numicon

Pupil Book 5 Answers

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www.oxfordprimary.co.uk/numicon

About Numicon

Numicon is a distinctive multi-sensory approach to children's mathematical learning that emphasizes three key aspects of doing mathematics: communicating mathematically, exploring relationships and generalizing.

Numicon was founded in the daily experience of intelligent children having real difficulty with maths, the frequent underestimation of the complexity of the ideas that young children are asked to face when doing maths and recognition of the importance of maths to them and to society as a whole.

Numicon aims to facilitate children's understanding and enjoyment of maths by using structured imagery that plays to children's strong sense of pattern. This is done through research-based, multi-sensory teaching activities.

Numicon takes into account the complexity of abstract number ideas and seeks to foster the self-belief necessary to achieve in the face of challenge or difficulty.

Through the combination of communicating mathematically (being active, talking and illustrating), exploring relationships and generalizing, children are given the support to structure their experiences: a vital skill for both their mathematical and their overall development.

A multi-sensory approach, particularly one that makes use of structured imagery, provides learners with the opportunity to play to their strengths, thereby releasing their potential to enjoy, understand and achieve in maths. By watching and listening to what children do and say, this enjoyment in achievement is also shared by teachers and parents.

Numicon strives to support teachers' subject knowledge and pedagogy by providing teaching materials, Professional Development and on-going support that will help develop a better understanding of how to encourage all learners in the vital early stages of their own mathematical journey.

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Written by Jayne Campling, Andrew Jeffrey,
Adella Osborne and Dr Tony Wing

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Using Numicon Pupil Books

Introduction

The Numicon Pupil Books have been created to help children develop mastery of the mathematics set out in Numicon Teaching Resource Handbook (TRH) activities. The questions in the Pupil Books extend children's experiences of live TRH activities, giving them the opportunity to reason and apply what they have learned, deepen their understanding, take on challenges and develop greater fluency.

Just like the teaching activities, all Pupil Book pages are designed to stimulate discussion, reasoning, and rich mathematical communicating. The Numicon approach to teaching mathematics is about dialogue. It is about encouraging children to communicate mathematically using the full range of mathematical imagery, terminology, conventions and symbols.

All questions in the Pupil Books relate to specific Numicon TRH activities. At the top of each Pupil Book page you can find details of the Activity Group the page relates to (for example Calculating 1). The number after the decimal point tells you which focus activities the page accompanies (so Calculating 1·2 goes with focus activity 2). It is crucial to teach the relevant focus activities before children work on the questions. The Pupil Book questions are designed for children who are succeeding with specific TRH activities, and will invite them to think more deeply about a topic. If you find that children are struggling with a focus activity, details are given in the Teaching Resource Handbooks of other live activities, provided earlier in the progression, which you can work through together to support them until they are ready to move on.

There's a recommended order to teach the Activity Groups in and the Pupil Book materials follow this order of progression too, as you'll see from the contents page. You can use this order to help children see how their ideas and understanding builds upon what they have learned before.

These Pupil Book questions have been developed as a large bank that you can select from to best meet the changing needs of the children in your class. You can decide which questions are suitable for which children at which time, and no child is expected to find every question useful. How you choose to use the questions might also vary, for example you may find that particular questions are useful to discuss and work through together as a class.

Intelligent practice

The 'Practice' sections target two areas. Routine practice is used to promote fluency with particular aspects or techniques. Non-routine practice questions offer challenges in varied ways designed both to improve fluency and to deepen and extend understanding. Practice for simple fluency usually comes first and the questions on each page become progressively more challenging.

Going deeper

'Going deeper' questions are designed to develop children's growing mastery of an area, challenging their understanding beyond routine exercises. In these sections children are commonly asked to check, explain and justify their strategies and thinking. Trying to explain something clearly helps promote, and is a key indicator of, developing mastery.

Using the Pupil Books

Doing mathematics involves much more than logic, and children's emotions are crucially important. Thoughtful progress is more likely to happen through encouraging curiosity and good humour, and engaging with children in a polite and calm way. This is why the phrasing and tone of Numicon Pupil Book questions is deliberately different to many mathematical textbooks. For example, we often begin questions for children with "Can you…?" If any child says simply "yes" or "no" in response, we'd suggest replying with "Can you show me how …?" or "That's interesting, can you say anything about why not?" These invitations are effective beginnings to the kinds of open conversation and discussions that are at the heart of the Numicon approach.

Some Pupil Book questions have a pair work symbol to signal that these require specific work with a partner, and help with classroom management. These are not the only questions where working with a partner is likely to be beneficial however. All Pupil Book questions should be seen as opportunities for rich mathematical communicating between anyone and everyone in the classroom at all times, and this should be actively encouraged wherever you think appropriate. The Numicon approach is crucially about dialogue – action, imagery and conversation.

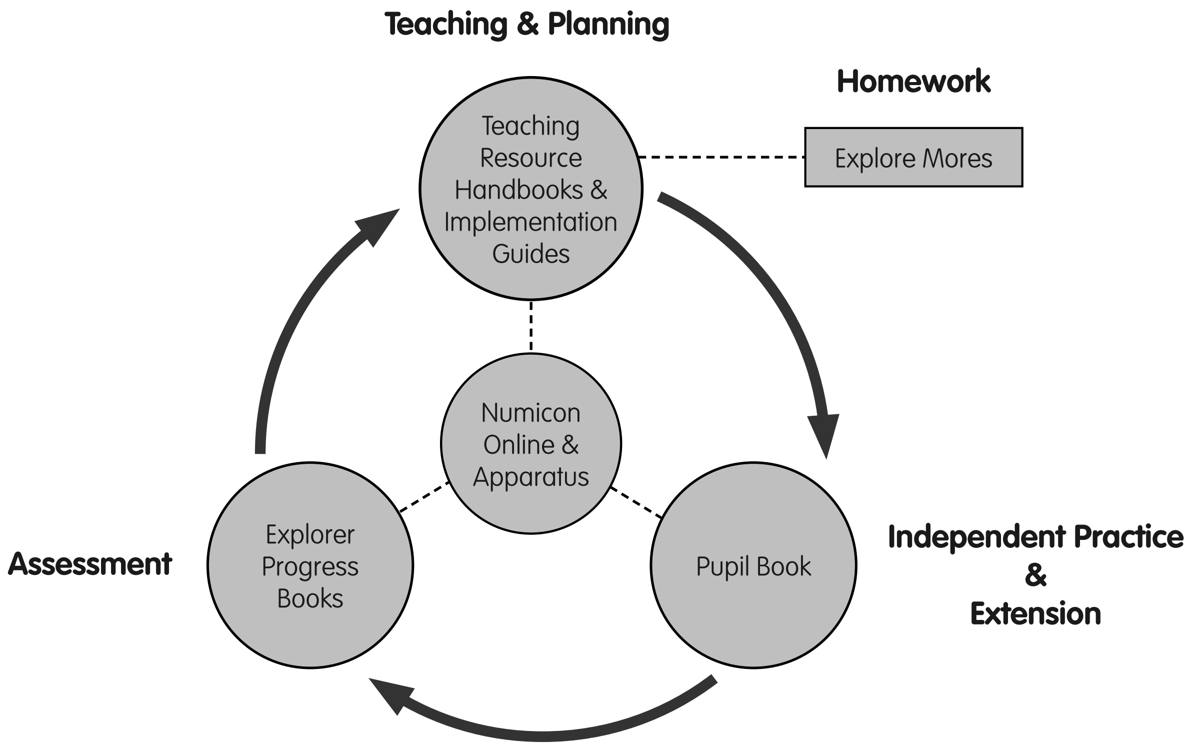
Finally, the Pupil Book questions are there to be enjoyed. Children who are supported, and who are succeeding, generally relish challenge and further difficulty. We hope you as teachers will also enjoy the journeys and pathways that these books will take children and their teachers jointly along.

Dr Tony Wing

A guide to the Numicon teaching resources

Numicon Pupil Books fit with the other resources shown here to fully support your teaching. You can also find additional resources, including an electronic copy of this answer book, on Numicon Online. This is available on the Oxford Owl website (www.oxfordowl.co.uk).

Numicon resources



Teaching Resource Handbooks

There is a Number, Pattern and Calculating and a Geometry, Measurement and Statistics Teaching Resource Handbook for each year group. The teaching in these handbooks is carried out through activities. You will find detailed support for planning and assessment here, along with vocabulary lists, the key mathematical ideas covered and photocopy masters.

Implementation Guides

Each Teaching Resource Handbook comes with an Implementation Guide. These provide guidance on the Numicon approach, how to implement this in the classroom and valuable information to support subject knowledge, including explanations of the key mathematical ideas covered and a glossary of mathematical terms used.

Explore More Copymasters

The Explore More Copymasters provide homework that enables children to practise what they are learning in school. For Geometry, Measurement and Statistics these are given in the back of the Teaching Resource Handbook. For Number, Pattern and Calculating these are provided in a separate book.

A homework activity is included for every Activity Group. Each one includes information for the parent or carer on the mathematics that has been learned in class beforehand and how to use the work together. These activities can also be used in school to provide extra practice.

Explorer Progress Books

There are four Explorer Progress books for each year group (one for Geometry, Measurement and Statistics and three for Number, Pattern and Calculating). There are two pages in the Explorer Progress Books for each Activity Group which can be used to assess children's progress, either immediately after the Pupil Book questions or at a later point to find out what learning has been retained. These progress books give children the opportunity to apply what they have learned to a new situation.

Apparatus

Physical apparatus	Apparatus on the Interactive Whiteboard Software

A wide range of apparatus and structured imagery is used in Numicon to enable children to explore abstract mathematical ideas. You can find digital versions of this apparatus in the Interactive Whiteboard Software available through Numicon Online. Here you can manipulate the apparatus from the front of the class and save anything you have set up for future use.

Numicon Online for planning and assessment support

Many other resources are provided on Numicon Online to support your planning, teaching and assessment. There are editable planning documents, photocopy masters and videos to support teaching. Assessment resources here include assessment grids for the Explorer Progress Books and milestone tracking charts to monitor children's progress throughout the year. You can access all these resources, along with the Interactive Whiteboard Software, through the Oxford Owl website (www.oxfordowl.co.uk).

Planning chart

The chart below shows you how the Activity Groups in the Teaching Resource Handbooks and the Pupil Book pages fit together and the key learning that is covered. The order follows the recommended teaching progression.

Key to abbreviations used on the chart
NPC: *Number, Pattern and Calculating Teaching Resource Handbook*
GMS: *Geometry, Measurement and Statistics Teaching Resource Handbook*

NNS: Numbers and the Number System
Geo: Geometry
Calc: Calculating
PA: Pattern and Algebra
Mea: Measurement

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
NNS 1: Working with numbers up to a million (Number, Pattern and Calculating 5, pages 82–87)	**pp. 2–5**	**NPC Milestone 1** • Read, write, and convert between column and quantity values of numbers up to 1 000 000 • Count in steps of powers of 10 forwards and backwards from any number, and explain which digit changes when a place value boundary is crossed.
NNS 2: Exploring equivalence with fractions (Number, Pattern and Calculating 5, pages 88–93)	**pp. 6–9**	**NPC Milestone 1** • Explain equivalences between improper fractions and mixed numbers • Use knowledge of factors and multiples to recognize and explain equivalences between proper fractions
NNS 3: Understanding decimals (Number, Pattern and Calculating 5, pages 94–100)	**pp. 10–13**	**NPC Milestone 1** • Read, write and order numbers with up to three decimal places • Recognize and explain decimal and common fraction equivalents, e.g. $0{\cdot}268 = \frac{268}{1000}$ including familiar common fraction equivalents, e.g. $\frac{1}{5} = 0{\cdot}2$
Geo 1: Measuring angles (Geometry, Measurement and Statistics 5, pages 25–33)	**pp. 14–17**	**GMS Milestone 1** • Estimate and classify angles between 0° and 360° • Measure angles using a protractor, correct to the nearest degree • Find missing angles at a point and in one full turn, or at a point on a straight line and in a half turn, using knowledge that the total angle is 360° or 180°, respectively

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
Calc 1: Developing fluency with adding and subtracting calculations and understanding inverse relationships (Number, Pattern and Calculating 5, pages 128–133)	**pp. 18–21**	**NPC Milestone 1** • Choose appropriate and effective mental or written methods to solve adding and subtracting number problems involving whole numbers up to 1000 • Solve adding and subtracting problems involving fractions and decimal fractions efficiently
Calc 2: Strategies for bridging when adding and subtracting mentally (Number, Pattern and Calculating 5, pages 134–139)	**pp. 22–25**	**NPC Milestone 1** • Choose appropriate and effective mental or written methods to solve adding and subtracting number problems involving whole numbers up to 1000 • Solve adding and subtracting problems involving fractions and decimal fractions efficiently
NNS 4: Estimating and rounding (Number, Pattern and Calculating 5, pages 101–107)	**pp. 26–29**	**NPC Milestone 2** • Round whole numbers to the nearest multiple of 10, 100, 1000, 10 000, or 100 000 • Round numbers with up to two decimal places to the nearest whole number and to one decimal place
Calc 3: Further strategies for adding and subtracting (Number, Pattern and Calculating 5, pages 140–146)	**pp. 30–33**	**NPC Milestone 2** • Convert an adding or subtracting calculation to an easier equivalent calculation
PA 1: Exploring sequences and number patterns (Number, Pattern and Calculating 5, pages 44–48)	**pp. 34–37**	**NPC Milestone 2** • Find the term-to-term rule for a linear sequence involving whole numbers, fractions or decimals, and work out missing terms
Geo 2: Transformations (Geometry, Measurement and Statistics 5, pages 34–42)	**pp. 38–41**	**GMS Milestone 1** • Identify lines of symmetry in given figures • Find or show images of points and polygons under translation or reflection in the first quadrant (with lines of symmetry parallel to the x- and y-axes), describing the transformation and how the position and coordinates of the point or polygon have changed • Explain that more than one transformation can map one shape onto another
NNS 5: Working with negative numbers (Number, Pattern and Calculating 5, pages 108–113)	**pp. 42–45**	**NPC Milestone 2** • Read, write and order positive and negative numbers • Calculate the difference between a positive and a negative number

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
Calc 4: Developing fluency with multiplying and dividing (Number, Pattern and Calculating 5, pages 147–153)	**pp. 46–49**	**NPC Milestone 2** • Use multiplying and dividing facts and knowledge of factors and multiples to solve problems • Solve problems effectively by finding fractions of amounts, making use of multiplying and dividing facts • Multiply and divide decimals to one decimal place
NNS 6: Comparing and ordering fractions (Number, Pattern and Calculating 5, pages 114–118)	**pp. 50–53**	**NPC Milestone 3** • Use knowledge of factors and multiples to find equivalent fractions and to simplify fractions to their lowest terms • Compare and order fractions with denominators which are multiples of the same number
PA 2: Using inverse relationships to solve problems (Number, Pattern and Calculating 5, pages 49–54)	**pp. 54–57**	**NPC Milestone 3** • Use the inverse relationships between adding and subtracting, and multiplying and dividing, to complete calculations with missing numbers
Calc 5: Written methods of adding (Number, Pattern and Calculating 5, pages 154–159)	**pp. 58–61**	**NPC Milestone 3** • Use efficient written column methods for adding and subtracting whole numbers up to 10 000 and decimals with up to 3 decimal places
Calc 6: Written methods of subtracting (Number, Pattern and Calculating 5, pages 160–165)	**pp. 62–65**	**NPC Milestone 3** • Use efficient written column methods for adding and subtracting whole numbers up to 10 000 and decimals with up to 3 decimal places
Calc 7: Multiplying and dividing by 10, 100 and 1000 (Number, Pattern and Calculating 5, pages 166–172)	**pp. 66–69**	**NPC Milestone 3** • Use known multiplying facts to multiply and divide whole numbers and decimals by 10, 100, and 1000
Mea 1: Metric and imperial units (Geometry, Measurement and Statistics 5, pages 53–62)	**pp. 70–73**	**GMS Milestone 1** • Convert between metric units of measure, e.g. between metres and kilometres • Recognize and use common imperial measures, e.g. miles, feet, inches, pints • Understand and use approximate equivalences between metric units and common imperial units
PA 3: Properties of number (Number, Pattern and Calculating 5, pages 55–61)	**pp. 74–77**	**NPC Milestone 4** • Find the lowest common multiple of two or more numbers • Find the highest common factor of two or more numbers • Explain the difference between prime and composite numbers and identify them by testing accordingly

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
Calc 8: Using mental methods for multiplying and dividing (Number, Pattern and Calculating 5, pages 173–177)	**pp. 78–81**	**NPC Milestone 4** • Use the distributive property of multiplying and dividing to break down calculations into parts and complete them using mental or informal methods
Calc 9: Division with remainders (Number, Pattern and Calculating 5, pages 178–183)	**pp. 82–85**	**NPC Milestone 4** • Interpret answers to dividing calculations which are not whole numbers appropriately, in context
Geo 3: Exploring angles (Geometry, Measurement and Statistics 5, pages 43–51)	**pp. 86–89**	**GMS Milestone 2** • Accurately identify and describe parallel sides in polygons, including from conventional symbols • Identify the interior and exterior angles of a polygon, and explain the difference and relationship between them • Explain in simple terms why the exterior angle sum of any polygon is 360° • Identify and draw diagonals in polygons • Distinguish between regular and irregular polygons • Use angle sum facts and understanding of the angle properties of polygons to calculate the size of unknown angles
Calc 10: Proportion and ratio (Number, Pattern and Calculating 5, pages 184–189)	**pp. 90–93**	**NPC Milestone 4** • Use multiplying and dividing to solve problems involving scaling
Calc 11: Percentages (Number, Pattern and Calculating 5, pages 190–194)	**pp. 94–97**	**NPC Milestone 4** • Explain percentage as the number of parts per hundred • Find and explain percentage, fraction and decimal equivalents in order to solve problems in context
Mea 2: Interpreting charts and graphs (Geometry, Measurement and Statistics 5, pages 63–76)	**pp. 98–101**	**GMS Milestone 2** • Complete, read and interpret information in tables, charts and graphs • Read and use scales on charts and graphs in a variety of contexts, including with negative numbers • Recognize that choosing which is an appropriate method for representing data depends on the type of data and the question being asked
NNS 7: Solving problems with fractions, decimals and percentages (Number, Pattern and Calculating 5, pages 119–125)	**pp. 102–105**	**NPC Milestone 5** • Know percentage equivalents of commonly used fractions, e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ • Use percentages to express simple proportions, e.g. 24 out of 32 as 75% • Find percentages of amounts, including measures

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
PA 4: Looking for patterns and generalizing (Number, Pattern and Calculating 5, pages 62–68)	**pp. 106–109**	**NPC Milestone 5** • Know and be able to use simple tests of divisibility • Explain what square and cube numbers are
Mea 3: Calculating area and perimeter (Geometry, Measurement and Statistics 5, pages 77–84)	**pp. 110–113**	**GMS Milestone 2** • Describe area as a measure of flat space and perimeter as a measure of length • Recognize that area is measured in square units • Calculate the areas and perimeters of rectangles, in square centimetres and centimetres, respectively, given pairs of dimensions • Find missing side lengths and calculate areas and perimeters of composite rectilinear shapes, in square centimetres and centimetres
Calc 12: Written methods of multiplying (Number, Pattern and Calculating 5, pages 195–199)	**pp. 114–117**	**NPC Milestone 5** • Use efficient written methods to multiply numbers with up to 4 digits by 2-digit numbers
Mea 4: Estimating volume and capacity (Geometry, Measurement and Statistics 5, pages 85–94)	**pp. 118–121**	**GMS Milestone 3** • Explain the difference between volume and capacity and the equivalence between cubic centimetres and millilitres • Build cuboids of a given volume using cubes or rods • Create 2D drawings of 3D shapes • Give a reasonable estimate of the volume of an object, in cubic centimetres • Calculate volume or capacity given the dimensions of a cuboidal shape or space
Calc 13: Written methods of dividing (Number, Pattern and Calculating 5, pages 200–204)	**pp. 122–125**	**NPC Milestone 5** • Choose appropriate and effective mental or written methods to divide numbers with up to 4 digits by single-digit numbers
Calc 14: Calculating fractions of amounts (Number, Pattern and Calculating 5, pages 205–210)	**pp. 126–129**	**NPC Milestone 5** • Calculate fractions of amounts in practical problem-solving contexts

Activity group title and pages in the Teaching Resource Handbook	Accompanying Pupil Book pages	Milestone statements covered
Mea 5: Working with area and perimeter (Geometry, Measurement and Statistics 5, pages 95–100)	**pp. 130–133**	**GMS Milestone 3** • Find missing side lengths and calculate areas and perimeters of more complex composite rectilinear shapes, in square centimetres or metres and in centimetres or metres • Estimate areas of irregular shapes • Express the area or perimeter of a rectangle algebraically in order to find an unknown length, e.g. $5 \times b = 30$ so $b = 6$ and the unknown length is 6 cm
Mea 6: Scale drawing (Geometry, Measurement and Statistics 5, pages 101–107)	**pp. 134–137**	**GMS Milestone 4** • Explain the meaning of the term 'scale drawing' • Know that the scale of a drawing is the ratio of drawn to actual lengths • Identify the scale of a drawing by comparing drawn and actual lengths • Select an appropriate scale for a scale drawing
Calc 15: Calculating with fractions (Number, Pattern and Calculating 5, pages 211–217)	**pp. 138–141**	**NPC Milestone 6** • Organize work independently and communicate ideas fluently • Multiply proper fractions and mixed numbers by whole numbers • Relate finding a fraction of a number or quantity to multiplying by a fraction
Calc 16: Solving problems involving several steps (Number, Pattern and Calculating 5, pages 218–223)	**pp. 142–145**	**NPC Milestone 6** • Solve a range of multi-step problems by choosing appropriate operations, strategies and methods • Organize work independently and communicate ideas fluently
Mea 7: Solving problems involving time, money and measures (Geometry, Measurement and Statistics 5, pages 108–116)	**pp. 146–149**	**GMS Milestone 4** • Find sensible solutions to multi-step problems involving a variety of measures, e.g. time, money or mass, choosing appropriate operations, strategies and methods • Organize work independently and communicate ideas fluently
PA 5: Using equivalence to solve problems (Number, Pattern and Calculating 5, pages 69–74)	**pp. 150–153**	**NPC Milestone 6** • Explain how to solve missing number problems involving both equivalence and inequality • Approach problems involving number relationships systematically, logically and effectively
PA 6: Logic and reasoning (Number, Pattern and Calculating 5, pages 75–80)	**pp. 154–157**	**NPC Milestone 6** • Explain how to solve missing number problems involving both equivalence and inequality • Approach problems involving number relationships systematically, logically and effectively

Cover

Practice

The top of the flag is 8 metres above the treasure. Children can use the scale to see how many metres separate the flag and the treasure.

Going deeper

The treasure is now 5 m below the surface of the water. Children might work out and then check their answer in different ways, e.g. by using the scale to find the difference, and by calculating that the difference between $^{-}1$ m and $^{-}6$ m is 5 m.

Page 2: Working with large numbers (NNS 1·1)

Practice

1 Eight million, five hundred and fifty thousand, four hundred and five.

2 If the children have written and read out the 7-digit numbers correctly their numbers will match.

Going deeper

1 Answers will vary. One possible example is 3 286 741. This is a larger number than 3 286 471 because the first four digits from the left are the same but the hundreds digit is a 7 in the larger number rather than a 4. This means the number has seven hundreds compared with four hundreds. It is also smaller than 4 183 762 because it has a smaller millions digit.

