b Diameter – 10·51 cm

Radius – 5·26 cm

c Diameter – 9 cm

Circumference – 28·26 cm

Going deeper

1 $C = 3{\cdot}14 \times D$

$C = 3{\cdot}14 \times 2r$

2 Answers will vary.

Page 91: Using the relationship between diameter and circumference (Geo 2·3)

Practice

1 2·83 m

2 9·8 × 3·14 = 30·772 (whole circumference)

15·39 m (semicircle)

3 a 14·44 feet **b** 9·42 km **c** 31·4 mm

Going deeper

1 The outline of their diagram should measure 27·5 cm × 15·25 cm with each segment being approximately 9·2 cm wide.

2 a 8·03 m **b** 4·01 m

Page 92: Solving circle problems (Geo 2·4)

Practice

1 22·5 cm × 3·14 = 70·65 cm

2 16·5 cm × 3·14 = 51·81 cm

3 a Check children's diagrams. The diameter of the two largest circles should each be 17 cm.

b 17 cm × 3·14 = 53·38 cm

Going deeper

1 The radius is 9 cm so the diameter of the circular pie must be double this (18 cm). The circumference of the pie is about 3·14 × the diameter. 3·14 × 18 = 56·52 cm.

2 The circumference of the new pie is less than the circumference of the pie in Going deeper question 1, so you would expect the slice of pie to be smaller. Estimated lengths will vary. If children estimate that circumference is roughly 3 times the diameter, then 30 ÷ 3 gives a diameter of 10 cm. The radius (length of the pie slice) is half of this, so 5 cm long. If children calculate the length more precisely 30 ÷ 3·14 gives a diameter of 9·6 cm and radius (slice of pie length) of 4·8 cm.

Page 93: Solving more problems (Geo 2·4)

Practice

1 Look at what happens to the circumference as the diameter increases, e.g. 4 × 3·14; 8 × 3·14.

2 a Gear A circumference = 25·12 cm

b Gear B diameter = 4 cm

Gear B circumference = 12·56 cm

Going deeper

1 4 × 12·56 = 50·24 cm

2 25·12 + 6·28 = 31·4 cm

3 24 × 12·56 = 301·44 cm

GMS Milestone 3

- Recognize and name the radius, diameter and circumference of any circle
- Recognize that the diameter of any circle is twice the radius

Page 94: Solving non-routine problems using all four operations (Calc 13·1)

Practice

1 a Alistair was fastest; Isaac was slowest.

180 (3 minutes) ÷ 10 (lengths); 240 ÷ 12; 120 ÷ 8; 180 ÷ 15

b Answers will vary.

2 Approximate distances: Anita – 33 lengths; Isaac – 30 lengths; Shona – 40 lengths; Alistair – 50 lengths.

Going deeper

1 12 seconds (Alistair would beat Shona)

Various methods possible, e.g. Alistair's average speed = 180 (3 minutes) ÷ 15 (lengths) = 12 seconds per length, so four lengths = 48 seconds; Shona's average speed = 120 ÷ 8 = 15 seconds per length, so four lengths = 60 seconds. Alistair is quicker by 12 seconds.

Page 95: Solving non-routine problems using fractions and percentages (Calc 13·2)

Practice

1 D

2 £38·80

3 A £7·50

B £7·20

C £3

D £5

Going deeper

1 Emilia bought pair B, pair C and pair E.

Page 96: Solving non-routine problems – multiplying and dividing (Calc 13·3)

Practice

1 Rachel's, as the distance travelled in one revolution is pi times the diameter, so the larger the diameter, the further the bike will travel in one turn. As Mo has the larger wheel, he will travel further per revolution, so needs fewer revolutions to go the same distance.

2 Rachel – 1350 cm; Mo 1650 cm

3 Mo will be 30 m ahead.

Going deeper

1 **a** 741 revolutions **b** 606 revolutions

2 1 metre

Page 97: Further non-routine problems (Calc 13·4)

Practice

1 50 m

2 45 seconds

3 1 min 15 s

Going deeper

1 1·5 times

2 Leo will finish first; they both travel the same distance but Leo is running for a greater proportion of the distance than Rajesh.