2 A good strategy to read 7-digit numbers is to firstly say the millions part of the number and then to focus on the gaps which separate the rest of the digits into three place columns. The first gap occurs after the thousands so we can read this gap as 'thousands.' The last three digits are the hundreds, tens and ones. So 3 286 741 is read as three million, two hundred and eighty-six thousand, seven hundred and forty-one.

Page 3: Visualizing a million (NNS 1·3)

Practice

1 An approximate volume of a cup of rice is 50 teaspoons which is 2500 grains (50 grains × 50 grains). The volume of four cups of rice or 1 litre of rice is 10 000 grains (2500 grains × 4).

1 million grains of rice would roughly be 100 litres (10 000 grains × 100).

2 Answers could be roughly the same as a box of 100 1-litre bottles of water or cartons of juice, 100 litre cubes, a holdall or flight luggage suitcase measuring approximately 78 × 30 × 47 cm or a small wheelie bin.

Going deeper

1 There are 60 seconds in a minute so in an hour there are 3600 seconds (60 × 60) and 86 400 seconds in a day (3600 × 24). A rough estimate of this might be 80 000 (4000 × 20). So one million seconds is roughly 11 days because 11 × 80 000 = 880 000 or 11 × 86 000 = 946 000.

2 There are 365 days in a year so if you are ten you have lived for 3650 days. This number is a long way off a million days.

3 1000 years would be 365 000 days. This is roughly 360 000 and if you use this approximation you can estimate that if you were born about 3000 years ago you would be over 1 million days old.

4 One way to think about a million is to describe it as a thousand thousands. So that is a thousand base-ten thousand cubes.

5 Another way to describe a million is ten lots of a hundred thousand.

Page 4: Exploring place value (NNS 1·5 & 1·6)

Practice

1 If the starting number was 313 211: counting on in ten steps of 1000 gives:

314 211, 315 211, 316 211, 317 211, 318 211, 319 211, 320 211, 321 211, 322 211, 323 211. Only the thousand digit changes until the point where the boundary is crossed in the tens of thousands column as underlined in this example.

2 If the starting number was 54 312: counting back ten steps of 100 gives:

54 212, 54 112, 54 012, 53 912, 53 812, 53 712, 53 612, 53 512, 53 412, 53 312. Only the hundred digit changes until the point where the thousands boundary is crossed as underlined in this example.

Counting back ten steps of 100 is the same as counting back one step of a 1000. This changes the thousand digit, e.g. 54 312 changes to 53 312 in the example above.

3 Answers will vary depending on the numbers rolled on the dice.

Going deeper

1 If the starting number is 3 100 211 subtracting 90 000 removes the hundred thousand digit but leaves ten thousand 3 010 211.

2 There is no right/wrong answer for this question. The value is in children thinking about the possibilities and reasoning through their response.

Page 5: Working with Roman numerals NNS (1·8)

Practice

1 **a** 36 **b** 42 **c** 720

2 **a** An example of a date in Roman numerals is XXV.I.MMXVII which can be read 25.1.2017.

b Answers will vary.

c 4 July 1862 would read as IV.VII.MDCCCLXII in Roman numerals.

Going deeper

1 **a** The bigger number is CCCXLIV which is 344. The smaller number is CCLXVIII which is 268.

b DCXII This can be worked out by adding the hundreds (CCC + CC = D), then the tens (XL + LX = C) and ones (IV + VIII = XII).

c This can be worked out by subtracting the hundreds CCC – CC = C, then subtracting the tens from the remaining hundred and the tens CXL – LX = LXXX, then subtracting the ones from the remaining tens and ones LXXXIV – VIII = LXXVI.

2 It is easier to calculate with the numerals we use today because we can set out our calculations in place value columns.

3 The Romans would use a counting board with pebbles placed in columns or an abacus with beads.

NPC Milestone 1

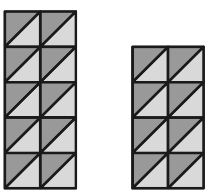
- Read, write, and convert between column and quantity values of numbers up to 1 000 000
- Count in steps of powers of 10 forwards and backwards from any number, and explain which digit changes when a place value boundary is crossed

Page 6: Exploring equivalence with fractions (NNS 2·2 & 2·3)

Practice

1 **a** $\frac{15}{2}$ **b** $7\frac{1}{2}$ **c** $\frac{1}{2}$.

2 **a**



b $\frac{18}{2} = 9$ **c** $\frac{1}{2} \times 18 = 9$ and $18 \div 9 = 2$

3 **a** $11\frac{1}{2}$ **b** $\frac{25}{2}$ **c** 19 **d** $\frac{51}{2}$

Going deeper

1 **a** A 1-shape, made of tiles which are each divided into 4 sections with one section shaded.

b A 3-shape, made of tiles with each tile divided into 4 sections. One section of each tile should be shaded.

c A 10-shape, made of tiles with each tile divided into 4 sections. One section of each tile should be shaded.

2 A 9-shape, made of tiles with each tile divided into 4 sections. One section of each tile should be shaded.

3 **a** $\frac{9}{4}$ can be read as nine quarters. To illustrate this we need to shade a section, or quarter, of a tile, in each of the nine tiles that make up the 9-shape.

b A 10-shape and 9-shape, made of tiles with each tile divided into 4 sections. One section of each tile should be shaded.

4 **a** Quarters can be shown best with the 4-shape.

b To illustrate $\frac{15}{4}$ with 4-shapes you could use four Numicon 4-shapes with four counters filling three of the Shapes and three counters filling three holes on one of the Shapes so that each of 15 quarters is filled by a counter. 3 and $\frac{3}{4} = \frac{15}{4}$.

5 **a** 4-rods would be best to for illustrating quarters.

b $2\frac{3}{4}$ can be shown with two 4-rods and three 1-rods or eleven 1-rods next to a number train of 4-rods.

Page 7: Converting mixed numbers and improper fractions (NNS 2·4)

Practice

1 $\frac{3}{8}$

2 You could use a 3-rod and an 8- rod or three 1-rods and an 8-rod.

3 **a** An example would be that $\frac{13}{8}$ can be illustrated with two 8-shapes filled with 13 counters or 13 1-rods and two 8-rods.

b These are all improper fractions.

4 **a** $1\frac{5}{8}$, $2\frac{4}{8}$, $3\frac{7}{8}$, $5\frac{3}{8}$

b Mixed numbers

c To convert improper fractions to mixed numbers you can divide the numerator by the denominator, e.g. for $\frac{13}{8}$ you can divide 13 by 8 which is 1 r5. This can be written as the mixed number 1 and $\frac{5}{8}$.

5 An example would be that 3 and $\frac{1}{6}$ can be illustrated with four 6-shapes and enough counters to fill the first three 6-shapes and 1 hole in the last 6-shape.

This can also be illustrated with three 6-rods and a 1-rod.

Going deeper

1 $\frac{31}{8}$ or $\frac{62}{16}$ or $3\frac{14}{16}$

2 It is easier to convert both fractions into improper fractions, e.g. $\frac{23}{5} + \frac{19}{5} = \frac{42}{5}$.
Alternatively you could convert both fractions into mixed numbers, e.g. $4\frac{3}{5} + 3\frac{4}{5} = 8\frac{2}{5}$.

3 You could convert both fractions to improper fractions, e.g. $\frac{23}{5} - \frac{19}{5} = \frac{4}{5}$.

Page 8: Exploring equivalent fractions (NNS 2·5)

Practice

1 To divide the tray of brownies into 18 equal parts you divide a half into ninths or a third into sixths.

2 **a** $\frac{1}{3}$ can be described as $\frac{6}{18}$, $\frac{2}{6}$, $\frac{3}{9}$, etc.
b $\frac{1}{2}$ can be described as $\frac{9}{18}$ or $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, etc.
c $\frac{1}{6}$ can be described as $\frac{3}{18}$, $\frac{2}{12}$, $\frac{4}{24}$, etc.
d $\frac{1}{4}$ can be describes as $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$, etc.

3 You could use a 9-rod and an 18-rod to show a $\frac{1}{2}$ of 18.
You could use a 6-rod and an 18-rod to show a $\frac{1}{3}$ of 18.
You could use a 3-rod and an 18-rod to show a $\frac{1}{6}$ of 18.
You could use a 2-rod and an 18-rod to show a $\frac{1}{9}$ of 18.

Going deeper

1 There are many ways to describe a $\frac{1}{2}$ as a fraction. You just need to choose a numerator that is half of the denominator, e.g. $\frac{5}{10}$, $\frac{10}{20}$, $\frac{50}{100}$.
There is no right/wrong answer for the explanation. The value is in children thinking about the possibilities and reasoning through their response.

2 There are many ways to describe $\frac{2}{3}$ as a fraction. You can write out a sequence of multiples of 2 and 3, e.g. $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, or use the fraction $\frac{2}{3}$ and multiply the numerator and denominator by the same number, e.g. if we multiply the numerator and denominator by 5 we get 2 × 5 = 10 and 3 × 5 = 15. So $\frac{10}{15}$ also describes $\frac{2}{3}$.

3 These fractions describe the same amount: $\frac{2}{5}$, $\frac{8}{20}$ and $\frac{20}{50}$, $\frac{6}{10}$, $\frac{9}{15}$, $\frac{30}{50}$ and $\frac{60}{100}$.

4 $\frac{5}{8} = \frac{15}{24}$
$\frac{3}{9} = \frac{4}{12}$
$\frac{2}{7} = \frac{6}{21}$
$\frac{3}{11} = \frac{12}{44}$

Page 9: Using fractions in everyday situations (NNS 2·6)

Practice

1 **a** To show the proportions in Tia's drink you can use a 1-rod for squash and a 5-rod for water or for the full jug you can use a 2-rod for squash and 10-rod for water.
b There is $\frac{1}{6}$ or $\frac{2}{12}$ of squash and $\frac{5}{6}$ or $\frac{10}{12}$ of water.

2 2 jugs would need $\frac{20}{24}$ of water and 3 jugs would need $\frac{6}{36}$ of squash and $\frac{30}{36}$ of water.

3 **a** In the squash column there is a multiple pattern of 2s and 12s and in the water column there is a multiple pattern of 10s and 12s.
b There is a $\frac{1}{6}$ fraction family in the squash column and a $\frac{5}{6}$ fraction family in the water column.

Going deeper

1 To illustrate the squash fraction family you could use a 6-shape with one peg inside a hole, two 6-shapes with two pegs, or three 6-shapes with three pegs, etc.

2 To illustrate the water fraction family using a number line you could first draw a line from 0–1 with six equally-spaced intervals. Then you can make a mark at $\frac{5}{6}$. Next draw another line the same length with twelve intervals and make a mark at $\frac{10}{12}$. You could also make another line the same length with 24 intervals and make a mark at $\frac{20}{24}$. The marks should all line up at the same distance along the number lines.

3 To illustrate $\frac{2}{5}$ you could use a 5-shape with two pegs inside, two 5-shapes with four pegs or three 5-shapes with six pegs.

4 To test if a fraction is part of the $\frac{3}{7}$ family you can divide the numerator in the fraction by 3, e.g. 222 ÷ 3 and the denominator by 7, e.g. 518 ÷ 7. If the answer to both divisions is the same then the fraction is in the same family, so $\frac{222}{518}$ is in the $\frac{3}{7}$ family because 222 ÷ 3 = 74 and 518 ÷ 7 = 74.

NPC Milestone 1

- Explain equivalences between improper fractions and mixed numbers
- Use knowledge of factors and multiples to recognize and explain equivalences between proper fractions

Page 10: Understanding decimals (NNS 3·1)

Practice

1 $37\frac{1}{4}$ as a decimal is 37·25 and $37\frac{1}{2}$ as a decimal is 37·5 or 37·50 so 37·35 is in between these two amounts in the same way that 35 is between 25 and 50.

2 Answers will vary but could include 3·27, 3·3, 3·33, 3·4, 3·45.

3 $37\frac{75}{100}$

Going deeper

1 Whole numbers do not need a decimal point but numbers that are part of a whole or those that are made up of whole numbers and parts need a decimal point to separate the whole number and the decimal parts, e.g. 0·5 or 2·5.

2 Some examples might include money, food nutrition labels, measurements or on petrol pumps.

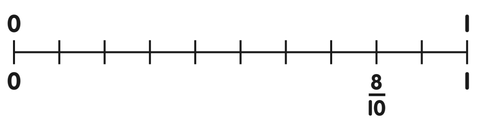
3 Some examples might include recipes, telling the time, shopping for things like cheese, describing capacity, e.g. in a petrol context you might hear someone describe that their petrol tank is about a quarter full.

Page 11: Converting fractions to decimals (NNS 3·2 & 3·3)

Practice

1 **a** The baseboard is covered with eight 10-shapes which means 80 out of one hundred parts are covered.

b This can be written as $\frac{80}{100}$ or $\frac{8}{10}$ as a fraction or 0·8 which means '8 tenths'.

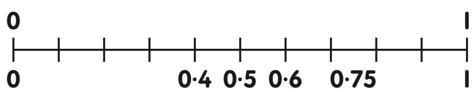
2 

This is the eighth interval of ten so represents 8 tenths.

3 Examples might include proper fractions like $\frac{2}{5}, \frac{1}{2}, \frac{6}{10}, \frac{3}{4}$.

a In size order these would be $\frac{2}{5}, \frac{1}{2}, \frac{6}{10}, \frac{3}{4}$.

b As decimals these are 0·4, 0·5, 0·6, 0·75.

c 

Going deeper

1 **a** To convert $12\frac{3}{4}$ to a decimal you might know that $\frac{1}{4}$ is $\frac{25}{100}$ and that if you multiply this $\frac{1}{4}$ by 3 you will find $\frac{3}{4}$ which is $\frac{75}{100}$ or 0·75. This means $12\frac{3}{4}$ as a decimal is 12·75.

b Answers will vary.

2 To write 18·4 as a fraction you can write the whole number 18 and then if you know that $\frac{1}{5}$ is equal to 0·2 or $\frac{2}{10}$ then you can work out that 0·4 is $\frac{2}{5}$ or $\frac{4}{10}$. So 18·4 is $18\frac{4}{10}$ or 18 and $\frac{2}{5}$.

3 To convert $\frac{2}{3}$ to a decimal you can divide 2 ÷ 3 on a calculator or divide 1 ÷ 3 and multiply this by 2. $\frac{2}{3}$ = 0·66666.

4 When you divide the numerator of a proper fraction by 3 or 9 the calculator shows a decimal with lots of repeating digits, e.g. 1 ÷ 3 = 0·33333333, 2 ÷ 3 = 0·66666666, 1 ÷ 9 = 0·11111111, 2 ÷ 9 = 0·2222222.

Page 12: Thousandths as decimals (NNS 3·6)

Practice

1 3·264 can be written on the number line labelled 3·26 to 3·27 at the fourth interval after 3·26.

2 The first number line should be labelled 2 to 3, then 2·5 to 2·6 and 2·58 to 2·59. 2·586 can be marked on the sixth interval after 2·58.

3 3·264 as a fraction can be written as $3\frac{264}{1000}$. When a decimal number has 3 digits after the decimal point these can be read as 264 thousandths and therefore can be written as the numerator with 1000 as the denominator.

Going deeper

1 If the number line is 0–1 then 0·586 is between 0·5 and 0·6. Just over half way along the line but closer to 0·6 than 0·5.

2 Label the number line 5·28 on the left and 5·29 on the right and mark 5·289 on the ninth interval after 5·28.

3 The exact middle point between 0 and 0·01 is 0·005, however children may have picked other points around this.

Page 13: Comparing and ordering decimals (NNS 3·4, 3·7 & 3·8)

Practice

1 Looking at the base-ten blocks you can see that to make 2·891 and 2·879 you need the same amount to make the whole number and tenths but for 2·891 you need nine hundredths compared with seven hundredths for 2·879.

This means that 2·891 is the bigger number.

2 a Another way to decide which decimal number is bigger is to write them as a fraction. As a fraction 2·343 is $2\frac{343}{1000}$ and 2·398 is $2\frac{398}{1000}$.

b As $\frac{398}{1000}$ is more than $\frac{343}{1000}$ this means that 2·398 is bigger than 2·343.

3 2·81, 2·879, 2·891, 2·908, 3·009, 3·147

The last three in the list are bigger than 2·9: 2·908, 3·009, 3·1471.

Going deeper

1 $\frac{38}{100} > \frac{364}{1000}$. If you write both fractions as thousandths $\frac{38}{100}$ becomes $\frac{380}{1000}$ which is more than $\frac{364}{1000}$.

2 When you write this number under place value headings you can see that 0·027<u>41</u> has $\frac{41}{100\,000}$ and 0·027<u>29</u> has $\frac{29}{100\,000}$ so 0·02741 is bigger.

Ones	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$	$\frac{1}{100\,000}$
0	0	2	7	4	1
0	0	2	7	2	9

NPC Milestone 1

- Read, write and order numbers with up to three decimal places
- Recognize and explain decimal and common fraction equivalents, e.g. 0·268 = $\frac{268}{1000}$, including familiar common fraction equivalents, e.g. $\frac{1}{5}$ = 0·2

Page 14: Measuring angles in degrees (Geo 1·1)

Practice

1 a Ravi's; the angle is smaller than a right angle but greater than half a right angle, so must be larger than 45°.

b Acute; it is smaller than 90.

2 a 90° b 30° c 120° d 300°

3 a Use a protractor to check answers.

b Answers will vary.

Going deeper

1 a 15° b 547·5° c 180° and 6570°

Page 15: Measuring angles with a protractor (Geo 1·2)

Practice

1 a 127°

b Answers will vary. Look for a clear explanation.

2 a acute; 37° b obtuse; 169°

3 Sketches of reflex angles 323° and 191°. Subtract answers from question 2 from 360°.

Going deeper

1 a 270° b 247·5° c 022·5°

Page 16: Measuring and drawing angles (Geo 1·3 and 1·4)

Practice

1 Walk 3 km. Turn 65 degrees anti-clockwise, and walk 8·5 km to the hill. Turn 145 degrees clockwise and walk 6 km to the tree. Turn 130 degrees anti-clockwise and walk 7 km to the finish.

2 a Use a protractor to check answers.

b Use a protractor to check answers.

c Use a protractor to check answers.

Going deeper

1 Following the instructions should reveal a regular hexagon.

2 Following the instructions should reveal a regular octagon.

3 With a larger angle it takes more turns to complete a revolution.

Page 17: Finding missing angles (Geo 1·5)

Practice

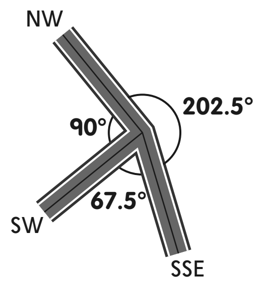
1 a 360° b 270°

2 a = 125°; because angles on a straight line total 180°.

b = 180°

Going deeper

The three angles are 202·5°, 67·5°, and 90°.



GMS Milestone 1

- Estimate and classify angles between 0° and 360°
- Measure angles using a protractor, correct to the nearest degree
- Find missing angles at a point and in one full turn, or at a point on a straight line and in a half turn, using knowledge that the total angle is 360° or 180°, respectively

Page 18: Developing adding and subtracting (Calc 1·1)

Practice

1 One example of a solution is 19 + 25 + 21. Children might refer to 20 + 20 + 20 = 60 as a good starting point and then adapt this calculation by adding the remaining 5 lengths either to one child or by dividing them between the three children, e.g. 20 + 20 + 25 or 21 + 23 + 21.

2 Some examples: Look for children adjusting the calculation systematically.

a 340 + 580 + 20 = 940
330 + 580 + 30 = 940
320 + 580 + 40 = 940
310 + 580 + 50 = 940
300 + 580 + 60 = 940

b 324 + 55 + 345 = 724
334 + 45 + 345 = 724
344 + 35 + 345 = 724
354 + 25 + 345 = 724
364 + 15 + 345 = 724

3 **a** 645 – 251 + 252 = 646 (645 is one more than 646 so if you add the 251 back + 1 more the answer will be 646).

b 58 + 76 = 53 + 81 (53 is 5 less than 58 so if you add 5 onto 76 this balances the calculation).

Going deeper

1 438 is 60 less than 498 so to make this calculation balance you need to subtract 60 more than 276 which is 336.

2 18 + 22 + 25
18 + 23 + 24
19 + 21 + 25
19 + 22 + 24
19 + 23 + 23
20 + 20 + 25
20 + 21 + 24
20 + 22 + 23
21 + 22 + 22
21 + 21 + 23

Answers will vary. Reasoning will indicate if children worked systematically checking for repeats.

3 One example might be to add 9 to 49 because 89 is 9 more than 80.

Page 19: Adding and subtracting in problem solving (Calc 1·2 & 1·3)

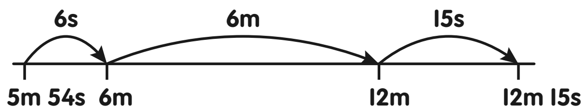
Practice

1 **a** £6·75 **b** £4·40 **c** £2·65 **d** £3·80 **e** £6·51 **f** £8·28

2 **a** 66·9 km **b** 37·2 km **c** 19·6 km

Going deeper

1 Some children may draw a number line to show how they worked out the answer to the problem.



Using inverse they might write four number facts or eight if they move the equals sign

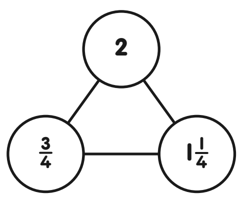
5m 54s + 6m 21s = 12m 15s   12m 15s – 6m 21s = 5m 54s
12m 15s = 5m 54s + 6m 21s   5m 54s = 12m 15s – 6m 21s
6m 21s + 5m 54s = 12m 15s   12m 15s – 5m 54s = 6m 21s
12m 15s = 6m 21s + 5m 54s   6m 21s = 12m 15s – 5m 54s

2 Answers will vary. Some children may mention the use of counting up or using a number line as above.

Page 20: Adding and subtracting fractions (Calc 1·4)

Practice

1



$\frac{3}{4} + 1\frac{1}{4} = 2$, $1\frac{1}{4} + \frac{3}{4} = 2$, $2 - \frac{3}{4} = 1\frac{1}{4}$, $2 - 1\frac{1}{4} = \frac{3}{4}$

2 Check children's answers. The lower circles should add up to the upper circle, e.g. $1\frac{2}{3} + 1\frac{1}{3} = 3$.

3 $2\frac{3}{5}$

Going deeper

1 Answers will vary.

Some children may say that they add the lower circles together if both numbers are given to find the number in the top circle and explain how they use the inverse to subtract one of the lower circles from the number in the top circle if one of the lower circle numbers are missing.

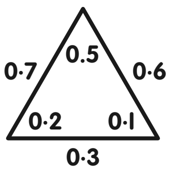
2 Answers will vary. Some children will change the calculation into improper fractions, e.g. $\frac{33}{7} - \frac{27}{7} = \frac{17}{7} - \frac{11}{7}$ and explain that 17 is 16 less than 33 and 27 is 16 more than 11.

Others will us the same reasoning with fractions, e.g. $2\frac{3}{7}$ is $2\frac{2}{7}$ less than $4\frac{5}{7}$ so $1\frac{4}{7} + 2\frac{2}{7}$ is $3\frac{6}{7}$.

Page 21: Adding and subtracting decimals (Calc 1·5 & 1·6)

Practice

1



2 **a** The first grid has one solution.

+	2·4	1·3
5·1	7·5	6·4
4·7	7·1	6·0

The second grid has a number of possibilities of which this is one example.

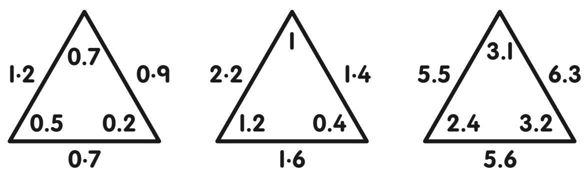
+	3·4	2·2
2·3	5·7	4·5
6·7	10·1	8·9

3 Another solution can be found by adjusting, e.g. 3·1 is 0·3 less than 3·4 and 2·6 is 0·3 more than 2·3.

+	3·1	1·9
2·6	5·7	4·5
7	10·1	8·9

Going deeper

1



2

0·7	0	0·5
0·2	0·4	0·6
0·3	0·8	0·1

3 7·35 – 3·53 + 3·54 = 7·36

An example of an explanation would point out that the difference between 7·35 and 7·36 is 0·01 and explain that the missing number needs to be 0·01 more than 3·53.

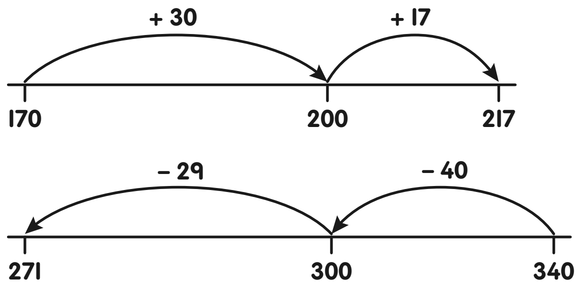
NPC Milestone 1

- Choose appropriate and effective mental or written methods to solve adding and subtracting number problems involving whole numbers up to 1000
- Solve adding and subtracting problems involving fractions and decimal fractions efficiently

Page 22: Using bridging strategies (Calc 2·1 & 2·2)

Practice

1 Answers will vary depending on which numbers are chosen e.g.



2 **a** 195 + 75 = 195 + 5 + 70 = 200 + 70

b 174 – 80 = 174 – 74 – 6 = 100 – 6 = 94

c 360 – 85 = 360 – 60 – 25 = 300 – 25 = 275

d 1600 + 540 = 1600 + 400 + 140 = 2000 + 140 = 2140

Going deeper

1 Answers will vary but should identify that when adding 80 + 50 + 7 that the 7 ones can be with either the 80 or the 50 and the answer will be the same. However with subtraction 80 – 57 = 23 and 87 – 50 = 37. The answers have a difference of 14 because 87 is 7 more than 80 and 50 is 7 less. They would be equal if we subtracted 7 more than 57 from 87, e.g. 80 – 57 = 87 – 64.

2 Answers will vary. Some children will suggest that bridging to the next 10 and then onto 100 is a good strategy to work out complements to 100. They may illustrate this with a number line, or explain the jumps, e.g. 84 to 90 = 6, 90 to 100 = 10 so 84 + 16 = 100.

3 Answers will vary and there is not a correct or incorrect strategy for each of these. Encourage children to talk about their preferences, e.g.

a 76 + 84 = 160. Children may say that they added 6 and 4 because this makes 10 and then 70, 80 and the extra 10 using the doubles fact 8 + 8 = 16.

b 154 + 132 = 286. Children may say that they prefer to use partitioning to answer this question as no place value boundaries are crossed.

c 193 + 57 = 250. Children may say that they prefer to use bridging to answer this question. They can start at 193 and add 7 to make 200 and then add 50.

d 293 + 198 = 491. Children may say that they prefer to adjust this question to 300 + 191 = 491 by adding 7 to 300 and subtracting 7 from 198.

e 368 – 207 = 161. Children may say that they prefer to use partitioning to answer this question as no place value boundaries are crossed.

f 240 – 122 = 118. Children may say that they prefer to subtract 100 from 200 and 22 from 40.

g 325 – 75 = 250. Children may say that they prefer to use bridging to answer this question. They can start at 325 and subtract 25 to make 300 and then subtract 50.

h 476 – 97 = 379. Children may say that they prefer to use subtract 100 and then add 3 because this is the same as subtracting 97.

Page 23: Bridging with time (Calc 2·3)

Practice

1 **a** 2:45 p.m.
b 2:30 p.m.
c 4:15 p.m.
d 5:10 p.m.
e 7:35 p.m.

2 **a** 12:25 p.m.
b 4:40 p.m.
c 2:50 p.m.
d 1:45 p.m.
e 7:20 a.m.

3 Illustrate on a number line with 4:10 p.m. at the end and a jump back of 2 hours to 2:10 p.m.

Next a jump back of 10 mins to 2:00 p.m. and another jump back 5 mins to 1:55 p.m.

Going deeper

1 Meat 11:40 a.m.
Potatoes 12:35 p.m.
Carrots 12:50 p.m.
Broccoli 1:24 p.m.
Gravy 1:05 p.m.

2 When Tia does a column calculation to add 2 hours 45 minutes and 1 hour 20 minutes she doesn't notice that 40 minutes and 20 minutes make an hour so instead of writing the answer as 3 hours and 65 minutes she could write it as 4 hours 5 minutes.

Page 24: Bridging with fractions (Calc 2·4)

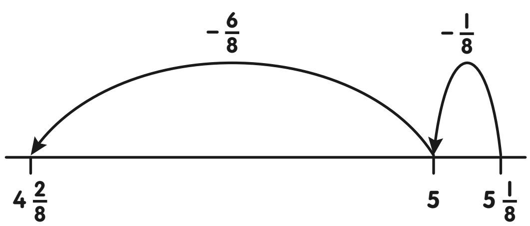
Practice

1 **a**, **b** Answers will vary. An example of an adding calculation $2\frac{1}{4} + \frac{7}{8} = 3\frac{1}{8}$.

2 Answers will vary. An example of a subtracting calculation: $3\frac{3}{8} - \frac{5}{8} = 2\frac{6}{8}$.

Going deeper

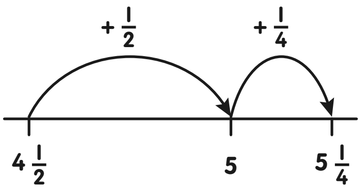
1 Answers will vary: some might use a number line to show bridging to subtract $\frac{7}{8}$ e.g.



Others will select examples like $4\frac{1}{2} - \frac{7}{8}$ and explain that $4\frac{1}{2}$ is the same as 4 and $\frac{4}{8}$ so $4\frac{4}{8} - \frac{7}{8} = 4\frac{4}{8} - \frac{4}{8} - \frac{3}{8} = 3\frac{5}{8}$.

Others will explain how to change the mixed numbers into and improper fraction, e.g. $2\frac{1}{4} = \frac{18}{8}$ so $\frac{18}{8} - \frac{7}{8} = \frac{11}{8}$.

2 Answers will vary: some might use a number line to show bridging to add $\frac{3}{4}$, e.g.



Others will select examples like $2\frac{1}{4} + \frac{3}{4}$ and explain that $2\frac{1}{4} + \frac{3}{4} = 2\frac{1}{4} + \frac{1}{4} + \frac{2}{4} = 3$.

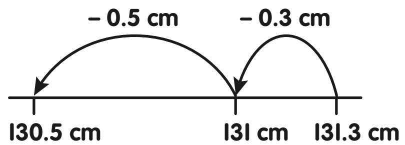
Others will explain how to change the mixed numbers into and improper fractions, e.g. $5\frac{1}{8} = 4\frac{1}{8}$ and $\frac{3}{4} = \frac{6}{8}$ so $4\frac{1}{8} + \frac{6}{8} = 4\frac{7}{8}$.

3 Answers will vary: some might notice that they can adjust the calculation from $\frac{5}{6} + \frac{13}{6}$ to $\frac{6}{6} + \frac{12}{6}$ so that they have $\frac{6}{6}$ which is equivalent to 1 and $\frac{12}{6}$ which is equivalent to 2. This will help them explain that $\frac{5}{6} + \frac{13}{6} = 1 + 2$.

Page 25: Bridging with decimals (Calc 2·5 & 2·6)

Practice

1 130·5



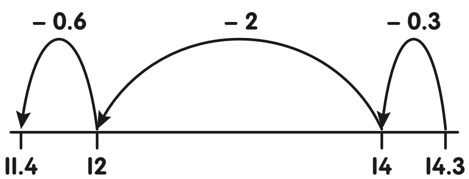
2 Answers will vary. An example of an adding calculation: 14·3 + 2·9 = 17·2.

An example of a subtracting calculation: 6·4 – 2·9 = 3·5.

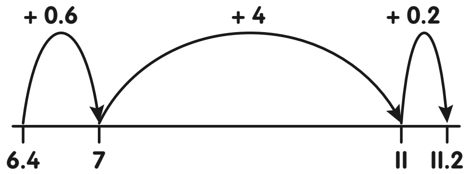
3 Answers will vary. One example is 1·7 + 0·9 = 2·6 and 5·4 – 2·8 = 2·6.

Going deeper

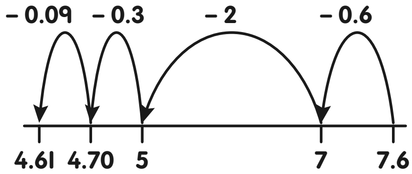
1 Answers will vary: some children might use a number line to explain how bridging helps them to subtract examples like 2·9 from 14·3.



2 Answers will vary: some children might use a number line to explain how bridging helps to add examples like 6·4 + 4·8.



3 Answers will vary: some might use a number line to explain how bridging helps to subtract 2·99 from 7·6.



Other children might use a money context to help them solve the calculation and count up from £2·99 to £7·60, e.g. £2·99 + 0·01 = £3, £3 + £4 = £7, £7 + 0·60 = £7·60. In total the amount added to £2·99 is £4·61.

Other children might subtract 3 or £3 from 7·60 or £7·60 by identifying that £3 is 0·01 or 1p more than 2·99, after they have subtracted 3 or £3 they then add on the 0·01 or 1p, e.g. £7·60 – £3 = £4·60, £4·60 + 1p = £4·61.

NPC Milestone 1

- Choose appropriate and effective mental or written methods to solve adding and subtracting number problems involving whole numbers up to 1000
- Solve adding and subtracting problems involving fractions and decimal fractions efficiently

Page 26: Everyday estimating (NNS 4·1)

Practice

1 a Strategies to estimate how many times the letter 'e' is used in a page of a reading book might include counting how many are in each line and multiplying by how many lines are on the page.

b To estimate how many times the letter 'e' is used in the book the previous estimation can be multiplied by the number of pages in the book.

2 You might check a paragraph of text in their books to find out that the letter 'e' is used more or frequently in words than 't'.

3 1000 penny coins would weigh roughly 3·5 g × 1000 = 3500 g or 3·5 kg.

Going deeper

1 Strategies to estimate the number of tables in school might include counting the number of tables in one class and then multiplying this number by the number of classes in the school and then adding on other rooms or places that have tables, e.g. dinner hall, corridors, group rooms, etc.

2 To estimate the number of sweets in the jar you could count the number of sweets in one layer and then multiply this by the number of layers in the jar.

Page 27: Rounding to multiples of 10 (NNS 4·2, 4·3 & 4·4)

Practice

1 24 242 is close to 24 250 and this can be shown on the number line about halfway between 24 200 and 24 300.

2 Answers will vary. An example of a number that would round up to 24 200 to the nearest 100 is 24 160 and an example that would round down is 24 240.

3 If the shoe weighs 446 g the nearest 100 grams would be 400 g.

4 If the width of the table is 1103 mm, rounding this to the nearest 10 mm is 1100 mm.

5 The equator is 40 000 km to the nearest 1000 km.

6 9990 is the closest to 10 000. It is only 10 away.

Going deeper

1 **a** 1 738 560 **b** 1 738 600 **c** 1 739 000

d 1 740 000 **e** 1 700 000

To round to the nearest 10 look at the ones digit. If this digit is 5 or larger the number will round up to the nearest 10 but if the ones digit is less than 5, the number will round down. Similarly if you round to the nearest 100, look at the tens and ones and decide if this is 50 or larger and if rounding to the nearest 1000, look at the hundreds, tens and ones and decide if this is 500 or larger.

2 If the crowd is 45 000 to the nearest hundred,

a the smallest number this could be is 44 950

b the largest number is 45 049.

Page 28: Rounding with decimals (NNS 4·5 & 4·6)

Practice

1 An estimate of the area could be $4\,m \times 3\,m = 12\,m^2$ or $4\,m \times 2{\cdot}8\,m = 11{\cdot}2\,m^2$.

2 The wild flower bed is roughly $12\,m^2$ and therefore needs 24 g of seed. This leaves 26 g in the bag.

3 The edging needs to be 14 m. This requires 10 packs of edging. 10 × 1·5 m = 15 m which at £3·69 for a pack is roughly £37.

Going deeper

1 If Josh is 140 cm tall he would get between 500 and 600 £2 coins. 500 coins is 500 × 2·5 mm = 125 cm tall and 600 coins is 600 × 2·5 mm = 150 cm. This is roughly worth between £1000 and £1200. His weight is 35 kg which is 35 000 g. If a £1 coin weighs roughly 10 g then we can work out that 35 000 ÷ 10 is 3500 so we would have about £3500. So his weight in £1 coins would be the best choice.

2 An estimate of how many 10p coins in 1 km or 100 000 cm would be 40 000, which is 100 000 ÷ 2·5. 40 000 × 7 g = 280 000 g = 280 kg.

Page 29: Estimating calculations (NNS 4·7 & 4·8)

Practice

1 If you choose 290 and 2, you can write the calculation 290 ÷ 2 = 145. This calculation makes the closest answer to 150. With addition the closest answer is 129 (9 + 120), with multiplication the closest answer is 240 (2 × 120), with subtraction the closest answer is 118 (120 – 2), with division the next closest calculation is 420 ÷ 3 = 140.

2 Answers will vary. The following are just examples of ways to estimate each calculation.

4600 + 500 = 5100

8320 – 500 = 7820

25 × 84 = 2100

3200 ÷ 40 = 80

Going deeper

1 **a** Answers will vary. One example of a calculation that will produce a number between 300 and 500 using the numbers in the list is 4509 ÷ 12.

b One example of a calculation that will produce a number between 1000 and 1500 using the numbers in the list is 1003 + 303.

c One example of a calculation that will produce a number between 2000 and 5000 using the numbers in the list is 4509 + 20.

2 1003 – 303 – 5 × 20 = 600

3 **a** To write a calculation with the largest possible number you need to multiply the two largest numbers together, 4509 × 1003.

b To write a calculation with the smallest possible number you need to divide the smallest number by the largest, 5 ÷ 4509.

Multiplying two whole numbers makes a larger number than adding.

Dividing small numbers by large numbers makes very small numbers.

NPC Milestone 2

- Round whole numbers to the nearest multiple of 10, 100, 1000, 10 000, or 100 000
- Round numbers with up to two decimal places to the nearest whole number and to one decimal place

Page 30: Further adding and subtracting (Calc 3·1)

Practice

1 456 + 100 = 556

556 + 40 = 596

596 + 3 = 599

Or 456 + 143 = 599

2 599 + 4 = 603

3 603 + 400 = 1003

4 1003 + 100 000 + 30 000 + 2000 = 133 003

5 133 003 – 10 000 = 123 003

6 123 003 – 40 = 122 963

Going deeper

1 12 631 – 800 = 11 831

You can work this out by comparing the numbers. The tens and ones are the same and you can count up in hundreds from 11 831 to 12 631. This would be a jump of 200 to 12 031 and then 600 to 12 631. Adding these jumps gives 800 which is the difference between 11 831 to 12 631 or the amount subtracted from 12 631 to end up with 11 831. Alternatively you could count back in steps of 100 from 12 631 to 11 831. This would be a jump of 600 from 12 631 to 12 031 and then a jump of 200 to reach 11 831. Again this equals 800.

2 Answers will vary. Some might mention that crossing place value boundaries makes the subtraction harder, e.g. to subtract 40 000 from 133 003 you can subtract 33 000 to reach 100 003 and then 7000 more or to subtract 4 from 133 003 you can subtract 3 to reach 133 000 and then 1 more from 133 000 which leaves 132 999.

133 003 – 4 = 132 999

133 003 – 40 = 132 963

133 003 – 400 = 132 603

133 003 – 4000 = 129 003

133 003 – 40 000 = 93 003

Page 31: Balancing with adding and subtracting (Calc 3·3)

Practice

1 580 + 60 = 600 + 40

2 Answers will vary, e.g.

288 – 169 = 290 – 171

187 + 236 = 200 + 223

3 **a** 695 + 40 = 700 + 35 **b** 695 – 40 = 700 – 45

4 **a** 589 + 43 = 600 + 32 **b** 632 – 197 = 630 – 195

Going deeper

1 **a** Answers will vary.

An example of an adding calculation made easier using balancing is 397 + 65 = 400 + 62.

An example of an adding calculation that is not made easier using balancing is 150 + 234 = 384.

b Answers will vary.

Some children might say that the first calculation involves crossing a place value boundary, e.g. 400 and it is close to this boundary.

Using a balancing calculation to solve the second calculation is not as easy as using a partitioning method Adding the ones together, then the tens, then the hundreds.

2 Answers will vary.

Some children might say that when numbers are near to place value boundaries that balancing calculations can make subtractions easier, e.g. 593 – 271 = 600 – 278.

Page 32: Adding and subtracting decimals by partitioning (Calc 3·4)

Practice

1 **a** A to B is 2·6, A to C is 5., A to D is 6·8, A to E is 10, A to F is 10·8

b 13·1 km

2

3·8 – 0·5 = 3·3	5·6 – 0·5 = 5·1	4·9 – 0·5 = 4·4	3·7 – 0·5 = 3·2
3·8 – 1·3 = 2·5	5·6 – 1·3 = 4·3	4·9 – 1·3 = 3·6	3·7 – 1·3 = 2·4
3·8 – 2·4 = 1·4	5·6 – 2·4 = 3·2	4·9 – 2·4 = 2·5	3·7 – 2·4 = 1·3
3·8 – 0·6 = 3·2	5·6 – 0·6 = 5	4·9 – 0·6 = 4·3	3·7 – 0·6 = 3·1

Going deeper

1 **a** Answers will vary. Some children will explain that they would partition both numbers in the calculation into ones and tenths, and then add the ones followed by the tenths before recombining, e.g.

3 + 2 = 5   0·4 + 0·8 = 1·2   5 + 1·2 = 6·2

Alternatively they might prefer to partition the smaller number and then add the ones followed by tenths to the larger number, e.g.

3·8 + 2 = 5·8 and then 5·8 + 0·4 = 6·2

b Answers will vary. Some children will explain that they would partition both numbers in the calculation into ones and tenths, and then subtract the ones followed by the tenths before recombining, e.g.

3 – 2 = 1   0·8 – 0·4 = 0·4   1 + 0·4 = 1·4

Alternatively they might prefer to partition the number that they want to subtract and then subtract the ones followed by the tenths, e.g.

3·8 – 2 = 1·8 and then 1·8 – 0·4 = 1·4

2 **a** Answers will vary and may be similar to Q1a.

b Answers will vary and may be similar to Q1b.

3 Answers will vary.

Page 33: Adding and subtracting in columns (Calc 3·5 & 3·6)

Practice

1 Answers will vary. Some children might explain how they could use a balancing calculation, e.g. 170 + 153 others might explain how they could partition the smaller number and then separately add the hundreds, tens and ones, e.g. 167 + 100 = 267, 267 + 50 = 317, 317 + 6 = 323.

Look for children who decide that they prefer the column method to explain this in relation to the exchanging involved.

2 Answers will vary. Some children might explain how they could use a balancing calculation, e.g. 480 – 382; others might explain how they could partition the smaller number and then separately subtract the hundreds, tens and ones, e.g. 484 – 300 = 184, 184 – 80 = 104, 104 – 6 = 98.

Look for children who decide that they prefer the column method to explain this in relation to the exchanging involved.

3 **a** Answers will vary.

Examples might include

Pizza and sandwich £9·85 + £2·89 = £12·74

Pasta and jacket potato £6·99 + £3·45 = £10·44

Pasta and fish fingers £6·99 + £5·15 = £12·14

b Answers will vary.

Examples might include

Pizza and sandwich £3·49

Pasta and jacket potato £5·79

Pasta and fish fingers £4·09

Going deeper

1 **a**

$$\begin{array}{ccccc} & 2 & 3 & 2 & 6 \\ + & 3 & 4 & 7 & 2 \\ \hline & 5 & 7 & 9 & 8 \\ \hline \end{array}$$

b

$$\begin{array}{ccccc} & 2 & 2 & 1 & 4 \\ + & 1 & 7 & 8 & 6 \\ \hline & 4 & 0 & 0 & 0 \\ \hline & {}_1 & {}_1 & {}_1 & \end{array}$$

c

$$\begin{array}{ccccc} & 6 & 8 & 9 & 8 \\ - & 4 & 5 & 5 & 3 \\ \hline & 2 & 3 & 4 & 5 \\ \hline \end{array}$$

d

$$\begin{array}{ccccc} & 7 & {}^{7}\cancel{8} & {}^{9}\cancel{0} & {}^{1}3 \\ - & 3 & 0 & 8 & 4 \\ \hline & 4 & 7 & 1 & 9 \\ \hline \end{array}$$

NPC Milestone 2

- Convert an adding or subtracting calculation to an easier equivalent calculation

Page 34: Sequences and patterns (P&A 1·1 & 1·2)

Practice

1 2010, 2004, 1998, 1992, 1986

2 Leap years will be 2020, 2024, 2028, 2032, 2036

3 Answers will vary.

4 3243, 3255, and 3279

Going deeper

1 Answers will vary.

2 168 – 123 = 45 so the gaps between possible numbers will be factors of 45 smaller than 45 itself, that is, 1, 3, 5, 9 and 15. So possible missing sequences will be: 124, 125, 126, … ; 126, 129, 132, … ; 128, 133, 138, … ; 132, 141, 150, … ; 138, 153.

3 In general, a good strategy is to subtract the smaller number from the larger and then investigate the factors of the difference; these will specify the sizes of the possible gaps between the intermediate numbers. In the special case that the difference between the original numbers is a prime number, the only possibility is that the intermediate numbers each increase by 1 each time.

In making up their own sequence problems children could either simply specify two numbers with an interesting difference, or present a sequence with several gaps, as in Practice Q4.

Page 35: Decimal sequences (P&A 1·5)

Practice

1 The orange to green ratio (orange:green) is 3:10, so possible values for individual rods include: (3, 10), (6, 20), (9, 30), etc. but see also following questions offering decimal values.

2 Green rods would be worth 0·3, so the sequence of the illustrated rods would be: 0·3, 0·6, 0·9, 1·2, 1·5, 1·8, 2·1, 2·4, 2·7, 3·0.

3 0·5, 0·8, 1·1, 1·4, 1·7, 2·0. The first whole number will be 2·0 because the difference between 0·5 and 2·0 is 1·5, and that is a multiple of 0·3.