NPC Milestone 5

- Solve non-routine problems using all four operations

Page 98: Exploring the four quadrants (Geo 3·1)

Practice

1 Lighthouse (5, 7)

Cafe (2, 3)

Food shop (5, 1)

Post office (2, ⁻2)

Beach shop (4, ⁻4)

Campsite (⁻5, 4)

Landing jetting (⁻5, 2)

Swimming pool (⁻3, ⁻1)

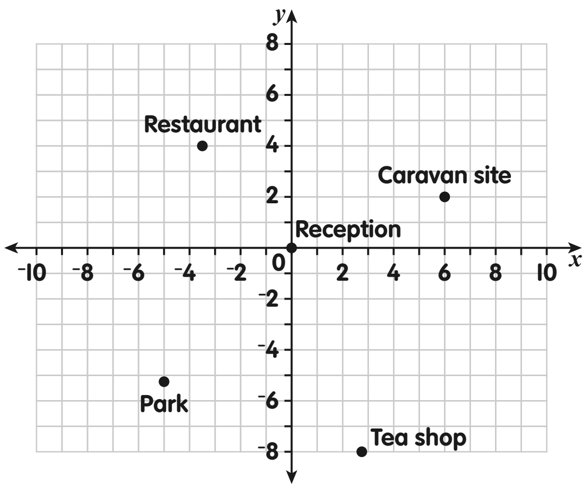
Hotel (⁻5, ⁻3)

Tennis courts (⁻3, ⁻5)

2 (0, 5), (9, 0), (0, ⁻6), (⁻8, 0)

Going deeper

1

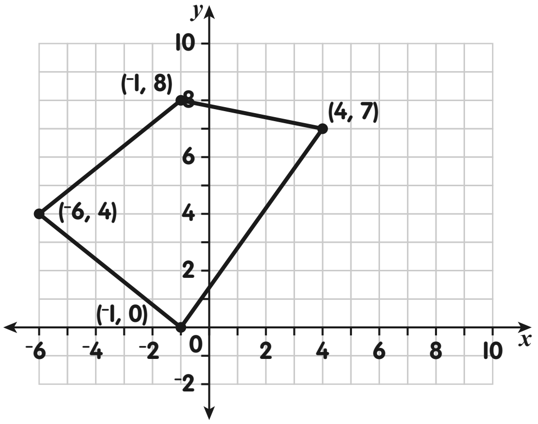


2 Look for children using the correct vocabulary and describing the quadrants correctly.

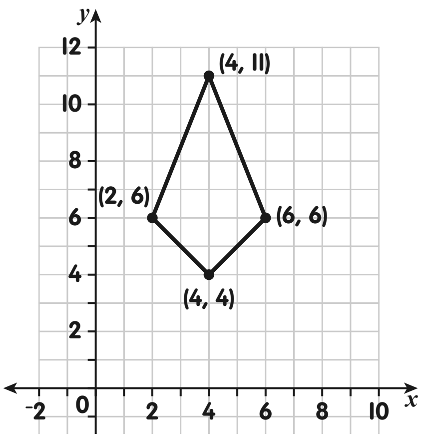
Page 99: Translation in four quadrants (Geo 3·3)

Practice

1



2 a and **b** Answers will vary. Example below. Children should be able to translate across and up and down.

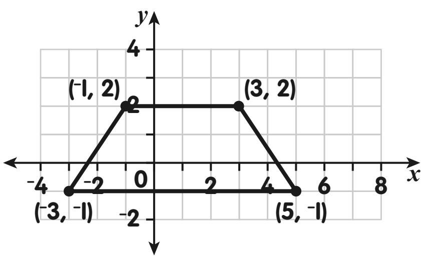


3 (5,2), (2,6), (8,-3)

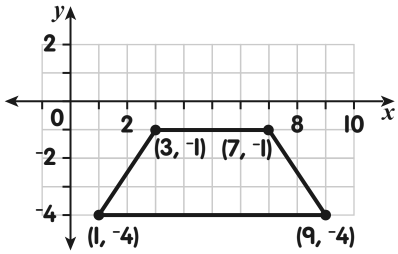
Going deeper

1 (-2,-1), (3,-5)

2 a



b and **c** Answers will vary. Example below, translated by $(x + 4), (y - 3)$.