4 6·5, 6·9, 7·3, 7·7, 8·1, …

5 Answers will vary.

Going deeper

1 This time an orange rod is worth 0·1, so a green rod will be worth 0·03. Numbers in boxes will therefore be: 0·03, 0·06, 0·09, 0·12, 0·15, 0·18, 0·21, 0·24, 0·27, 0·3.

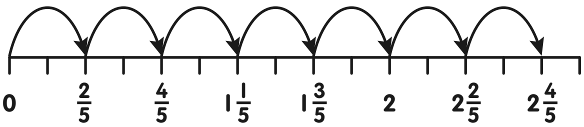
2 The sequence rule is 'add 0·07', so the black (7) and orange (10) number rods in a format similar to Q1 can be used to illustrate the sequence.

Page 36: Fraction sequences (P&A 1·6)

Practice

1 Answers will vary but may include the use of number rods, Numicon Shapes, objects with a number circled, etc.

2 If number rods are used, a combination of red and yellow rods would be suitable.

3 

4 Answers will vary but the number line is a good way to help with counting on and back in fractions.

5 a $\frac{2}{3}, 1\frac{1}{3}, 2, 2\frac{2}{3}, 3\frac{1}{3} \ldots$

b $\frac{5}{6}, 1\frac{2}{3}, 2\frac{1}{2}, 3\frac{1}{3}, 4\frac{1}{6} \ldots$

c $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{8}, 1\frac{1}{2}, 1\frac{7}{8} \ldots$

Going deeper

1 Each gap in the sequence is $\frac{7}{9}$ and there will be 9 gaps between 0 and the 10th term. $9 \times \frac{7}{9} = 7$, so the 10th term will be 7.

2 Children may check and explain this sequence in any number of ways including writing out a sequence, starting at 0 and adding on $\frac{7}{9}$ each time.

Page 37: Connecting sequences with equivalent fractions (P&A 1·7)

Practice

1 $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24} \ldots$

2 Keep adding 2 to the numerator and adding 3 to the denominator.

3 They are all equivalent to $\frac{2}{3}$.

4 a $\frac{1}{3}, \frac{2}{6}, \frac{3}{9} \ldots$

b $\frac{2}{7}, \frac{4}{14}, \frac{6}{21} \ldots$

c $\frac{5}{8}, \frac{10}{16}, \frac{15}{24} \ldots$

d $\frac{3}{5}, \frac{6}{10}, \frac{9}{15} \ldots$

Going deeper

1 Each equivalent fraction in a family sequence can be simplified to its lowest form by dividing top and bottom by a common factor, that is, cancelling.

2 Answers will vary but may include generating the fractions in a less systematic way. The method used in Practice Q1 is the easiest, and most systematic way.

3 All fractions in a fraction family correspond to the same point on a number line, because they are all equivalent. Showing different fractions on separate number lines but lining all the number lines up one below the other should illustrate this.

4 Answers will vary, e.g. illustrating a fraction family with two Numicon Shapes (one for the numerator, another for the denominator) will mean that the multiple of each Shape required for the numerator and the denominator of any fraction, will be the same. This could follow directly from the rule given for Practice Q2.

5 The 10th member of the family will be 3×10 (for the numerator) and 8×10 (for the denominator) $= \frac{30}{80}$.

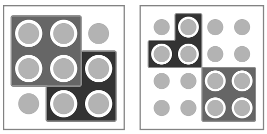
NPC Milestone 2

- Find the term-to-term rule for a linear sequence involving whole numbers, fractions or decimals, and work out missing terms

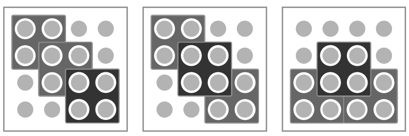
Page 38: Transformations (Geo 2·1)

Practice

1 There are two ways, excluding reflections or rotations of either:



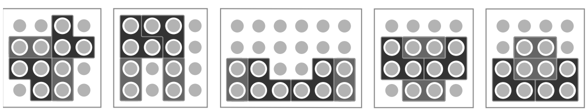
2 Answers will vary. Several ways depending on which Shape is added Here are some examples if the added Shape is a 3-shape:



3 Answers will vary. Many possibilities. Check that any offered solutions are indeed symmetrical.

Going deeper

1 Answers will vary. There are very many possibilities. Here are some examples:



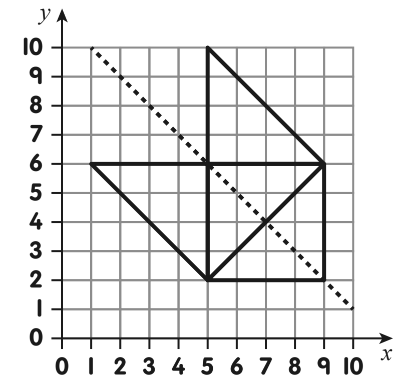
2 Check that the patterns match.

Page 39: Reflections on a coordinate grid (Geo 2·2)

Guidance: You can save time by giving children photocopy master 16 from the Geometry, Measurement and Statistics 5 Teaching Resource Handbook of a blank coordinate grid, instead of asking for this to be drawn.

Practice

1 Completed diagram should look like this: (Line of symmetry has been added).



2 Triangles have the same area, angles and side lengths. Orientation is different.

3 Hexagon – symmetry line shown dotted.

Going deeper

1 Check that images are reflections.

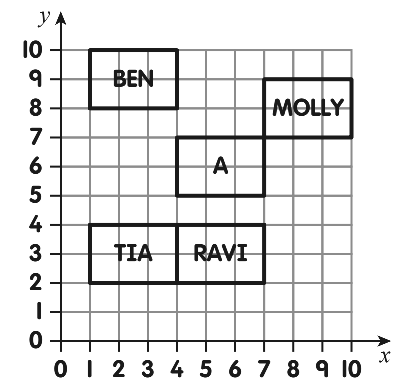
2 Answers will vary. Many possible answers.

Page 40: Describing translations using coordinates (Geo 2·3)

Guidance: You can save time by giving children photocopy master 16 from the Geometry, Measurement and Statistics 5 Teaching Resource Handbook of a blank coordinate grid, instead of asking for this to be drawn.

Practice

1 **a**



b Ben: 3 left, 3 up — Molly: 3 right, 3 up

Ravi: 3 down — Tia: 3 left, 3 down

2 Ben: 3 right, 3 down — Molly: 3 left, 2 down

Ravi: 3 up — Tia: 3 right, 3 up

Going deeper

1 It's Ravi's rectangle. In any order:

Left 16, up 3

Right 10, down 7

Right 6, up 7

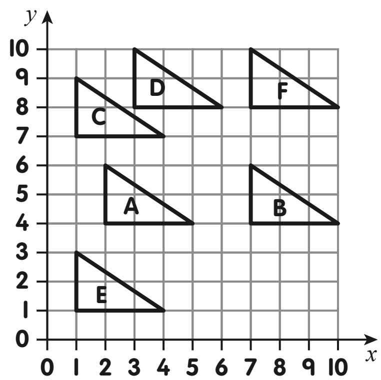
2 Ben's and Tia's rectangles reflect in line $x = 6$; the original rectangle and Ravi's rectangle reflect in line $x = 4{\cdot}5$; Ravi's and Tia's rectangles reflect in line $y = 4$.

Page 41: Exploring translations on a coordinate grid (Geo 2·4)

Guidance: You can save time by giving children photocopy master 16 from the Geometry, Measurement and Statistics 5 Teaching Resource Handbook of a blank coordinate grid, instead of asking for this to be drawn.

Practice

1



2 **a** A to B **b** E to C **c** C to D **d** F to D

e E to B **f** B to E **g** F to E

Going deeper

1 (3,3), (3,1) and (6,1)

2 Answers will vary. Check that images are in correct position.

GMS Milestone 1

- Identify lines of symmetry in given figures
- Find or show images of points and polygons under translation or reflection in the first quadrant (with lines of symmetry parallel to the x- and y-axes), describing the transformation and how the position and coordinates of the point or polygon have changed
- Explain that more than one transformation can map one shape onto another

Page 42: Working with negative numbers (NNS 5·1)

Practice

1 **a** $^{-}8$ m **b** $^{-}3$ m **c** $^{-}5$ m

2 The numbers decrease in positive value because the diver is moving in the direction towards zero.

3 As a diver hits the water they will be at zero height and zero depth.

4 The numbers increase in negative value because the diver is moving in the direction away from zero.

Going deeper

1 Above sea level refers to the distance from the height of the sea to the point of measure.

2 If the sea level were to rise by one metre the depth would increase to 8351.

3 Illustrate this on a time line from 384 BCE with a jump of 60 years to 324 BCE. The numbers decrease in size as they approach the zero.

Page 43: Negative and positive temperatures (NNS 5·2 & 5·3)

Practice

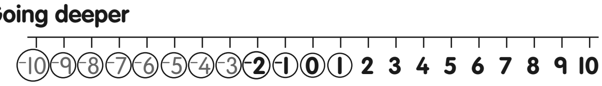
1 $^{-}9$°C, $^{-}7$°C, $^{-}3$°C, 1°C, 2°C, 4°C, 11°C

2 5°C, 4°C, 3°C, 2°C, 1°C, 0°C, $^{-}1$°C, $^{-}2$°C, $^{-}3$°C, $^{-}4$°C, $^{-}5$°C

3 $^{-}32$°C, $^{-}21$°C, $^{-}17$°C, $^{-}14$°C, $^{-}11$°C, $^{-}9$°C, $^{-}6$°C, $^{-}3$°C

Going deeper

1



2 **a** $^{-}23$

b $^{-}17$

c $^{-}4$

3 Negative numbers can be used in association with many different measures, e.g. with owing money, coordinates or latitudes south of the equator and longitudes west of the prime meridian.

Page 44: Temperature differences (NNS 5·4 & 5·5)

Practice

1 **a** 120°C

b 90°C

c 21°C

d 68°C

e 29°C

2 Death Valley, Kharga, Athens

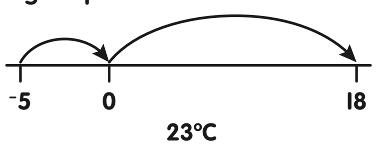
Lapland, Luhansk, Yukan

3 −58°C, −47°C, −37°C, −21°C, −18°C

4 Examples of pairs of numbers with a difference 12°C are 10°C and −2°C or 8°C and −4°C.

Going deeper

1



Illustrate with two jumps on the number line. The first from −5 to 0 and the second from 0 to 18. Label the difference as 23°C.

One strategy to work out the difference between positive and negative numbers is to bridge on the number line from the negative number to zero and then from zero to the positive number. Add up the two jumps to find the difference.

2 The difference between both numbers on a number line can be shown as a length and a length cannot be negative.

Page 45: Negative numbers and direction (NNS 5·6)

Practice

1 3

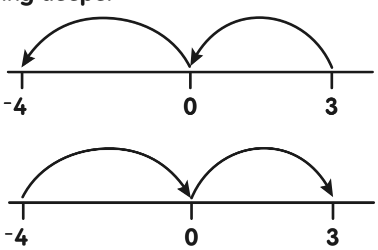
2 −7

3 8, 5, 2, −1, −4, −7, −10, −13, −16, −19

4 −10, −6, −2, 2, 6, 10, 14, 18, 22, 26

Going deeper

1



−4 + 7 = 3

Illustrate jumps on a number line from 3 to 0 and 0 to −4. Repeat with jumps from 4 to 0 and 0 to −3. Add the two jumps to show that the difference is the same. This is always the same.

2 −15, −10, −5, 0, 5, 10

3 $-3\frac{3}{4}, -2\frac{1}{2}, -1\frac{1}{4}, 0, 1\frac{1}{4}, 2\frac{1}{2}, 3\frac{3}{4}$

NPC Milestone 2

- Read, write and order positive and negative numbers
- Calculate the difference between a positive and a negative number

Page 46: Developing multiplying and dividing (Calc 4·2 & 4·3)

Practice

1 2- and 5-shapes

2 5-, 6-, 7- and 8-shapes

3 7, 9, 11 and 12

Going deeper

1 Usually, if we call four different Numicon Shapes a, b, c and d, 'a' combines with b, c and d (to give 3 products), 'b' then combines with c and d (to give 2 more products), and 'c' combines with d to give the final product. 3 + 2 + 1 = 6 products.

2 There would be fewer than 6 products either if some Shapes were repeated, or some products are duplicated, as with the 2, 3, 4 and 6 collection of Shapes (3 × 4 = 2 × 6 = 12, so the product '12' is duplicated).

3 Yes, it is possible, e.g. the 4-, 5-, 6- and 8-shapes. The only way to obtain an odd product is to multiply two odd numbers together, so as long as there are either no odd values, or just one odd value in the set, all the products will be even.

4 Yes, it is possible, e.g. 3, 5, 7, and 9. If all four values are odd, then all the products will be odd.

Page 47: Short written methods: multiplying and dividing (Calc 4·5 & 4·6)

Practice

1 322 × 4 = 1288 (greatest product) and 234 × 2 = 468 (smallest product)

2 432 ÷ 2 = 216 (greatest quotient) and 223 ÷ 4 = 55 r 3 (smallest quotient)

3 2432 × 5 = 12 160

4 2496 ÷ 6 = 416 and 2490 ÷ 6 = 415 are both possible answers.

5 Answers will vary.

Going deeper

1 To make the greatest product use the highest digit as the multiplier, and arrange the other digits to make the highest possible multiplicand. To make the smallest product use the smallest digit as the multiplier, and arrange the other digits to make the smallest possible multiplicand.

To make the greatest quotient divide by the smallest digit, and arrange the other digits to make the dividend as large as possible. To make the smallest quotient divide by the largest digit, and arrange the other digits to make the dividend as small as possible.

2 In general larger numbers are harder to calculate successfully mentally, particularly where there are no, or few, zeroes, e.g. 38 784 × 7 is probably best done with a written method, whereas 10 000 ÷ 5 is easy to do mentally.

Page 48: Fractions of an amount (Calc 4·7)

Practice

1 Answers will vary, e.g. $\frac{1}{8}$ and $\frac{1}{3}$ would be easy to calculate mentally because they are each represented by a whole number of squares. But see fuller explanation under Q4 below.

2 2, 5, 12, 8, 3 and 9 respectively, e.g. 2 is $\frac{1}{7}$ of 14 can be shown by drawing a 7 × 2 array and colouring in 2 squares.

3 **a** 3 × 9 = 27   27 ÷ 3 = 9,
b 6 × 7 = 42   42 ÷ 6 = 7,
c 8 × 9 = 72   72 ÷ 8 = 9,
d 4 × 16 = 64   64 ÷ 4 = 16

Going deeper

1 **a** 120   **b** 228   **c** $\frac{1}{9}$ **d** $\frac{1}{4}$

2 As the denominator of the fraction doubles, the product halves. The next line would be: $\frac{1}{24}$ of 24 = 1.

3 Answers will vary, e.g. $\frac{1}{2}$ of 64 = 32

$\frac{1}{4}$ of 64 = 16

$\frac{1}{8}$ of 64 = 8

4 (See also Q1 above) All the easy fractions would be: $\frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{6}, \frac{5}{6}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}, \frac{1}{24}, \frac{5}{24}, \frac{7}{24}, \frac{11}{24}, \frac{13}{24}, \frac{17}{24}, \frac{19}{24}$ and $\frac{23}{24}$. Notice that the denominators are all factors of 24, which means that whole squares can be coloured. The numerators are all prime numbers that are not factors of 24, which means that these fractions are all in their lowest form. Fractions that would be difficult to colour in would be all other fractions, e.g. $\frac{3}{5}, \frac{1}{7}, \frac{5}{9}$.

Page 49: Multiplying and dividing decimals (Calc 4·8 & 4·9)

Practice

1 3 × 0·4 = 1·2 km

2 2·8 ÷ 4 = 0·7 km

3 **a** 0·6, 1·0, 1·2   **b** 0·9, 1·5, 1·8   **c** 1·5, 2·5, 3·0
d 1·8, 3·0, 3·6   **e** 2·4, 4·0, 4·8   **f** 2·7, 4·5, 5·4

4 Answers will vary, e.g.

a 1·5 ÷ 3 = 0·5, 1·5 ÷ 5 = 0·3
b 2·4 ÷ 3 = 0·8, 2·4 ÷ 6 = 0·4
c 3·6 ÷ 9 = 0·4, 3·6 ÷ 6 = 0·6
d 4·2 ÷ 7 = 0·6, 4·2 ÷ 6 = 0·7
e 5·2 ÷ 2 = 2·6, 5·2 ÷ 4 = 1·3

Going deeper

1 2·8 ÷ 4 = 0·7 and 28 ÷ 4 = 7, or 2·8 ÷ 0·7 = 4

2 4 × 0·6 = 2·4   4 × 6 = 24   0·6 × 4 = 2·4   6 × 4 = 24

3 13·2 ÷ 11 = 1·2   12 × 11 = 132

NPC Milestone 2

- Use multiplying and dividing facts and knowledge of factors and multiples to solve problems
- Solve problems effectively by finding fractions of amounts, making use of multiplying and dividing facts
- Multiply and divide decimals to one decimal place

Page 50: Using proper fractions (NNS 6·1)

Practice

1 **a**, **b** The stronger lemonade was made with $\frac{1}{2}$ lemon juice and $\frac{1}{2}$ water. The weaker lemonade is $\frac{1}{3}$ lemon juice and $\frac{2}{3}$ water.

2 $\frac{1}{2}$ is bigger than $\frac{3}{8}$.

$\frac{3}{8}$ can be illustrated with an 8-rod with a 3-rod on top and $\frac{1}{2}$ can be illustrated with an 8-rod and a 4-rod.

3 $\frac{5}{6}$ is bigger than $\frac{2}{3}$.

$\frac{2}{3}$ is equivalent to $\frac{4}{6}$ (Illustrate with two Numicon 6-shapes. The first 6-shape needs 5 pegs or counters and the second 6-shape needs 4 pegs or counters.

4 $\frac{6}{10}$ and $\frac{12}{20}$ are equivalent fractions and so equal in value. The numerator and denominator have both been multiplied by 2. Illustrate with Numicon Shapes or rods.

Going deeper

1 17 out of 20 is her best score. To compare these scores they can all be written as fractions with the same denominator, e.g. $\frac{4}{5}$ can be written as $\frac{16}{20}$, $\frac{7}{10}$ can be written as $\frac{14}{20}$. These are both less than $\frac{17}{20}$.

2 It would be better to be right $\frac{56}{64}$ times, which is the same as $\frac{7}{8}$, than $\frac{3}{4}$ times which is $\frac{6}{8}$ or $\frac{27}{32}$. This is less than $\frac{28}{32}$ which is equivalent to $\frac{7}{8}$.

3 $\frac{7}{12}$, $\frac{2}{3}$, $\frac{73}{108}$, $\frac{49}{72}$, $\frac{25}{36}$

4 Answers will vary. An example of a set of three fractions in order of size where the denominators have a common factor is $\frac{2}{6}$, $\frac{9}{24}$, $\frac{7}{12}$.

Page 51: Common denominators (NNS 6·2)

Practice

1 One way is to convert the number of beats each person plays into fractions with the same denominator, e.g. $\frac{2}{9} = \frac{8}{36}$, $\frac{4}{6} = \frac{24}{36}$ and $\frac{1}{2} = \frac{18}{36}$. This shows that Ravi plays the most times and Molly plays the fewest.

2 An example of a set of three fractions in order of size is: $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$. These can be compared as fractions with a common denominator of 12, e.g. $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$.

3 Another example of a set of three fractions in order of size, beginning with the largest is $\frac{3}{4}$, $\frac{7}{12}$, $\frac{3}{8}$. Illustrate on a number line with 24 intervals. Plot points $\frac{18}{24}$, $\frac{14}{24}$, $\frac{9}{24}$.

Going deeper

1 $\frac{3}{4}$ is the fraction closest to 1.

2 **a** There are 60 possible sets of three fractions. Children should work systematically to find all the sets.

b The highest possible total is $\frac{3}{4} + \frac{7}{12} + \frac{2}{3}$ or $\frac{9}{12} + \frac{8}{12} + \frac{7}{12} = \frac{24}{2} = 2$.

3 Clara has dug the most and Dev has dug the least. These fractions can be written with common denominators to compare, e.g. $\frac{1}{4} = \frac{6}{24}$ $\frac{1}{3} = \frac{8}{24}$ $\frac{3}{8} = \frac{9}{24}$. The combined amount dug is $\frac{6}{24} + \frac{8}{24} + \frac{9}{24} = \frac{23}{24}$ so there is $\frac{1}{24}$ still to be dug.

Page 52: Using 'greater than' and 'less than' signs (NNS 6·3)

Practice

1 Answers will vary. An example of a pair of fractions from the list compared using the < sign is: $\frac{11}{18} = \frac{22}{36}$ so $\frac{7}{36} < \frac{11}{18}$.

2 Answers will vary. An example of a pair of fractions from the list compared using the > sign is: $\frac{3}{4} = \frac{9}{12}$ so $\frac{3}{4} > \frac{7}{12}$.

3 The fractions can be written in order with 36 as common denominator.

$\frac{7}{36}$, $\frac{1}{2} = \frac{18}{36}$, $\frac{5}{9} = \frac{20}{36}$, $\frac{7}{12} = \frac{21}{36}$, $\frac{11}{18} = \frac{22}{36}$, $\frac{2}{3} = \frac{24}{36}$, $\frac{3}{4} = \frac{27}{36}$, $\frac{5}{6} = \frac{30}{36}$

Going deeper

1 $\frac{9}{18} > \frac{5}{12}$

2 $\frac{5}{8} < \frac{6}{9}$ or $\frac{5}{8} < \frac{9}{6}$

Page 53: Simplifying proper fractions (NNS 6·4)

Practice

1 The proportions are $\frac{15}{40}$ or $\frac{3}{8}$ lemon juice and $\frac{25}{40}$ or $\frac{5}{8}$ water.

2 Answers will vary. An example of six fractions that are equivalent to $\frac{3}{5}$ are $\frac{6}{10}$, $\frac{9}{15}$, $\frac{12}{20}$, $\frac{15}{25}$, $\frac{18}{30}$, $\frac{21}{35}$.

3 $\frac{3}{4}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{4}{5}$, $\frac{13}{24}$, $\frac{7}{8}$, $\frac{16}{17}$

Going deeper

1 This fraction can't be simplified because 53 is a prime number and therefore it can't be divided by anything other than itself and 1.

2 You could simplify $\frac{48}{52}$ to $\frac{12}{13}$ or $\frac{48}{54}$ to $\frac{8}{9}$.

3 A good strategy to simplify fractions is divide by the highest common factor of the numerator and denominator.

NPC Milestone $\boxed{3}$

- Use knowledge of factors and multiples to find equivalent fractions and to simplify fractions to their lowest terms
- Compare and order fractions with denominators which are multiples of the same number

Page 54: Inverse relationships (P&A 2·1 & 2·2)

Practice

1 In the first arithmagon the numbers in the circles add to give the numbers in the squares. In the second arithmagon the numbers in the circles multiply to give the numbers in the squares.

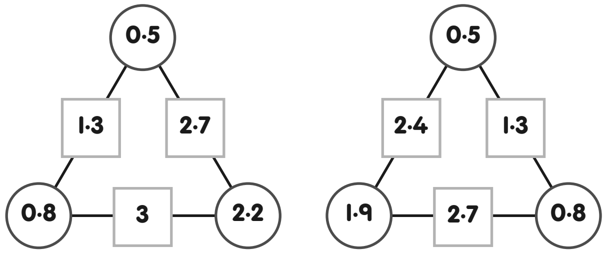
2 **a** Circle 8, square 56 **b** Top 6, lower left 12, lower right 5

3 It doesn't matter where the 'circle' numbers go, but 'square' numbers must be placed so that: $\frac{3}{4} + 1\frac{1}{4} = 2$, $1\frac{1}{4} + 1\frac{1}{2} = 2\frac{3}{4}$, and $\frac{3}{4} + 1\frac{1}{2} = 2\frac{1}{4}$.

Going deeper

1 Look for common factors in the 'square' numbers, then experiment with possibilities. In 2b, all the 'square' numbers are multiples of 6 so try putting 6 in one of the circles.

2 Two different adding arithmagons are possible, depending upon whether 2·7 is placed next to, or opposite, 0·5.



3 One circle and two adjacent squares may be left blank; whatever is put into one of these three boxes will determine the others uniquely.

4 40, 60 and 96 can be the 'square' numbers in a multiplying arithmagon. 18, 25, and 29 can be the 'square' numbers in an adding arithmagon.

Page 55: Missing number calculations (+ and –) (P&A 2·3 & 2·4)

Practice

1 298 + 445 = 743. Start with the ones column: which number, when added to 5 gives an answer with 3 ones? The tens column is now easily calculated.

2 **a** 15 + 75 = 90

b 376 + 538 = 914

c 513 – 285 = 228

d 120 – 69 = 51

3 **a** 636 + 195 = 831

b 470 – 119 = 351

c 787 + 115 = 902

4 Answers will vary.

5 3456 – 2778 = 678

Going deeper

1 604 – 422 = 182. Check by doing 422 + 182 = ? (604), and by doing 604 – 182 = ? (422).

2 5774 = 8959 – 3185. Method 1: Set out the problem as a column subtraction, and then reason from the ones column. Method 2: 5774 + 3185 = ? (8959).

Page 56: Missing number calculations (× and ÷) (P&A 2·4)

Practice

1 174 × 5 = 870. Look at the hundreds column; missing number here must be 1 since the product only contains 8 hundreds. Next look at the tens column; 7 × 5 = 35, so 2 must have been carried from the ones column – this implies that the ones empty box must be 4. Alternative strategy: divide 870 by 5.

2 336 ÷ 8 = 42   455 ÷ 7 = 65

3 Answers will vary.

Going deeper

1 Method 1: Multiply your proposed multiplicand (top number) by 3; is the product 6048? Method 2: Divide 6048 by 3.

2 Method 1: Multiply 1586 by 6 to give 9516 (one digit in each box). Method 2: Rewrite the calculation setting it out using a short dividing method and work backwards, starting with the ones. Notice that there is no remainder; this means that the final calculation has to be 36 ÷ 6 = 6. This implies that the tens calculation had to be 51 ÷ 6 = 8 remainder 3. This in turn implies that the hundreds calculation had to be 35 ÷ 6 = 5 remainder 5. Which then implies that the thousands calculation had to be 9 ÷ 6 = 1 remainder 3.

Page 57: Solving problems by working backwards (P&A 2·6 & 2·7)

Practice

1 Tia adds Ben's total spend (£7·58) to the amount he has left over (£2·42).

2 £12

3 £5, since £5 ÷ 2 = £2·50.

Going deeper

1 Method involves doing the inverse calculations in reverse order. 8 × 7 = 56, 56 – 6 = 50, 50 ÷ 5 = 10, so 10 was the original number.

2 Let 'a' be the first number thought of, then perform each step in order.

$a \times 5 = 5a$

$(5a + 25) \div 5 = a + 5$

This is because $5a \div 5$ is just a, and $25 \div 5$ is 5, giving us altogether $(5a + 25) \div 5 = a + 5$

Take away a (which is the first number thought of) from $a + 5$ and this just leaves 5.

Any number that you think of in the first place is always removed in the calculating, thus always leaving 5 as the answer.

3 Agata has £39·50, Harry has £34·70.

NPC Milestone 3

- Use the inverse relationships between adding and subtracting, and multiplying and dividing, to complete calculations with missing numbers

Page 58: Mental and written methods of adding (Calc 5·1)

Practice

1 The totals are: Blue 4090; Green 3814; Red 3940; Orange 4801. Individuals' preferred methods will vary, but look for children adapting amounts and compensating in order to make calculations mentally solvable, e.g. with Red class, 1695 + 2245 = 1700 + 2240 = 3940.

2 Again, answers will vary. Look for, e.g. 2025 + 4775 = 2000 + 4800, and 1598 being used often.

Going deeper

1 **a**, **b** Answers will vary, but look for children subtracting either a March or an April amount from a total, and moving from mental to written or vice versa, e.g. with the Orange results, 4801 – 2834 = 1967, and/or having used a written method to add initially, noticing that 2834 + 1067 = 2801 + 2000 = 4801.

2 Individuals' methods will vary, but look for children rounding up and/or down to support their estimating. Year 6 collected the most.

Page 59: Using the written column method for adding (Calc 5·2 & 5·3)

Practice

1 The tennis match has the bigger crowd (of 15 731).

2 **a** 4369 + 3694 = 8063

b 4639 + 4369 = 9008

$$\begin{array}{rcccc} & 4 & 3 & 6 & 9 \\ + & 3 & 6 & 9 & 4 \\ \hline & 8 & 0 & 6 & 3 \\ \hline & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \end{array}$$

$$\begin{array}{rcccc} & 4 & 6 & 3 & 9 \\ + & 4 & 3 & 6 & 9 \\ \hline & 9 & 0 & 0 & 8 \\ \hline & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \end{array}$$

Going deeper

1 Notice that no 4-digit number made with these digits and, beginning with either a '6' or an '8' can be part of a pair totalling under 8000, begin by listing all other possible combinations, in order:

2568 5268

2586 5286

2658 5628

2685 5682

2856 5826

2865 5862

By inspection, note that each number starting with a '2' can combine with each other '2' number, giving 15 possible number pairs. And each of the lowest four '2' numbers can combine with each of the lowest two '5' numbers, giving a further 8 possibilities.

2 Begin by listing all possible numbers made with these digits that begin with either a 3 or a 4, in order:

3469 4369

3496 4396

3649 4639

3694 4693

3946 4936

3964 4963

Now look for pairs of numbers that will produce an '8' in the ones place of their total, i.e. only those pairs in which both numbers end with a '9'. Systematic trialling will then reveal that only the pair 3649 + 4369 = 8018.

Page 60: Adding money (Calc 5·4)

Practice

1 He could have bought Yoghurt, Oats, or Rice with his Apple Juice.

2 Sally could have bought: Raisins, Oats and Yoghurt, Rice, Oats and Yoghurt, or Apple Juice, Oats and Yoghurt. Only combinations that include the two lowest amounts will total less than £6. If you take the least expensive item (the yoghurt) and combine this with the third and fourth lowest cost items, the total is more than £6, so only the two lowest amounts, combined with any other item will be less than £6.

Going deeper

1 Individual prices will have to be between £9 and £9·50 if pairs of prices are to lie between £18 and £18·50. The only individual price possibilities (apart from '£9' with '£9·50') then are:

£9·30	£9·32	£9·35
£9·20	£9·23	£9·25
£9·02	£9·03	£9·05

Each row 1 price can be combined with any row 3 price, giving 9 possibilities.

Each row 1 price can be combined with any row 2 price, giving 9 more possibilities.

Row 2 prices may be combined with each other, giving 3 more possibilities.

Row 3 prices may be combined with each other, giving 3 more possibilities.

There are therefore 1 + 9 + 9 + 3 + 3 = 25 possible pairs.

2 There are various ways in which possibilities may be explored here. Both price final digits must be '2' since there is no other way to arrive at '4' in the ones total with the numbers available. Similarly, looking at possible bonds to 12 systematically reveals that the only way to achieve £12 in the total is with one price as £3… and the other as £9… The only £9… possibilities are £9·02, £9·32, or £9·52. By systematically subtracting each of these from £12·84 we discover that only £9·32 + £3·52 = £12·84.

Page 61: Adding decimals and measures (Calc 5·5)

Practice

1 Her journey would be shorter if she travelled directly (102·87 vs 116 km).

2 101·51 km

3 Calling in at three towns involves adding pairs of distances together, so you need to look for pairs of distances that together total less than 90 km. Using capital letter abbreviations for the towns, the following three town journeys are possible:

(A→E→B, 72·38 km)

(B→C→D, 76·08 km)

(B→C→E, 85·57 km)

(B→D→C, 80·94 km)

(B→E→A, 72·38 km)

(B→E→C, 77·47 km)

(B→E→D, 81·71 km)

Going deeper

1 **a** 2·478 + 4·095 = 6·573

b (2·478 + 5·209) balance (3·188 + 4·095) The difference is 0·404 kg.

c 14·97 kg

d Individual answers to this question will vary, and will reveal how comfortable children are with decimal fractions as opposed to larger numbers.

NPC Milestone 3

- Use efficient written column methods for adding and subtracting whole numbers up to 10 000 and decimals with up to 3 decimal places

Page 62: Mental and written methods of subtracting (Calc 6·1)

Practice

1 Answers will vary.

2 Answers will vary. Children may use a written column method, although this is not required; mental methods are also acceptable. Answers can be checked either by using another method, or by adding, e.g. if you have 904 – 189 = 715 then this may be checked with 715 + 189 = 904.

3 Answers will vary.

Going deeper

1 Answers will vary. All can be done using a written column method, and most can also be done mentally, e.g. 904 – 189 = 905 – 190 = 915 – 200 = 715.

2 **a** Every number in List B can be subtracted from every number in List A, so there are 6 × 6 = 36 different subtracting calculations possible.

b Answers will vary.

3 Answers will vary.

Page 63: Using the written column method for subtracting (Calc 6·2)

Practice

1 4345 – 629 = 3716. This may be checked by adding: 3716 + 629 = 4345.

2 3902 – 3395 = 507 miles

Going deeper

1 5134 – 3145 = 1989

2 Answers will vary.

Page 64: Subtracting money (Calc 6·3)

Practice

1 £24·61

2 **a** £36·43

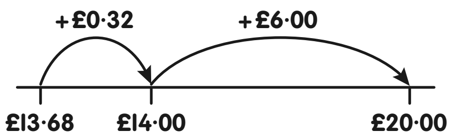
b Between weeks 4 and 5, difference is £27·63.

Going deeper

1 £92·57 – £78·83 = £13·74

2 First three weeks' total: £270·79; second three weeks' total: £281·72. Difference: £10·93.

3 £20·00 – £13·68 = £6·32. Mentally, this calculation is probably most easily accomplished by 'bridging' through £14·00.



Page 65: Subtracting decimals and measures (Calc 6·4 & 6·5)

Practice

1 3·665 litres

2 **a** Subtract the smallest volume from the largest: 22·641 – 5·438 = 17·203

b By estimating and trialling, the smallest difference is 10·206 – 8·007 = 2·199 litres.

Going deeper

1 By estimating and trialling, 18·889 – 10·206 = 8·683 is closest to 9.

2 You may use a written column method, or a variety of mental methods, e.g. by 'rounding and compensating'

84·46 – 75·57 = 84·89 – 76·00 = 88·89 – 80·00 = 8·89 litres

3 Answers will vary.

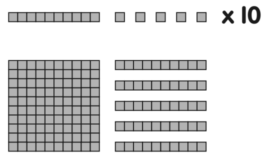
NPC Milestone 3

- Use efficient written column methods for adding and subtracting whole numbers up to 10 000 and decimals with up to 3 decimal places

Page 66: Multiplying and dividing by 10 (Calc 7·1 & 7·2)

Practice

1 150



2 **a** 80 **b** 240 **c** 350

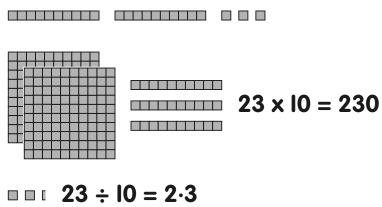
3 16 g

4 **a** 21·5 **b** 15p or £0·15 **c** 100 g or 0·1 kg

Going deeper

1 **a** £7 or 700p **b** £1·20

2 The correct rule for multiplying by 10 is that all the digits move one position to the left and for dividing by 10, one position to the right. Make sure that children are moving the digits, not the decimal point.



Page 67: Multiplying by 100 and 1000 (Calc 7·3)

Practice

1 rounders balls: 15 000

ping pong balls: 20 000

tennis balls: 10 000

2 **a** 27 g **b** 0·6 g

3 **a** 0·005 kg (or 5 g) **b** £1·50 **c** £120 **d** 25 g

Going deeper

1 Total weight = 74 kg

Cost per ball: £1·72

2 £150 × 10 + £230 × 10

£1500 + £2300 = £3800

Page 68: Multiplying and dividing decimals by 10 (Calc 7·4 & 7·5)

Practice

1 5 ℓ

red rod = 1 litre

white rod = 0.5 litres

2 **a** 46 **b** 7 **c** 3·5 **d** 6·2

3 220 ml

Going deeper

1 75 g risotto rice

150 ml hot vegetable stock

60 g peas

25 g mushrooms

2 **a** 3·2 ÷ 10 – move all the digits one place to the right to give 0·32

b 0·24

3 **a** Incorrect, it should be 0·09 – they have × 10 instead.

b Correct

c Correct

d Incorrect, it should be 0·7 – they have divided by 100.

Page 69: Multiplying and dividing decimals by 100 and 1000 (Calc 7·6)

Practice

1 **a, b** Hook a duck: 100 tickets = £120; 1000 tickets = £1200

Bouncy castle: £150; £1500

Raffle: £250; £2500

Coconut shy: £180; £1800

2 **a** 3·17 **b** 2·005 **c** 4580

3 Divide by 1000

4 Divide by 100

Going deeper

1 45

2 × 10, ÷100, ×1000

3 Answers will vary. They may include:

15 × 100 = 1500

150 ÷ 100 = 1·5

1500 ÷ 1000 = 1·5

1·5 × 10 = 15

1·5 ÷ 10 = 0·15

4 Answers will vary.

NPC Milestone 3

- Use known multiplying facts to multiply and divide whole numbers and decimals by 10, 100, and 1000

Page 70: Exploring different units of measurement (Mea 1·1)

Practice

1 25 cm = 250 mm = 0·25 m; 2·5 cm = 25 mm = 0·025 m

2 0·25 cm is odd one out. It has no equivalent measures in the list.

Going deeper

1 Molly won by 40 cm.

2 Jenny came first, Ali came second and Max came third.

Page 71: Exploring metric and imperial units (Mea 1·2 & 1·3)

Practice

1 **a** 180 cm **b** 10·8 m **c** 108 m **d** 27 m

2 **a** 12 inches (also 1 foot)

b 1·09 yards

c 0·5 yards

d 4 inches

Going deeper

1 7 feet = 7 × 12 inches = 84 inches = 84 × 2·5 cm = 210 cm, so Tia's fence is likely to be taller.

2 If 90 cm is equivalent to 1 yard, then 400 yards is around 400 × 90 cm, 36 000 cm, or 360 m. So Cook's Cakes is 360 metres away.

Cook's Cakes is nearer.

It is 60 m nearer than Root's Fruits.

Page 72: Converting between miles and kilometres (Mea 1.4)

Practice

1 42 km (to nearest whole km)

2 Running at 9 mins per mile, in 60 minutes Tia's mum will cover 6·7 miles. $6{\cdot}7 \times \frac{8}{5} = 10{\cdot}7$ km, so the answer will be around 10·7 km.

Going deeper

1 $70 \times \frac{5}{8} = 43{\cdot}75$ km

2 **a** 26·2 × 1·61 = 42·182, so still 42 km to the nearest km.
b 9 minutes for 1·61 km means 10·73 km in 1 hour.

3 4·49 km, 4500 m, 3 miles, 5 km, 3·7 miles, 4 miles

Page 73: Converting units of mass and volume (Mea 1·5)

Practice

1 11 full glasses

2 **a** Ben – 18·9 kg Molly – 14·85 kg Ravi – 19·8 kg Tia – 24·75 kg
b Molly could take 4·75 kg of items from Tia's suitcase into her own.

Going deeper

1 The cost of buying 38 litres of petrol in USA is \$20. The cost of buying 38 litres of petrol in UK is £43·70, which is approximately \$54·63. It will therefore cost approximately \$34·63 (or £27·70) less in the US than in the UK.

2 **a** If 1 lb is 450 g, $\frac{1}{2}$ lb is about 225 g, so 250 g is more than $\frac{1}{2}$ lb.

GMS Milestone 1

- Convert between metric units of measure, e.g. between metres and kilometres
- Recognize and use common imperial measures, e.g. miles, feet, inches, pints
- Understand and use approximate equivalences between metric units and common imperial units

Page 74: Properties of number (P&A 3·1 & 3·2)

Practice

1 60

2 Assuming a systematic approach, you need to ask five questions: Is it a multiple of 2? (yes), Of 3? (yes), Of 4? (no), Of 5? (no), Of 7? (yes). You need to ask five questions to rule out other possibilities than 42 logically.

3 Working systematically,
a 5 questions: Is it a multiple of 2, 3, 5, 7, 9?
b 8 questions: Is it a multiple of 2, 3, 4, 5, 7, 9, 11, 13?
c 4 questions: Is it a multiple of 2, 3, 5, 7?
d 9 questions: Is it a multiple of 2, 3, 4, 5, 7, 8, 9, 16, 32?
e 6 questions: Is it a multiple of 2, 3, 5, 7, 9, 11?
f 7 questions: Is it a multiple of 2, 3, 4, 5, 7, 8, 28?
g 8 questions: Is it a multiple of 2, 3, 5, 7, 9, 11, 27, 81?

4 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

Going deeper

1 The best overall strategy is to rule out possibilities systematically, rather than trying to 'guess and get lucky'. Start with the question 'Is it a multiple of 2?' to rule out half of the initial possibilities immediately. This systematic strategy may not always be fastest – sometimes someone may make a lucky guess – but it guarantees the correct result through reasoning. It is also important that children recognise which questions do not need asking, e.g. if the hidden number is both a multiple of 2 and of 3, then it will also be a multiple of 6.

2 Leaving aside the fact that normally multiples of 2 and of 6 both 'go on forever', the game context is restricted to numbers 1–100. Kyle is wrong because every multiple of 6 is also a multiple of 2, and there are other multiples of 2 that are not multiples of 6 (2, 4, 8, 10, etc.). Therefore, within any limited range, e.g. 1–100, there have to be more multiples of 2 than of 6.

3 There are either 13 dogs with 3 spots and 2 with 7, making 15 dogs in the group, or there are 6 dogs with 3 spots and 5 with 7, making 11 dogs in the group.

Page 75: Lowest common multiples (P&A 3·3)

Practice

1 The lowest common multiple of 2, 3, and 5 is 30, so the answer is £30.

2 **a** Answers will vary, e.g. choose 6 and 15. Multiples of 6: 6, 12, 18, 24, 30 … Multiples of 15: 15, 30 … LCM of 6, 15 is 30.
b 30, 60, 90, 120, 150

3 Working systematically through the possibilities, there are the following number pairs:

(2,3) (2,4) (2,5) (2,6) (2,7) (2,8) (2,9) (2,10) (2,12) (2,14) (2,16) (2,18)

(3,4) (3,5) (3,6) (3,9) (3,12) (3,15) (3,18)

(4,6) (4,8) (4,12) (4,16)

(5,10) (5, 15)

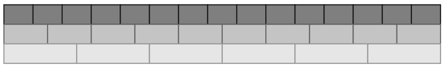
(6,9) (6,12) (6,18)

(7,14)

(8,16)

(9,18)

Going deeper

1 

2 There are more sophisticated methods involving prime factors that children will meet in secondary school, but at this stage listing multiples of 2, 3 and 5 and then comparing these lists is likely to be the preferred method. Answer is 30.

3 (2,4) because their LCM (4) is the smallest of all the possible LCMs of number pairs within the range 2–20. Hence the common multiples of (2,4), i.e. 4, 8, 12, 16, will lie closer together than those of any other number pair; this means there will be more of them in the range 50–150.

4 To compare and order fractions that have different denominators, you must first convert the fractions into equivalents with a common denominator; the lowest common denominator is usually the simplest to use, e.g. Which is bigger, $\frac{3}{8}$ or $\frac{1}{3}$? Using the LCM of 3 and 8 (24) you can convert these fractions to $\frac{9}{24}$ and $\frac{8}{24}$ respectively and then see that $\frac{3}{8}$ is bigger than $\frac{1}{3}$.

Page 76: Highest common factors (P&A 3·5)

Practice

1 a The common factors of 18 and 42 are 2, 3, and 6, so they could put their cards into packs of these sizes.

b 6

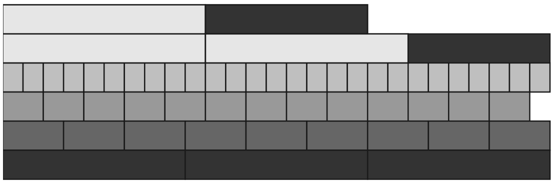
2 a Answers will vary, e.g. try 27 and 63. List the factors of each number; 27: 1, 3, 9, 27; 63: 1, 3, 7, 9, 21, 63. By inspection, the Highest Common Factor (HCF) of 27 and 63 is 9.

b For the example given in 2a the common factors are 1, 3, and 9.

3 Answers will vary, e.g. one pair would be (64, 72), HCF = 8. Children's familiarity with times tables will help them choose such pairs.

4 17

Going deeper

1 

Answer is 9.

2 There are more sophisticated methods involving prime factors that children will meet in secondary school, but at this stage listing the factors of 6, 15, and 24 and then comparing these lists is likely to be the preferred method.

3 The HCF of 8 and 12 is 4, so the biggest square tiles will be 4 m × 4 m.

4 When simplifying fractions, the most efficient method is to divide both numerator and denominator by their HCF, e.g. $\frac{64}{72} = \frac{64 \div 8}{72 \div 8} = \frac{8}{9}$.

Page 77: Prime and composite numbers (P&A 3·6 & 3·7)

Practice

1 12 has many factors, 11 has only two, 1 and 11.

2 2, 3, 5, 7, 11, 13, 17, 19, 23. These numbers have just two factors, themselves and 1.

3 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24. These numbers have more than two factors.

4 a Prime numbers **b** Composite numbers

Going deeper

1 Three: 1 × 12, 2 × 6, and 3 × 4.

2 Only one: 1 × 11.

3 a Only one

b Two: 1 × 21, and 3 × 7. 19 is prime, and 21 is composite.

4 A very important ancient Greek observation is behind this question: Every composite number can be expressed as a combination of prime factors, e.g. 24 = 2 × 2 × 2 × 3, and 16 = 2 × 2 × 2 × 2.