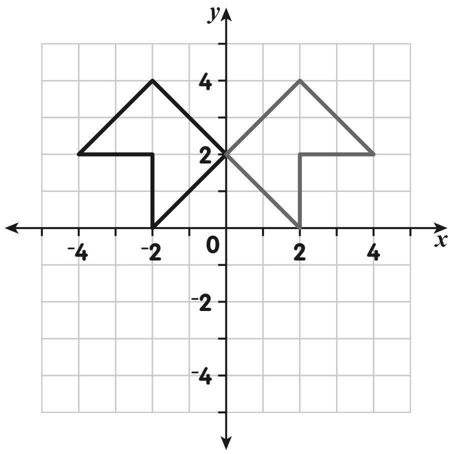


Page 100: Exploring reflections in the four quadrants (Geo 3·4)

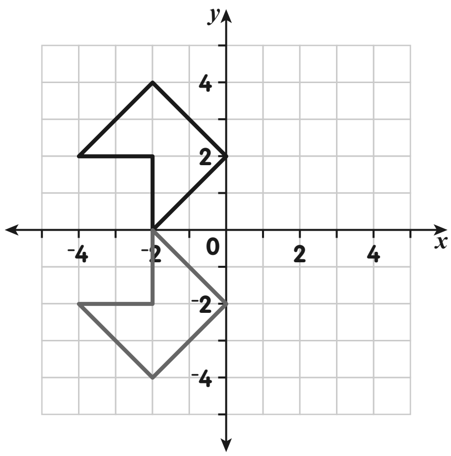
Practice

1 a y-axis

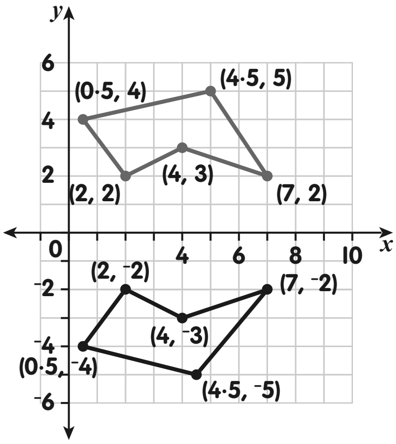
b



c (-2,0), (-2,-2), (-4,-2), (-2,-4), (0,-2)



2 Answers will vary. Any five-sided shape in the bottom right quadrant is correct. Example:

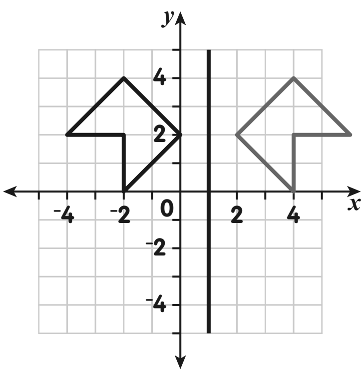


3 (3,2) and (3,⁻1), (0,2) (0,⁻1)

Going deeper

1 **a** (2,2), (4,0), (4,2), (6,2), (4,4)

b



2 $y = {}^-1$

Page 101: Exploring quadrilaterals on a coordinate grid (Geo 3·5)

Practice

1 **a** Answers will vary. Example (6,4).

b (10,4)

c (7,4)

d (5,6) – answers could vary here. Check the shape has one set of parallel lines.

Going deeper

1 (7,4)

2 Answers will vary.

GMS Milestone $\boxed{3}$

- Read and plot points using coordinates in all four quadrants
- Describe, draw and translate 2D shapes using the coordinates of their vertices
- Reflect points and shapes in both x and y axes using coordinates
- Describe the movements of shapes accurately using the language of transformations

Page 102: Using symbols and letters (P&A 4·1)

Practice

1 The number 'above' (or before) 26 is 9 fewer than 26, the number 'below' (or after) 26 is 9 greater.

2 The number above n is $n - 9$, the number below is $n + 9$.

3 The total is $n + (n - 9) + (n + 9) = 3n$.

Going deeper

1 If n is now 17, the total of all three numbers will be $n + (n + 9) + (n + 18) = 3n + 27$.

2 The total for the '15, 26, 37' pattern, using 'n' as the middle number, will be $n + (n - 11) + (n + 11) = 3n$.

3 Answers will vary, e.g. the totals are the same because these 'diagonal three' patterns of squares will always involve both adding and subtracting the same number (either 9 or 11) to the middle number.