NPC Milestone 4

- Find the lowest common multiple of two or more numbers
- Find the highest common factor of two or more numbers
- Explain the difference between prime and composite numbers and identify them by testing accordingly

Page 78: Exploring multiplying (Calc 8·1 & 8·2)

Practice

1 128

2 Answers will vary, e.g.

$16 \times 4 = 64$

$8 \times 8 = 64$

$32 \times 4 = 128$

$16 \times 80 = 1280$

3 **a** 75 **b** 102 **c** 126

Going deeper

1 Answers will vary. Look out for children who suggest multiplying by 20 and taking away one lot and those who suggest multiplying by 10 and then multiplying by 9.

2 Answers will vary, e.g. 8×17: double 17, double again and then again: 34, 68, 136.

$8 \times 10 = 80$, $8 \times 8 = 64$, $64 - 8 = 56$, $80 + 56 = 136$

$8 \times 5 = 40$ $40 \times 3 = 120$ $(17 = 5 + 5 + 5 + 2)$ $120 + (8 \times 2) = 136$

3 Answers will vary, e.g. 9×17.

$10 \times 17 = 170$, $170 - 17 = 153$

$9 \times 10 = 90$, $9 \times 7 = 63$

$10 \times 17 = 170$, $170 \div 2 = 85$ (5×17) + 4×17 (double 17, double again = 68) $85 + 68 = 153$

Page 79: Multiplying decimals (Calc 8·3)

Practice

1 **a** Molly 1·2 km × 5 = 6 km
Ravi 1·6 km × 5 = 8 km
Approaches will vary. Look out for children partitioning the ones and tenths and those who multiply by 10 and then halve their answer.

b Molly – 12 km
Ravi – 16 km

2 Lauren 1·5 km × 5 = 7·5 km

Tamal 0·9 km × 5 = 4·5 km

Paul 2·3 km × 5 = 11·5 km

Going deeper

1 £1·25 × 5 = £ 6·25

Approaches will vary and may include:

£1 × 5 = £5, 25p × 5 = £1·20 or £1·25 × 10 and then halve it.

2 9 × 1·6 cm + 6 × 2·3 cm

14·4 cm + 13·8 cm = 28·2 cm

Approaches will vary. Look out for children who multiply 1·6 cm by 10 and then take-away 1·6.

Page 80: Exploring dividing (Calc 8·4)

Practice

1 118 ÷ 9 = 13 r1. 14 times

2 Divisible by 5 – all those ending in 0 or 5

Divisible by 6 – 138, 156, 102, 114, 186, 174, 132

Divisible by 7 – 133, 154, 182, 196, 203, 147, 119, 161

Divisible by 8 – no odd numbers. 176, 152, 136, 184, 128

Divisible by 9 – all those whose digits total 9. 153, 171, 117

Going deeper

1 Look for children using the inverse and multiplying, e.g. $23 \times 6 = 138$.

2 Answers will vary. Look for children who sensibly partition the number, e.g. $119 \div 7 = (70 + 49) \div 7$; $10 + 7 = 17$.

3 Questions and answers will vary.

Page 81: Dividing further (Calc 8·5)

Practice

1 **a** £7·45 ÷ 3 = £2·48 with 1p left over.

b Strategies will vary but suggestions may include partitioning £7·45 into £6 and 145p.

2 £9·25 ÷ 5 = £1·85

3 £12·75 ÷ 7 = £1·82 with 1p left.

Strategies will vary but may include partitioning £12·75 into £7 and £5·75.

Going deeper

1 3·66 m ÷ 6 = 601 cm. Strategies will vary but suggestions may include dividing 3·6 by 6 and then dividing 0·06 by 6.

2 1·8 ℓ ÷ 6 = 300 ml

1·8 ℓ ÷ 8 = 225 ml

1·8 ℓ ÷ 9 = 200 ml

1·8 ℓ ÷ 10 = 180 ml

1·8 ℓ ÷ 12 = 150 ml

NPC Milestone 4

- Use the distributive property of multiplying and dividing to break down calculations into parts and complete them using mental or informal methods

Page 82: Dividing with remainders (Calc 9·1 & 9·2)

Practice

1 **a** 4 bars

b Answers will vary. Look for children who recognise they need to share the 2 bars between 4 of them, by cutting each one in half and getting $\frac{1}{2}$ a bar each or cutting each one into 4 and getting $\frac{1}{4}$ of each bar, so $\frac{2}{4}$s.

2 Answers will vary, but may include: 18 ÷ 5 gives 3 bars each with 3 left over. Divide each of the 3 bars, into 5 pieces so each child will get $3\frac{3}{5}$ altogether.

Going deeper

1 Answers will vary, e.g.

2 whole bars each = 10 bars

$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{10}{5}$

$\frac{10}{5}$ is equivalent to 2 whole bars.

They must have started with 12 bars.

2 **a** $3\frac{1}{3}$ **b** $2\frac{1}{2}$ **c** $1\frac{2}{3}$

3 Answers will vary.

Page 83: Short dividing (Calc 9·4)

Practice

1 Look for children suggesting taking the remainder and then dividing it equally between the divisor, e.g. with the example given, take the 1 left over and divide it by 4 leaving $\frac{1}{4}$.

2 Answers will vary. 318 ÷ 5 = 63 r3 which is $63\frac{3}{5}$.

3 **a**, **b**, **c** Answers will vary, e.g. 176 ÷ 5 will give a remainder of 1, 450 ÷ 9 will not give a remainder because 45 is in the 9 times table.

4 Answers will vary depending on questions chosen above, e.g. 176 divided by 5 = 35·2 because $\frac{1}{5}$ = 0·2.

Going deeper

1 Biggest possible answer: 592 ÷ 4

Smallest possible answer: 176 ÷ 9

2 **a** Answers will vary, e.g. 147 ÷ 5 (any number that is 2 more than a number in the 5 times table) or 654 ÷ 10 (any number that is 4 more than a number in the 10 times table).

b Answers will vary, e.g. 133 ÷ 6 gives 22 r1, when the divisor is 6 then r1 is equivalent to $\frac{1}{6}$ (any number that is 1 more than a number in the 6 times table).

Page 84: Linking improper fractions to dividing (Calc 9·6)

Practice

1 $\frac{5}{12}$

2 **a** $\frac{11}{12}$ **b** $\frac{17}{12}$ or $1\frac{5}{12}$ **c** 2 whole trays **d** 3 whole trays

3 7 boxes – children may show this by using number rods or Numicon Shapes and pegs.

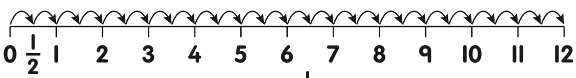
4 6 wholes

5 4 and 6 (how children show this will vary but may include using Numicon Shapes and pegs)

Going deeper

1 **a** $\frac{25}{5}$ **b** $\frac{36}{6}$ **c** $\frac{49}{7}$ **d** $\frac{32}{4}$ (answers will vary)

2 Answers will vary but could include: $\frac{24}{2}$, $\frac{36}{3}$, $\frac{60}{5}$.



24 lots of $\frac{1}{2}$ = 12

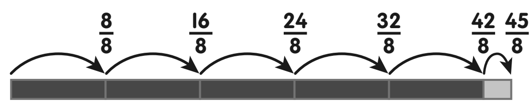
3 5 containers (4 of them full)

Page 85: Changing improper fractions to mixed numbers (Calc 9·7)

Practice

1 $4\frac{3}{6}$ or $4\frac{1}{2}$

2 **a**



b Look out for children who can explain that you need to count how many 8s in 45 and the left over can be expressed in eighths.

3 **a** $9\frac{5}{6}$ **b** $12\frac{5}{7}$ **c** $10\frac{3}{10}$ **d** $25\frac{3}{5}$

4 **a** $\frac{7}{2}$ **b** $\frac{31}{6}$ **c** $\frac{8}{3}$ (or any other equivalent fractions)

Going deeper

1 a $\frac{32}{3} = 10\frac{2}{3}$

b $\frac{9}{8} = 1\frac{1}{8}$

c There are 27 possible improper fractions. From smallest to largest these are:

$\frac{9}{8}, \frac{32}{28}, \frac{15}{12}, \frac{12}{9}, \frac{12}{8}, \frac{15}{9}, \frac{8}{5}, \frac{28}{15}, \frac{9}{5}, \frac{15}{8}, \frac{32}{15}, \frac{28}{12}, \frac{32}{12}$ & $\frac{8}{3}, \frac{12}{5}, \frac{15}{5}$ & $\frac{9}{3}, \frac{28}{9}, \frac{28}{8}, \frac{32}{9}, \frac{32}{8}$ & $\frac{12}{3}, \frac{15}{3}, \frac{28}{5}, \frac{32}{5}, \frac{28}{3}, \frac{32}{3}$.

NPC Milestone 4

- Interpret answers to dividing calculations which are not whole numbers appropriately, in context

Page 86: Understanding angles (Geo 3·1 and 3·2)

Practice

1 c, e, f, h, i are interior; a, b, d, g are exterior

2 a = 100°, b = 116°, c = 64°, d = 90°, e = 26°, f = 95°, g = 85°, h = 20°, i = 65°

Going deeper

1 a Find three angles that sum to 180° (interior) and three angles that sum to 360° (exterior).

b Interior: 48°, 30°, 102° Exterior: 150°, 132°, 78°

c Check that the triangle drawn has correctly labelled angle sizes.

Page 87: Exploring angles in triangles (Geo 3·3 & 3·4)

Practice

1 **a** equilateral **b** isosceles **c** scalene

d right-angled scalene **e** isosceles

2 **a** 60° **b** 70° **c** 33° **d** 57·6° **e** 24°

Going deeper

1 There are only two possible answers. Accept either an isosceles triangle with angles 120°, 30° and 30° or with angles 150°, 15° and 15°.

2 a Accept either an isosceles triangle with angles 45°, 45° and 90°, 60°, 60° and 60° or 75°, 75°, and 30°.

b The possible solutions are: 45°, 45° and 90°, 60°, 60° and 60° or 75°, 75°, and 30°.

Page 88: Exploring angles further (Geo 3·4)

Practice

1 a = 47°, b = 133°, c = 64°, d = 52°, e = 72·5°, f = 72·5°

2 **a:** angle sum of triangle

b: angles on a straight line

c: base angles of isosceles triangle

d: angle sum of triangle

e and **f:** angle sum of triangle then base angles of isosceles triangle

Going deeper

1 80°

2 52°

3 **a:** false – these make 180 by themselves

b: false – these make more than 180

c: false – three odd numbers cannot add up to 180 as it is even

d: could be true – any three even numbers that sum to 180

Page 89: Exploring angles in quadrilaterals (Geo 3·5)

Practice

1 a = 155, b = 135, c = 110, d = 90

Going deeper

1 40°, 80°, 120°, 120°

2 Any logical method can be applied, e.g. I started by seeing what would happen if the smallest angle was 10°, then 20°, and so on, or I started with the two larger angles, choosing angles that are a multiple of 30 so that a third angle can easily be found by dividing by 3 (120, 120 and 40). The final angle can be found by taking away the sum of these three angles from 360.

3 Only this one if using multiples of 10.

GMS Milestone 2

- Accurately identify and describe parallel sides in polygons, including from conventional symbols
- Identify the interior and exterior angles of a polygon, and explain the difference and relationship between them
- Explain in simple terms why the exterior angle sum of any polygon is 360°
- Identify and draw diagonals in polygons
- Distinguish between regular and irregular polygons
- Use angle sum facts and understanding of the angle properties of polygons to calculate the size of unknown angles

Page 90: Proportion and ratio (Calc 10·1)

Practice

1 120 g dried cranberries
400 g oats
80 g brown sugar
160 g toasted almonds
200 g wheat bran

2 240 g dried cranberries, 800 g oats
160 g brown sugar, 320 g toasted almonds

3 150 g dried cranberries, 100 g brown sugar
200 g toasted almonds, 250 g wheat bran

Going deeper

1 75 g dried cranberries
250 g oats
50 g brown sugar
100 g toasted almonds
125 g wheat bran

2 14 servings

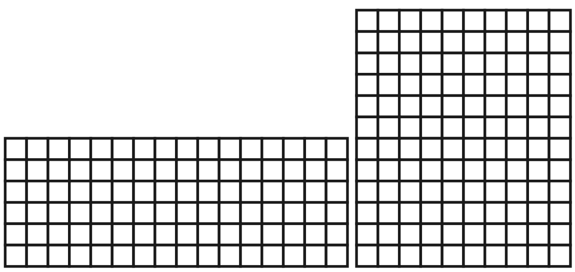
3 12 servings
Explanations will vary but look for children who recognise that 16 servings (2×) gives an amount greater than 720 g so it must be less than 16 servings, and half of 480 is 240 so we must need 8 servings + 4 servings.

4 200 out of 480 = $\frac{5}{12}$

Page 91: Making scale drawings (Calc 10·3)

Practice

1 a



2 You need to divide all the measurements by 10, so 35 cm becomes 3·5 cm, 42 cm becomes 4·2 cm, 43 cm becomes 4·3 cm, 77 cm becomes 7·7 cm and 85 cm becomes 8·5 cm.

Going deeper

1 Answers will vary.

2 Answers will vary. Look for children who are noticing a pattern between the dimensions of their rectangles and the areas.

Page 92: Solving problems involving rates (Calc 10·4)

Practice

1 £5

2 £5·60

3 £3

4 £2·40 + £1·50 = £3·90

Going deeper

1 $2\frac{1}{2}$ hours – both over 5

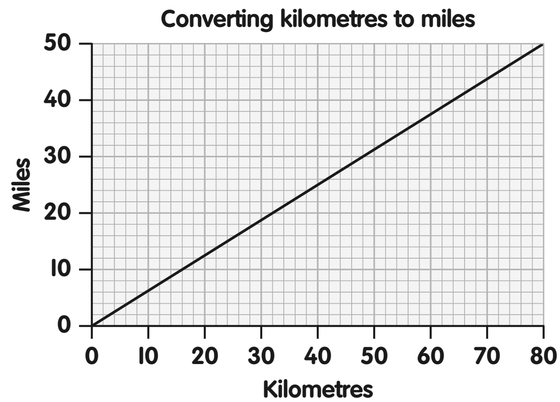
2 Answers will vary, e.g. 2 children aged over 5 for $1\frac{1}{2}$ hours at the weekend.
(£1·25 × 3 = £3·75, £3·75 × 2 = £7·50)

3 £5·75

Page 93: Solving conversion problems (Calc 10·5)

Practice

1 Children may explain that by drawing a line across from 10 miles until they reach the diagonal line and then drawing another line down to the km axis, will give the approximate number of km: 16 km.



2 30 miles

3 Answers will vary. Look for children who are reading off the graph correctly.

4 200 miles (using 32 km and reading off graph) and scaling up.

Going deeper

1 Approximately 108 km. Strategies could include working out 34 miles in km and doubling it or some children might extend the graph. Others may use additional strategies.

2 Calais to Nantes: 600 km

3 120 km + 320 km = 440 km

NPC Milestone 4

- Use multiplying and dividing to solve problems involving scaling

Page 94: Understanding 'per cent' (Calc 11·2 & 11·2)

Practice

1 10%

2 20%

3 50%

Going deeper

1 26–49

2 Answers will vary but could be anything less than 25%.

3 80% because $\frac{3}{4}$ is equivalent to 75%.

Page 95: Different ways to make 50% (Calc 11·4)

Practice

1 Answers will vary. Look for children who show $\frac{1}{2}$ of a whole.

2 Answers will vary. Look for children who show $\frac{1}{4}$ of a whole.

3 Answers will vary. Look for children who show $\frac{4}{10}$ or $\frac{2}{5}$ of a whole.

Going deeper

1 60 – multiply 25% value by 3.

2 50

3 Answers may vary but an example could include: 25% of a number is 5, what must 100% of that number be?

Page 96: Calculating percentages (Calc 11·5)

Practice

1 £12 because 50% is the same as $\frac{1}{2}$ and half of £24 is £12.

2 Socks £4, shirt £10, scarf £8, tie £7·50.

3 Jumper £6, socks £2, shirt £5, scarf £4, tie £3·75.

Look for children who recognise they need to find $\frac{1}{4}$ of the starting price and they can do that by halving the 50% off price.

Going deeper

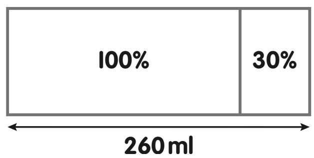
1

75% off all items	
Original Price	New Price
£28	**£7**
£40	£10
£56	£14
£35	**£8·75**

2 Children may draw a diagram to explain their thinking:

We know that the full price (100%) must be £20 + ? The £20 represents 80% and so 20% must be £20 ÷ 4 = £5. Therefore the original price must be £20 + £5 = £25.

3 Encourage children to draw a diagram to support them make sense of the problem:



We know 260 ml is 130% therefore 10% is 260 ml ÷ 13 = 20 ml.

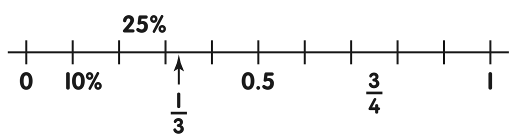
So 30% is 20 ml × 3 = 60 ml.

The original bottle must contain 260 ml – 60 = 200 ml.

Page 97: Decimal, fraction and percentage equivalents (Calc 11·6)

Practice

1



2 0·1 and $\frac{10}{100}$ or $\frac{1}{10}$

3 0·2 placed correctly on number line at the second marked interval.

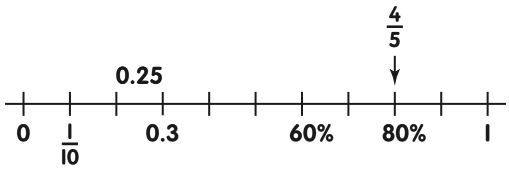
4 66% is two thirds and should come between the sixth and seventh marked intervals, slightly closer to the seventh one.

Going deeper

1 **a** 75% = 0·75, $\frac{75}{100}$, $\frac{3}{4}$ **b** 40% = 0·4, $\frac{4}{10}$, $\frac{2}{5}$

2 Any percentage between 31% and 39%.

3



NPC Milestone 4

- Explain percentage as the number of parts per hundred
- Find and explain percentage, fraction and decimal equivalents in order to solve problems in context

Page 98: Understanding charts and graphs (Mea 2·1)

Practice

1

Number rolled	Tally	Number of times	Score
1	||	2	2
2	||	2	4
3	||||	4	12
4	~~||||~~ ||	7	28
5	|||	3	15
6	||	2	12

2 Total score is 73.

3 Perfect probability is rare but children should expect approximately the same number of each to be rolled.

4 Answers will vary.

Going deeper

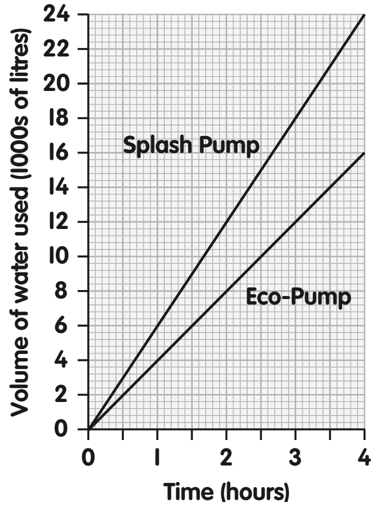
Name	Round 1	Round 2	Round 3	Total
Mike	17	18	12	47
Gita	15	17	13	45
Andrew	20	9	17	46

To work out Andrew's score we realise his total must be 46 subtract 9 and 17 from this to get 20.

Page 99: Using line graphs (Mea 2·2 & 2·3)

Practice

1, 2



3 **a** 2 hours **b** 3 hours

Going deeper

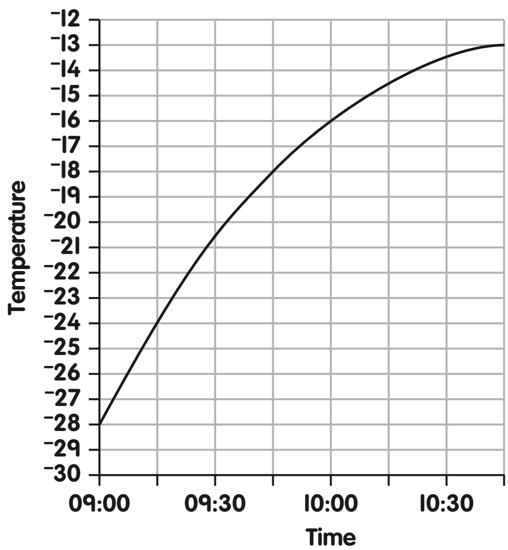
1 1 hour 15 minutes

2 6000 litres. This would take 1 hr with the Splash Pump and 1·5 hours with the Eco Pump.

Page 100: Measuring temperature over time (Mea 2·4)

Practice

1



2 Approximately 10:30 a.m. but accept any reasonable answer. The ice cream is ready to serve when it reaches $^{-}14$ degrees which happens shortly after 10:20 a.m.

3 Around 1·5 hours – from 09:00 to 10:30

4 It went up by 14·5 degrees.

Going deeper

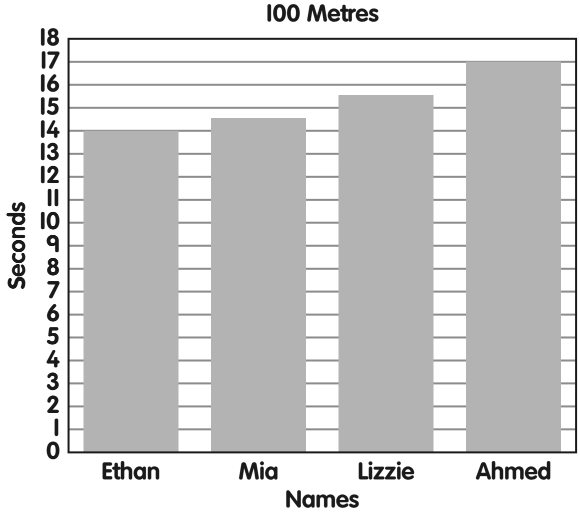
1 Accept any reasonable answer and explanation. They need to stop selling when the ice cream reaches ⁻12 degrees. Look for children examining the rate at which the ice cream is warming up over time. Children may suggest that the rate at which the ice cream is warming up has slowed down: it went up by 1 degree between 10:15 and 10:30 and then by 0·5 of a degree between 10:30 and 10:45. If you assume that the ice cream will continue to warm up by 0·5 degrees every 15 minutes the ice cream would reach ⁻12 degrees at 11:15.

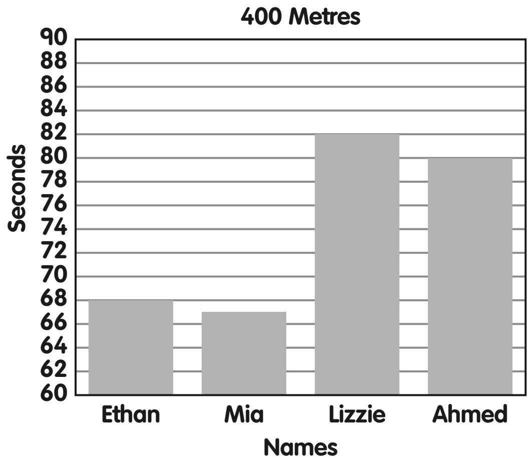
2 07:30 – it seems to take 90 minutes to get to the correct temperature.

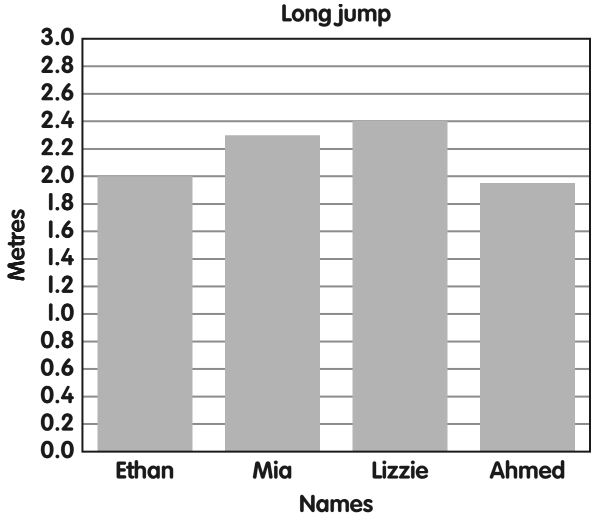
Page 101: Presenting and comparing data charts (Mea 2·6)

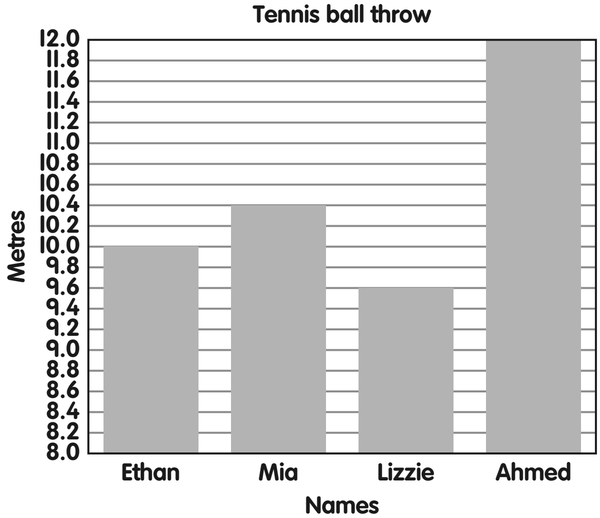
Practice

1







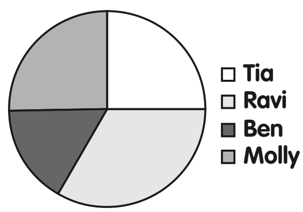


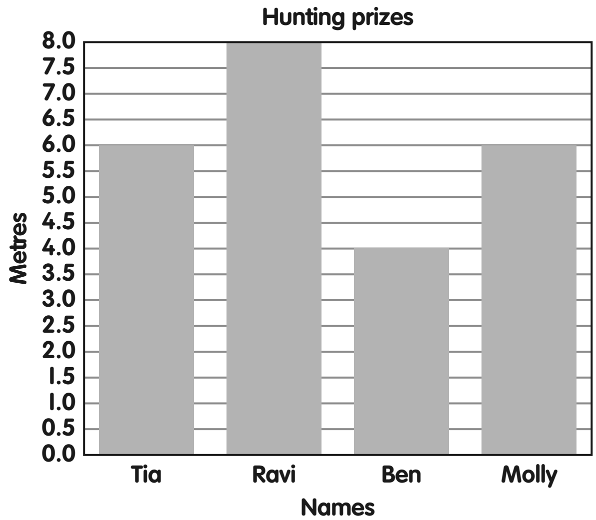
2 The bar charts show that Ethan and Mia are the best runners but Ahmed is a very strong thrower. The four children are quite evenly matched in the Long jump.

3 The best athlete – Mia has a first place in the 400 m and came second in all the other events, so is a good all-round candidate.

Going deeper

1 **Hunting prizes**





2 The bar chart shows who has the most, fewest, etc. The pie chart shows proportions more readily.

GMS Milestone 2

- Complete, read and interpret information in tables, charts and graphs
- Read and use scales on charts and graphs in a variety of contexts, including with negative numbers
- Recognize that choosing which is an appropriate method for representing data depends on the type of data and the question being asked

Page 102: Solving problems with fractions, decimals and percentages (NNS 7·1 & 7·2)

Practice

1 $\frac{30}{100}$ and $\frac{3}{10}$ are both examples of 30% as a fraction, and $\frac{3}{10}$ is the fraction in its simplest form. 30% is 0·3 as a decimal.

2 Answers will vary.

3 Answers will vary.

Going deeper

1 To order the fraction, decimals and percentages it helps to change them all to percentages. $\frac{2}{5}$ is $\frac{4}{10}$. This is equivalent to 40% which is also on the list, 0·42 is 42%, $\frac{3}{10}$ is 30%, 0·38 is 38%.

So the order is 30%, 35%, 38%, 40%, 42%.

2 **a** Answers will vary but may include:

- shops' discounts on products
- interest charged on loans or interest paid for money invested
- companies often describe their profits in terms of percentages
- a salesperson may be given a commission as a percentage of their total sales
- articles such as antiques or jewellery may increase in value as time goes by-appreciation.

b Percentages are usually the best way to describe an increase or a decrease in an amount. Some sale prices do use fractions, especially when the sale price is exactly half the original price.

Page 103: Percentages of amounts (NNS 7·3)

Practice

1 The 15% discount can be calculated by working out 10% and then 5%. To work out 10% you can divide the original cost by 10 and to work out 5% you can halve this. 15% can be found by adding 10% + 5%. In the case of a book costing £10, 10% is £1 and 5% is 50p so 15% is £1·50. To find the discounted price subtract £1·50 from the original £10 cost, which is £8·50.

2 Answers will vary, e.g. 80% of 450 g = 360 g, 10% of 450 g is 45g and 80% is 8 × 10% or 8 × 45 which is 360 g.

3 Answers will vary. Examples might include working out 20% and then subtracting this from the original amount to find 80%, e.g. 10% is 45 and 20% is 90 g. 450 g – 90 g is 360 g or, alternatively, working out that 1% is 4·5 and the multiplying this by 80.

Going deeper

1 One way to work out 75% is to work out 50% by halving the original amount and then halving this again to find 25%. Finally adding add these two amounts together for 75%.

2 65% can be found by finding 50%, 10% and 5%.

3 10% of 29 kg is 2·9 so 20% is 5·8 kg.

10% of 25 kg is 2·5 kg so 30% is 7·5 kg.

7·5 kg is heavier than 5·8 kg.

Page 104: Equivalents (NNS 7·4)

Practice

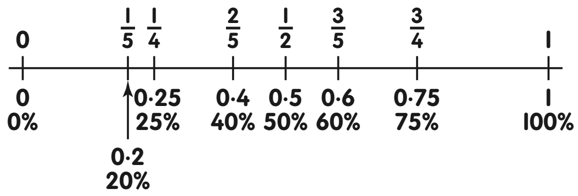
1 20% is $\frac{20}{100}$ which simplifies to $\frac{1}{5}$ when you divide the numerator and denominator by 20.

$\frac{20}{100}$ is equivalent to 0·2 or two tenths.

So 40% is $\frac{40}{100}$ which is equivalent to four tenths or 0·4. $\frac{40}{100}$ and simplifies to $\frac{2}{5}$.

60% is $\frac{60}{100}$ which is equivalent to six tenths or 0·6.

$\frac{60}{100}$ simplifies to $\frac{3}{5}$.



2

	50%	25%	75%
a £58	£29	£14·50	£43·50
b £4·64	£2·32	£1·16	£3·48
c 96p	48p	24p	72p
d 8p	4p	2p	6p
e £300	£150	£75	£225
f £48	£24	£12	£36

3 50% can be calculated by halving and 25% by halving and halving again.

75% is 50% and 25% added together.

4 To work out 17·5% you can first find 10% by dividing by 10 and then halve 10% to find 5%, and halve again to find 2·5%. Next add 10% + 5% + 2·5%.

Going deeper

1 **a** To illustrate 25% with rods you could use a 1-rod and a 4-rod or a 2-rod and 8-rod. If you use a 3-rod you would need a 12-rod which would be 3 number rods not 2.

b To illustrate 20% with rods you could use a 1-rod and a 5-rod or a 2-rod and 10-rod. If you use a 3-rod you would need a 15-rod which would be 3 number rods not 2.

2 If both children had the same amount of money, spending $\frac{1}{4}$ is equivalent to spending 25% which is more than 20%. However, as we don't know the amount that each child had to start with we cannot answer this question.

Page 105: Percentages as proportions, and as operators (NNS 7·5 & 7·6)

Practice

1 Illustrate with a diagram similar to example on page 105. Amounts as shown here.

100%	10%	5%	2·5%	20%	30%	50%	25%
23 ℓ	2·3 ℓ	1·15 ℓ	0·575 ℓ	4·6 ℓ	6·9 ℓ	11·5 ℓ	5·75 ℓ
36 m	3·6 m	1·8 m	0·9 m	7·2 m	10·8 m	18 m	9 m
80 kg	8 kg	4 kg	2 kg	16 kg	24 kg	40 kg	20 kg
£4	40p	20p	10p	80p	£1·20	£2	£1
72 mins	7·2 mins	3·6 mins	1·8 mins	14·4 mins	21·6 mins	36 mins	18 mins

2 Illustrate a copy of the diagram shown in Q2.

100%	10%	5%	2·5%	20%	30%	50%	25%
200 km	20 km	10 km	5 km	40 km	60 km	100 km	50 km

3 One way to work out the whole distance from knowing that $2\frac{1}{2}$% = 5 km is to work out 5% by doubling 2·5% and doubling again to work out 10%. 10% can then be multiplied by 10 to work out 100%.

Alternatively divide 5 by 2·5 to find out 1% and then multiply this by 100 to find 100%.

Going deeper

1 **a** $\frac{50}{100}$ or 50% **b** $\frac{50}{100}$ or 50% **c** $\frac{25}{100}$ or 25%
d $\frac{75}{100}$ or 75% **e** $\frac{16}{100}$ or 16% **f** $\frac{10}{100}$ or 10%

2 Answers will vary. Some answers will say that 29% and 69% can be calculated by working out 30% or 70% and subtracting 1% which makes it quite a simple percentage to calculate.

Children may give some examples of percentages that are more complicated to calculate, e.g. 83·7% with their own reasons.

NPC Milestone 5

- Know percentage equivalents of commonly used fractions, e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
- Use percentages to express simple proportions, e.g. 24 out of 32 as 75%
- Find percentages of amounts, including measures

Page 106: Looking for patterns and generalizing (P&A 4·1)

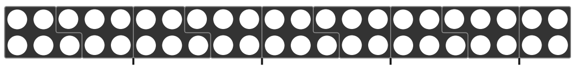
Practice

1 312 is divisible by 4. Since 100 is divisible by 4, any number of hundreds, (e.g. 300) is divisible by 4, so you only need to ask if the final two digits of any number are also divisible by 4. 12 is divisible by 4, so 312 is too.

2 Answers will vary, e.g. **a** 2034 **b** 2034
c 2036 **d** 2035 **e** 2030

3 288 is divisible by 6 because it is divisible by both 2 and by 3. Any number that is divisible by 2 and by 3 will also be divisible by the Lowest Common Multiple (LCM) of 2 and 3, which is 6. (See rod illustration which uses red 2-rods at the top and green 3-rods at the bottom.)

Going deeper

1 

2 See explanation in Q1 above.

3 The same rod illustration as in Practice Q3 may help children visualise this explanation. An <u>even</u> number of 3s (i.e. 2 × 3, 4 × 3, etc.) will always produce an even number; an <u>odd</u> number of 3s, (e.g. 3 × 3, 5 × 3, etc.) will always produce an odd number.

Page 107: Sequences of rod patterns (P&A 4·2 & 4·3)

Practice

1 **a** 12 **b** 31 **c** 22 **d** 30

2 **a** One rod of 'term number' length + 2 ones
b Three rods of 'term number' length + 1 one
c Two rods of 'term number' length + 2 ones
d Three rods of 'term number' length

3 **a** 6 + 2 **b** (3 × 5) + 1 **c** (2 × 5) + 2 **d** 3 × 5.

Going deeper

1 (2 × 10) + (2 × 12) = 20 + 24 = 44

2 **a** The hole is a square with a side length of the term number.
b 10 × 10

3 The area of the hole is the 'term number' length squared, i.e. the area of the hole in the nth term is n^2.

4 (4 × 20) + 4

5 The total rod length of the nth term is: $2n + 2(n + 2) = 4n + 4$.

Page 108: Square numbers (P&A 4·4, 4·5 & 4·6)

Practice

1 **a** The length of the red rod is 2 times its width.
The length of the light green rod is 3 times its width.
The length of the dark pink rod is 4 times its width, etc.
b If you arrange a set of number rods in order of size, smallest first, then the length of each rod is the same number of times its width as its position in the sequence, e.g. the length of the 5th rod in the sequence is 5 times its width.

2 The length of a 12-rod would be 12 times its width.

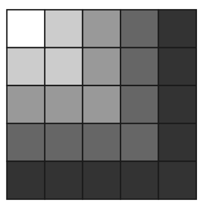
3 **a** As the length of the dark pink rod is 4 times its width, 4 of these rods placed next to each other side by side will form a square in which length = width = 4.
b This is the same for all number rods. As the length of an nth rod is n times its width, n of these rods placed next to each other side by side will form a square in which length = width = n.

4 $5^2 = 5 \times 5 = 5 + 5 + 5 + 5 + 5 = 25$

5 9 and 36

6 4, 16, and 25

Going deeper

1 
Most explanations will depend upon this kind of illustration. Children's explanations will be in their own words.

2 Factors of any number work in pairs because any factor has to divide into a given number a certain number of times, e.g. 6 is a factor of 24 because it divides into 24 exactly 4 times. So 6 is 'paired with' 4, and 4 is 'paired with' 6, as they multiply together to produce 24. So in general, since factors typically combine in pairs, any number should have an even number of factors; every factor should have a 'partner'.

In the special case of square numbers (4, 9, 16, etc.), there is always one factor that is 'paired with itself' (2, 3, 4, etc.), thus a square number will always have an odd number of factors.

Page 109: Cube numbers (P&A 4·7)

Practice

1 1, 8, 27, 64, 125, 216, 343

2 64, 343

3 This needs 8^3 as well. $27 + 216 + 512 = 3^3 + 6^3 + 8^3 = 755$.

4 4^3, $4 \times 4 \times 4$, $4^2 \times 4$, $4 + 4 + 4 + \ldots$ (16 times), 4×4^2, 64

Going deeper

1 **a** Model of a cube made with 25 yellow rods **b** 125

2 **a** Yes **b** Their total rod lengths are the cube numbers.

3 **a, b** Cube numbers are calculated by multiplying square numbers by the length of the side of the square. Using the illustration at the top of Pupil Book 5, page 109, you can say that each 'layer' of rods within a cube is a square, and that the number of 'layers' in a cube is the same as the side length of each square, giving a perfectly $n \times n \times n$ sized cube.

NPC Milestone 5

- Know and be able to use simple tests of divisibility
- Explain what square and cube numbers are

Page 110: Area and perimeter (Mea 3·1)

Practice

1 62 cm (1 by 30), 34 cm (2 by 15), 26 cm (3 by 10) or 22 cm (5 by 6)

2 122 cm, 64 cm, 46 cm, 38 cm, 34 cm, 32 cm

3 It could have six different perimeters; 182 cm, 94 cm, 66 cm, 46 cm, 42 cm, 38 cm.

Going deeper

1 Tia can make 4: 1×64, 2×32, 4×16 and 8×8

Ben can make 4: 1×66, 2×33, 3×22 and 6×11

Ravi can make 3: 1×68, 2×34 and 4×17

Molly can make 6: 1×60, 2×30, 3×20, 4×15, 5×12 and 6×10

2 Assuming Molly draws lengths that are whole numbers of centimetres, the two possible rectangles are 3 cm by 6 cm and 4 cm by 4 cm. If she is permitted lengths that are not whole numbers, then there are other possibilities, (e.g. 7 cm by 2·8 cm).

Page 111: Using area to explore factors, square numbers and prime numbers (Mea 3·2)

Practice

1 There are 5 possibilities: 1×100, 2×50, 4×25, 5×20, 10×10.

2 There are 6 possibilities: 1×72, 2×36, 3×24, 4×18, 6×12, 8×9.

3 1×64, 2×32, 4×16 and 8×8

4 37 is the worst to choose as it can only be arranged in a single straight line (prime number).

Going deeper

1 You always get a square number. If you create this with number rods and add a single cube you will be able to make a square. Algebraically, this is because $n(n + 2) + 1$ is the square of $n + 1$.

Page 112: Area, perimeter and decimals (Mea 3·3)

Practice

1 $(1 \times 1) + (2 \times 1{\cdot}5) + (3 \times 1{\cdot}5) + (4 \times 1{\cdot}5) = 14{\cdot}5\,m^2$. You know you have found them all if you worked systematically, starting with 1 as one of the side lengths, then 2 as one of the side lengths, etc. When you reach 6×7 you know there are no more possibilities because the dimensions are just repeated in reverse order, e.g. 6×7 becomes 7×6 and so on.

2 $(2 \times 2{\cdot}5) + (4{\cdot}5 \times 3) + (3{\cdot}5 \times 4) + (2{\cdot}5 \times 2{\cdot}5) + (5 \times 2{\cdot}5) = 51{\cdot}25\,m^2$, which, when multiplied by £20, is £1025.

3 Multiply the length and width of each room to find the area of carpet needed per room, then add all the areas together to find the total area of carpet needed. Finally multiply this by £20 to find the total cost.

Going deeper

1 Area could be $12\,m^2$, $22\,m^2$, $30\,m^2$, $36\,m^2$, $40\,m^2$ or $42\,m^2$. You know you have found them all if you worked systematically, starting with 1 as one of the side lengths, then 2 as one of the side lengths, etc. When you reach 6×7 you know there are no more possibilities because the dimensions are just repeated in reverse order, e.g. 6×7 becomes 7×6 and so on.

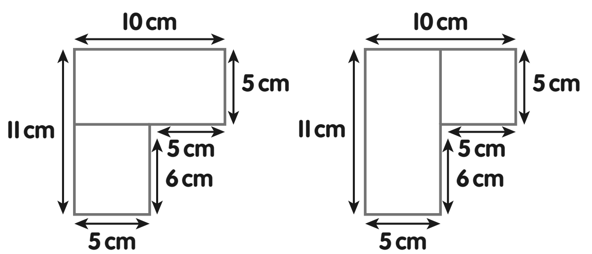
2 Two possibilities: 3 cm by 8 cm or 4 cm by 5 cm.

Page 113: Finding the area and perimeter of composite shapes (Mea 3·4)

Practice

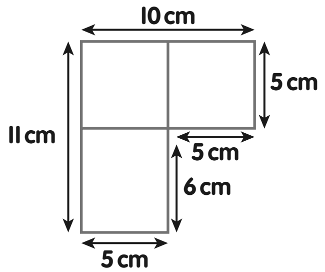
1 42 cm

2



The first diagram shows $30 + 50 = 80\,cm^2$. In the second diagram, $25 + 55 = 80\,cm^2$.

3

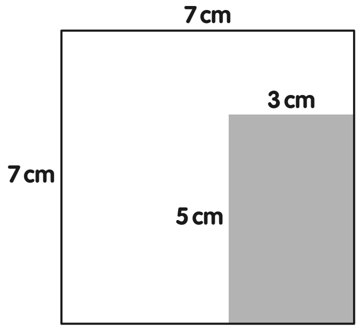


$(5 \times 6) + (5 \times 5) + (5 \times 5) = 30 + 25 + 25 = 80\,cm^2$

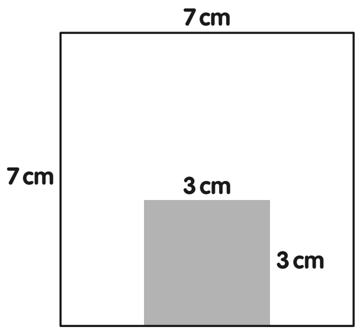
Going deeper

1 He is right – just the overall height and width (11 cm and 10 cm) are enough to work out the perimeter.

2



3 To increase the perimeter, we must cut off a 3 cm by 3 cm rectangle, but not at the corner.



GMS Milestone 2

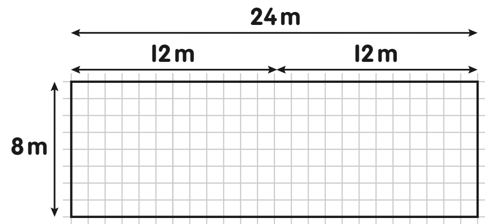
- Describe area as a measure of flat space and perimeter as a measure of length
- Recognize that area is measured in square units
- Calculate the areas and perimeters of rectangles, in square centimetres and centimetres, respectively, given pairs of dimensions
- Find missing side lengths and calculate areas and perimeters of composite rectilinear shapes, in square centimetres and centimetres

Page 114: Developing written methods of multiplying (Calc 12·1)

Practice

1 Answers will vary but should give an answer of 192, e.g.

$12 \times 8 = 96$, $96 \times 2 = 192\,m^2$.



2

$$\begin{array}{r} 24 \\ \times \quad 8 \\ \hline 192\ m^2 \\ {\scriptstyle 3} \end{array}$$

3 Chosen methods will vary but answers will be:

a $90\,m^2$ **b** $192\,m^2$ **c** $333\,m^2$

Going deeper

1 **a** 1480 g or 1·48 kg

b 1480 g × 7 = 10 360 g or 10·36 kg

2 **a** 210 g

b 1470 g or 1·47 kg

c 5880 g or 5·88 kg

Page 115: Multiplying decimals using short multiplication (Calc 12·2)

Practice

1 23·4 cm

2 26·6 cm

3 **a** 39·2 **b** 19·2 **c** 17·1

Going deeper

1 **a** 3·35 ℓ **b** 5·22 pints **c** 192·8 kg

2 **a**

×	5	0·6
6	30	3·6

b

×	3	0·4
6	18	2·4

c

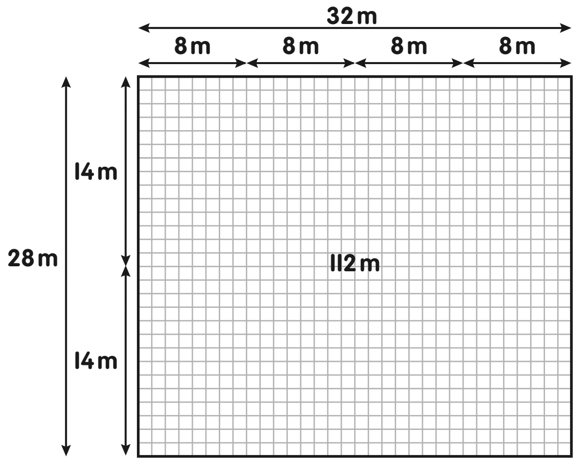
×	2	0·3	0·05
8	16	2·4	0·4

Page 116: Multiplying 2-digit numbers (Calc 12·4)

Practice

1 896 m^2. Answers will vary in terms of how children have partitioned their rectangle.

2 Answers will vary, e.g. 14 × 8 = 112; 112 × 8 = 896.



3 **a** 2530 **b** 972

Going deeper

1 Answers will vary.

Page 117: Using long multiplication (Calc 12·5)

Practice

1

×	40	5
10	400	50
5	200	25

2 The total of each row of Molly's calculation matches each row of Ben's long multiplying.

3 **a** 308 **b** 351 **c** 1008

Going deeper

1 108

2 **a** 300 **b** 960 **c** 644 **d** 360
e 2000 **f** 121

Encourage children to solve at least half of them mentally (suggest a, b, d, f).