Page 103: Generalizing sequences (P&A 4·2)

Practice

1 The term-to-term rule is 'Add 7'.

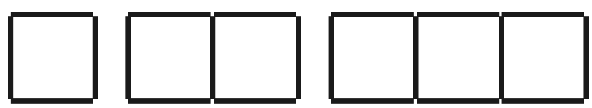
2 The 10th term will be $1 + (7 \times 10) = 71$.

3 The nth term will be $1 + (7 \times n) = 7n + 1$.

4 Answers will vary.

Going deeper

1 Answers will vary, e.g.



2 The nth term will be $12 + (9 \times n) = 9n + 12$ or $3(3n + 4)$.

3 Answers will vary, but could follow the steps of Activity 2 from *Number, Pattern and Calculating 6 Teaching Resource Handbook*, page 63.

Page 104: Exploring function machines (P&A 4·4)

Practice

1 The machine is multiplying each input by 7.

2 $y = 7x$

3 $y = 3x + 8$

Going deeper

1 If the output is 23, then $23 = x - 6$, so x (the input) = 29.

2 $x = y + 6$

3 **a** $y = 2x + 1$

b The nth term of the sequence 3, 5, 7, 9… is $2n + 1$.

(Example explanation) The nth term of the sequence in b is calculated using the same rule that the outputs in a are calculated from their inputs, i.e. by the rule, 'double it plus 1'. So the inputs to the function machine in a are effectively equivalent to the term numbers of the sequence in b, and the same 'rule' is applied to both, giving identical 'outputs'.

Page 105: Rules of arithmetic (P&A 4·6)

Practice

1 If you are adding three numbers together, it doesn't matter which pair you add first, the total will always be the same. (This is the Associative Law of Addition.)

2 $(a + b) + c = d$; $a + (b + c) = d$; $(a + c) + b = d$

3 **a** If you are multiplying three numbers together, it doesn't matter which pair you multiply first, the product will always be the same. (This is the Associative Law of Multiplication.)

b $(a \times b) \times c = d$; $a \times (b \times c) = d$; $(a \times c) \times b = d$

Going deeper

1 **a** For example, because 200 on the right-hand side of the number sentence is 10 more than 190 on the left-hand side, but 27 on the right is 10 fewer than 37 on the left. So the right is both '10 more' and '10 fewer' than the left, which makes them both the same.

b Masam is correct because the +10 and ⁻10 cancel out each other leaving $a + b = a + b$.

2 For example, because 150 on the right-hand side is 5 more than 145 on the left-hand side, and 100 on the right is also 5 more than 95 on the left, the differences between the numbers on the left and between the numbers on the right will be the same.

$(a - b) = (a + 5) - (b + 5) = a + 5 - b - 5 = a - b$

3 This may be difficult for some children, but only generalises important information that they should already know about adding fractions.

The first key step is to realise that in order to add fractions together they must be expressed with a common denominator. So, if adding $\frac{1}{a}$ and $\frac{1}{b}$ their common denominator will be ***'ab'***.

The next key step is to convert both fractions to their equivalent forms with the common denominator. So $\frac{1}{a}$ becomes $\frac{b}{ab}$ and $\frac{1}{b}$ becomes $\frac{a}{ab}$.

Then $\frac{1}{a} + \frac{1}{b} = \frac{b}{ab} + \frac{a}{ab} = a + \frac{b}{ab}$.

Children may like to check that this matches exactly what happens in the example of adding $\frac{1}{2}$ and $\frac{1}{3}$ given in their book.

NPC Milestone 5

- Use symbols and letters to represent variables and unknowns in mathematical situations

Notes

Notes

Notes

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