NPC Milestone 5

- Use efficient written methods to multiply numbers with up to 4 digits by 2-digit numbers

Page 118: Volume and capacity (Mea 4·1)

Practice

1 1 cm × 1 cm × 32 cm; 1 cm × 2 cm × 16 cm; 1 cm × 4 cm × 4 cm; 2 cm × 2 cm × 8 cm; 2 cm × 4 cm × 4 cm

All the cuboids have a volume of 32 cm^3.

2 1 cm × 1 cm × 72 cm; 1 cm × 2 cm × 36 cm; 2 cm × 2 cm × 18 cm; 2 cm × 3 cm × 12 cm; 2 cm × 6 cm × 6 cm; 3 cm × 4 cm × 6 cm. Suggested resources: use rods to help find them all.

3 Answers will vary, e.g. **a** 5 cm × 3 cm × 2 cm, **b** 3 cm × 3 cm × 4 cm, **c** 2 cm × 5 cm × 4 cm, **d** 1 cm × 5 cm × 9 cm.

Going deeper

1 Answers will vary.

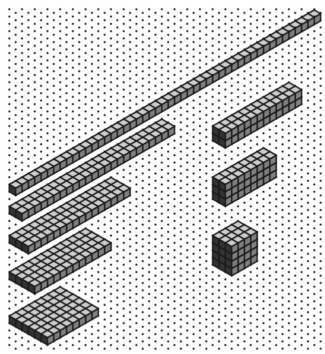
2 They could use 60 white rods, 30 red rods, 20 light green rods, 15 purple rods, 12 yellow rods, 10 dark green rods or 6 orange rods.

Page 119: Drawing 3D shapes in 2D (Mea 4·2)

Practice

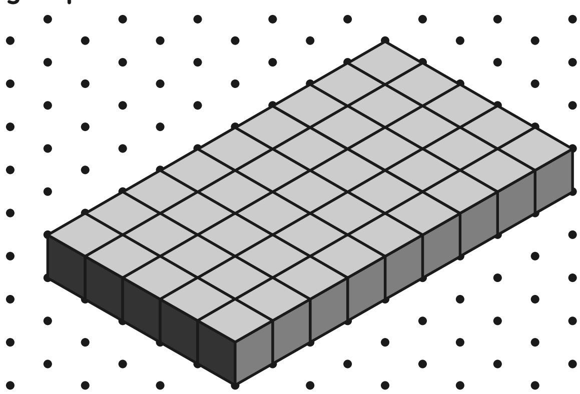
1 **a** There are a further eight ways to do it.

b



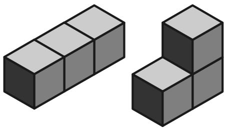
Going deeper

1 **a**



b This is not the only possibility, because he could also create a cuboid that is 3 cm by 3 cm by 5 cm.

2 There are only two ways. All other ways are these same shapes but at different orientations.



Page 120: Cubic centimetres and millilitres (Mea 4·4 & 4·5)

Practice

1 360 white rods

2 180 red rods, 120 light green rods, 90 purple rods, 72 yellow rods, 60 dark green rods, not possible with black rods, 45 brown rods, 40 blue rods or 36 orange rods.

3 Black – it isn't possible to get the water level to exactly 500. This is because we need to raise it exactly 360 ml, and 7 is not a factor of 360.

Going deeper

1 Four possible combinations: 20 5-rods and no 6-rods; 14 5-rods and five 6-rods; eight 5-rods and ten 6-rods; two 5-rods and 15 6-rods.

2 Not possible – no multiples of 6 and 7 add up to 29.

Page 121: Volume and capacity problems (Mea 4·6)

Practice

1 6 cm

2 The walls could be made using 10-rods and 8-rods. The two longer walls also made of 10-rods (two for each wall). The shorter walls will then each be made of two 8-rods.

3 **a** 72 cm^3 **b** 4 × 10-rods + 4 × 8-rods

Going deeper

1 6 × 6 × 6 = 216, so the cube is 6 m by 6 m by 6 m. A 1 m wall on each side means the interior will be 4 m × 4 m × 4 m = 64 m^3.

2 48 m^3

GMS Milestone 3

- Explain the difference between volume and capacity and the equivalence between cubic centimetres and millilitres
- Build cuboids of a given volume using cubes or rods
- Create 2D drawings of 3D shapes
- Give a reasonable estimate of the volume of an object, in cubic centimetres
- Calculate volume or capacity given the dimensions of a cuboidal shape or space.

Page 122: Written methods of dividing (Calc 13·1)

Practice

1 108 ÷ 6 = 18

2 **a** 14 **b** 24

Going deeper

1 Stories will vary and may involve sharing or grouping, e.g. I want to put 15 coloured pencils in a pot. There are 120 pencils, how many pots do I need?

2 Answers will vary.

Page 123: Short written method of dividing (Calc 13·2)

Practice

1 Answers will vary. Look for children explaining that the 1000 block cannot be shared by 6 so it needs to be exchanged for ten 100 flats giving 12 100 flats to share between 6. This gives two 100s each and then continue the rest of the calculation in the same way.

2 **a** 215 **b** 199 **c** 324 **d** 426 r4 **e** 452 r5

Going deeper

1 It should be 270 r5. He has made a place value error.

2 **a** In the final part of the calculation we are left with 63 ÷ 7 which is 9 not 8. Also the answer has been written in the incorrect place value columns. There should be nothing written in the thousands column and the rest of the digits need to move one place to the right.

b They have probably thought that because you can't divide two 100 flats they will just give them another 10 of them making it 12 flats to divide between 3. This has then confused the rest of the calculation. Also the answer is bigger than the number they started with.

3 **a** 239 **b** 93 Both answers can be checked using an inverse calculation.

Page 124: Dividing with the answer as a decimal (Calc 13·3 & 13·4)

Practice

1 **a** 67 cm **b** 66·7 cm or 667 mm

2 0·43 m

3 0·625 litres

Going deeper

1 **a** 0·6666 **b** 1·4286 **c** 0·3333 **d** 0·25 **e** 0·7777 **f** 0·6

2 Answers will vary but look for children who recognise the dividend needs to NOT be an exact multiple of the divisor, e.g. 21 ÷ 5 will give a remainder.

Page 125: Dividing money using the short written method (Calc 13·5)

Practice

1 Yes it will because £6·51 × 3 = £19·53.

2 £7·16

3 **a** £4·85 **b** £5·65 **c** £9·45

Going deeper

1 Encourage children to think about how to solve the ones suggested mentally.

a £18 ÷ 3 = £6 (mental)
£18 ÷ 4 = £4·50 (mental)
£18 ÷ 5 = £3·60 (mental)

b £28·80 ÷ 3 = £9·60 (written)
£28·80 ÷ 4 = £7·20 (mental)
£28·80 ÷ 5 = £5·76 (written)

c £45·60 ÷ 3 = £15·20 (written)
£45·60 ÷ 4 = £11·40 (mental)
£45·60 ÷ 5 = £9·12 (mental)

2 Answers will vary, but may include:
£21 ÷ 3, £24 ÷ 3
£28 ÷ 4, £40 ÷ 4
£35 ÷ 5, £200 ÷ 5

NPC Milestone 5

- Choose appropriate and effective mental or written methods to divide numbers with up to 4 digits by single-digit numbers

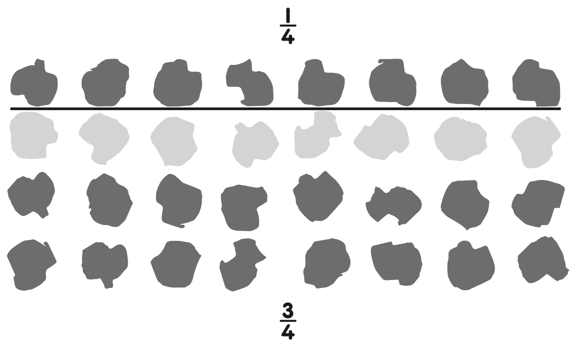
Page 126: Finding fractions of amounts (Calc 14·1)

Practice

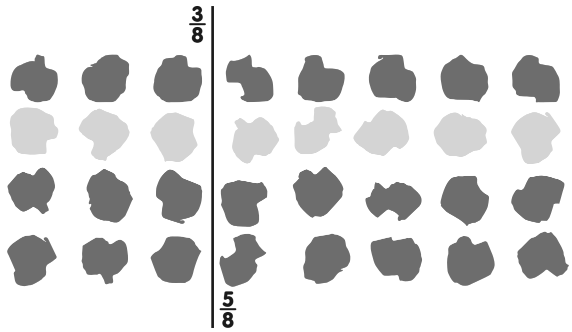
1 **a** Yes, because one row out of 4 rows is yellow.

b 32 ÷ 4 = 8

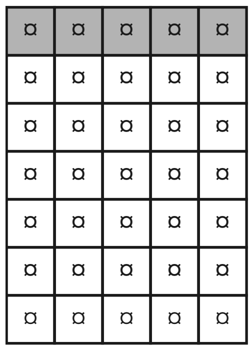
2 **a** 24



b 12



3 **a**



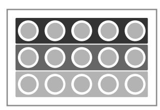
b Answers will vary but might include:

$\frac{2}{5}$ of 35 = 14, $\frac{3}{5}$ = 21, $\frac{4}{5}$ = 28

$\frac{1}{7}$ of 35 = 5, $\frac{2}{7}$ = 10, $\frac{3}{7}$ = 15 ….

Going deeper

1



$\frac{1}{3}$ of 15 = 5, $\frac{2}{3}$ = 10

$\frac{1}{5}$ of 15 = 3, $\frac{2}{5}$ = 6

b Only 15 × 1 because 3 and 5 are the only factors of 15.

2 a 1×12 array

2×6 array, $\frac{1}{2}$ of 12 is 6, $\frac{1}{6}$ is 2, $\frac{2}{6}$ is 4, $\frac{4}{6}$ is 8, $\frac{5}{6}$ is 10

3×4 array, $\frac{1}{3}$ of 12 is 4, $\frac{2}{3}$ is 8, $\frac{1}{4}$ of 12 is 3, $\frac{3}{4}$ of 12 is 9

b Yes because 12 has more factors than 15.

Page 127: Finding fractions of quantities (Calc 14·2)

Practice

1 Blue: 25

Red: 50

Silver: 60

Black: 15

2 75p/hour so for 3 hours it will cost £2·25.

Going deeper

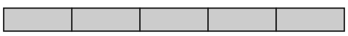
1 720 cars (120×6)

2 Answers will vary.

Page 128: Measurement and finding fractions (Calc 14·4)

Practice

1 105 cm



175 cm

$\frac{1}{5}$ of 175 or one piece above = 35 cm

35 cm × 3 gives middle piece … = 105 cm

2 1·5 m

3 0·96 m

4 **a** 3·75 kg **b** 1·5 ℓ **c** £7·50

Going deeper

1 1·8 m

(2·7 ÷ 6 = 0·45, 0·45 × 4)

2 Problems will vary that children make up but answers should be:

a 3·2 ℓ

b 2·25 kg

c £10·50

Page 129: Sharing things equally (Calc 14·5)

Practice

1 **a** 7 divided by 3

They will each get 2 whole sandwiches each and $\frac{1}{3}$ of a sandwich.

b 7 divided by 4

They will get 1 whole sandwich each and then $\frac{3}{4}$ of another.

2 **a** 10 divided by 3 = $3\frac{1}{3}$

b 20 divided by 3 = $6\frac{2}{3}$

Going deeper

1 11

($3 \times 3 = 9$, $\frac{2}{3} \times 3 = \frac{6}{3} = 2$)

2 28 because 28 ÷ 4 gives a whole number (7) and 28 ÷ 3 = 9 r1.

NPC Milestone 5

- Calculate fractions of amounts in practical problem-solving contexts

Page 130: Working with area and perimeter (Mea 5·1)

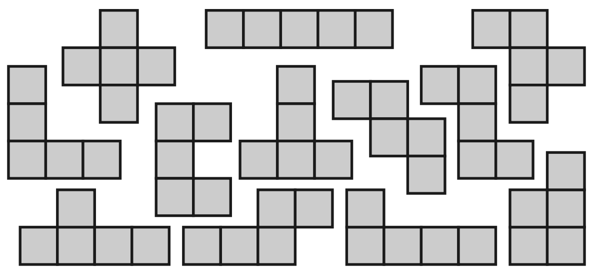
Practice

1 10 panels

2 In a square 5 m by 5 m. It can contain 25 chickens.

Going deeper

1 There are 12 in total. They all have the same perimeter:



2 A shape made from 6 equal squares.

3 **a** The largest possible perimeter is 14 cm.

b Experimentation – also the first square placed gives 4 edges, and each extra square can only increase this total by 2. 4 plus 5 lots of 2 gives 14.

Page 131: Area of composite shapes (Mea 5·2)

Practice

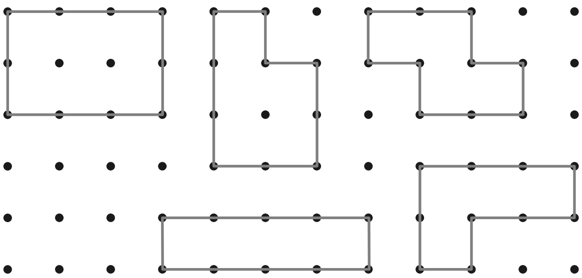
1 $(4{\cdot}5 \times 3) + (7 \times 3) = 34{\cdot}5\ \text{cm}^2$

$(3{\cdot}5 \times 10) + (3{\cdot}5 \times 7) = 59{\cdot}5\ \text{cm}^2$

Going deeper

1 $400\ \text{cm}^2$

2 **a**



b Areas are $1{\cdot}5\ \text{m}^2$, $1{\cdot}25\ \text{m}^2$, $1{\cdot}0\ \text{m}^2$, $1{\cdot}0\ \text{m}^2$, $1{\cdot}0\ \text{m}^2$ (clockwise from top left).

Page 132: Calculating the area and perimeter of oblongs (Mea 5·3)

Practice

1 **a** perimeter = 16 cm and area = $12\ \text{cm}^2$,

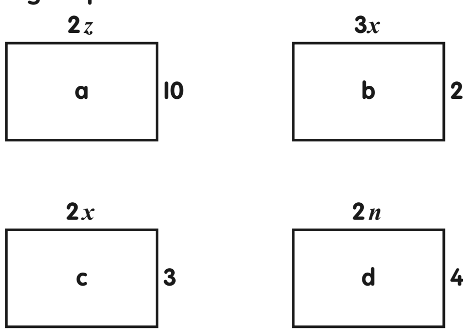
b perimeter = $4 + 2n$ and area = $2n\ \text{cm}^2$,

c perimeter = $2n + 2a$ and area = $na\ \text{cm}^2$

2 3 cm

Going deeper

1



2 **a** Divide $20z$ by $2z$

b Subtract 4 then halve

c Divide $6x$ by 3

d Subtract $4n$ then halve

Page 133: Side length, area and perimeter (Mea 5·4)

Practice

1 Area = purple × orange or 40 square units. Perimeter = 2p + 2o

2 **a** Area = 45, perimeter = 28

b Area = 20, perimeter = 24

c Area = 54, perimeter = 30

Going deeper

1 **a** 2·5 **b** x **c** $2b$ **d** 20

2 **a** 9 **b** $4x$ **c** $4b + 10$ **d** 64

b Answers will vary.

GMS Milestone 3

- Find missing side lengths and calculate areas and perimeters of more complex composite rectilinear shapes, in square centimetres or metres and in centimetres or metres
- Estimate areas of irregular shapes

Express the area or perimeter of a rectangle algebraically in order to find an unknown length, e.g. $5 \times b = 30$ so $b = 6$ and the unknown length is 6 cm

Page 134: Understanding scale (Mea 6·1)

Practice

1 **a** 30 cm × 50 = 15 m, 60 cm × 50 = 30 m

b $15\ \text{m} \times 30\ \text{m} = 450\ \text{m}^2$

2 **a** 120 ÷ 6 = 20 so scale is 1:20

b 5·5 cm × 20 = 1 m 10 cm

Going deeper

1 Scale is 1:50 so carriage will be $\frac{1}{50}$ of 12 m, or 24 cm.

2 1:200 or similar; this will make the scale drawing 10 cm high.

Page 135: Making scaled drawings (Mea 6·2)

Practice

1 **a** 3 cm, 4 cm, 5 cm

b Draw triangle with sides 9 cm, 12 cm, 15 cm.

c 9 times – the square of the scale factor

2 Answers will vary according to the size of room.

Going deeper

1 Based on the cover being approximately 20 cm by 26 cm, the drawing should be 5 cm by 6·5 cm.

$520 \div 4 \div 4 = 32{\cdot}5\,\text{cm}^2$

2 Converting the length and height of the table into centimetres makes it 300 cm wide by 150 cm long. A scale factor of 1:10 means the drawing would need to be 30 cm by 15 cm. A4 paper is 29.7 cm by 21 cm, so this drawing would not fit. A scale of 1:15 or similar can be used which makes the scale drawing 20 cm by 10 cm.

Page 136: Using scale drawings to find actual sizes (Mea 6·3)

Practice

1 Approximately 140 m

2 Approximately 160 m

Going deeper

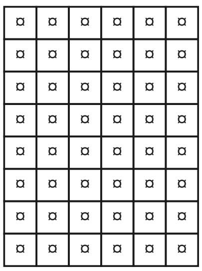
1 1:750 000

2 82·5 km (11 cm × 750 000)

Page 137: Exploring the effects of scaling (Mea 6·4)

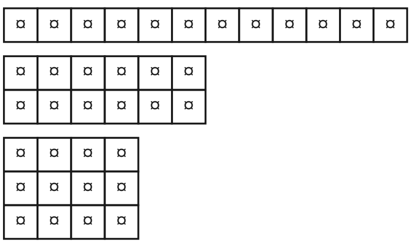
Practice

1 **a** 6 panels in a rectangular array like the one shown. Each panel should be drawn as 2 cm by 3 cm.



b $6\,\text{m}^2$ **c** $48\,\text{m}^2$ **d** $6\,\text{cm}^2$ **e** $48\,\text{cm}^2$

2 Three possibilities. Each square in the drawing should measure 2 cm by 2 cm.



Going deeper

1 Scale factor is 1:30 as 80 cm × 30 = 24 m.

So pump is 30 × 32 cm, or 9·6 m.

2 **a** Scale factor was 3. The scale drawing would have been 4 cm × 1 cm. Each side was multiplied by 3, giving a rectangle with side lengths 12 cm × 3 cm (area $36\,\text{cm}^2$).

b Answers will vary.

GMS Milestone 4

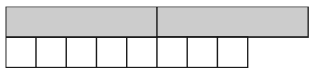
- Explain the meaning of the term 'scale drawing'
- Know that the scale of a drawing is the ratio of drawn to actual lengths
- Identify the scale of a drawing by comparing drawn and actual lengths
- Select an appropriate scale for a scale drawing

Page 138: Calculating with fractions (Calc 15·1 and 15·2)

Practice

1 **a** $\frac{3}{5}$ eaten, $\frac{2}{5}$ left

b $\frac{1}{5} + \frac{2}{5} + \frac{2}{5} + \frac{3}{5}$ goes over one whole.



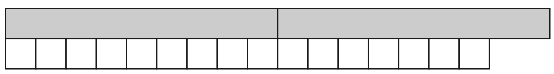
c $\frac{8}{5}$ or $1\frac{3}{5}$

2 **a** $\frac{7}{8}$

b $\frac{3}{8}$

3 $\frac{12}{8}$, $1\frac{1}{2}$ or $1\frac{4}{8}$

4 **a** $\frac{16}{9}$ or $1\frac{7}{9}$



b $\frac{8}{9}$

Going deeper

1 Answers will vary, e.g.

a $\frac{3}{8} + \frac{3}{8} = \frac{3}{4}$

b $\frac{7}{8} - \frac{1}{8} = \frac{3}{4}$

2 Answers will vary, e.g.

a $\frac{1}{4} + 1\frac{1}{4}$

$\frac{7}{8} + \frac{5}{8}$

$\frac{2}{3} + \frac{5}{6}$

b $3\frac{1}{2} - 2$

$\frac{9}{3} - \frac{12}{8}$

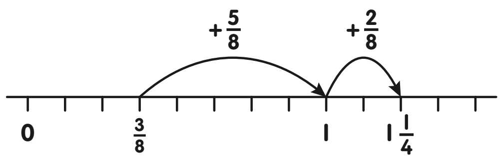
$1\frac{3}{4} - \frac{1}{4}$

Page 139: Adding and subtracting fractions on a number line (Calc 15·3)

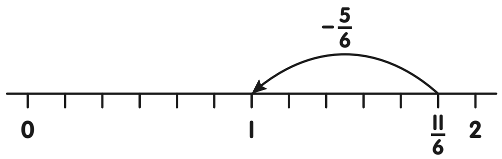
Practice

1 $2\frac{3}{8}$ can be written as $\frac{19}{8}$ as an improper fraction. $\frac{19}{8} - \frac{5}{8} = \frac{14}{8}$ or $1\frac{6}{8}$.

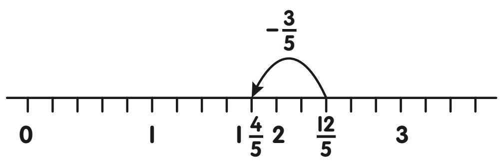
2 a $\frac{3}{8} + \frac{7}{8} = 1\frac{1}{4}$



b $\frac{11}{6} - \frac{5}{6} = 1$



c $\frac{12}{5} - \frac{3}{5} = 1\frac{4}{5}$



Going deeper

1 a The children should recognise that the pattern repeats $1\frac{3}{8}$, $1\frac{7}{8}$, $2\frac{3}{8}$, $2\frac{7}{8}$.

b The pattern repeats … $7\frac{1}{3}$, $6\frac{2}{3}$, $5\frac{1}{3}$, $4\frac{2}{3}$.

2 Answers will vary.

Page 140: Further adding and subtracting fractions (Calc 15·4)

Practice

1 a $\frac{5}{8}$

b $\frac{3}{8}$ because $\frac{5}{8} + \frac{3}{8}$ is equivalent to a whole.

2 a $\frac{1}{4}$

b $\frac{3}{8}$

c $\frac{4}{12}$ or $\frac{1}{3}$

Going deeper

1 Answers will vary but could include:

$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

$\frac{1}{4} + \frac{5}{8} = \frac{7}{8}$ (easy as in same fraction family)

$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ (easy as the same denominator)

$\frac{3}{10} + \frac{2}{5} = \frac{7}{10}$ (easy as $\frac{2}{5}$ is the same as $\frac{4}{10}$)

$\frac{1}{6} + \frac{2}{3} = \frac{5}{6}$ (easy as $\frac{2}{3}$ is the same as $\frac{4}{6}$)

2 a $\frac{1}{2} + \frac{1}{5} + \frac{3}{10}$

$\frac{1}{5} = \frac{2}{10}$ so yes, Ben is correct, this will total a whole.

b Answers will vary. Examples:

$\frac{1}{4} + \frac{1}{8} + \frac{5}{8}$

$\frac{1}{3} + \frac{3}{9} + \frac{2}{6}$

Page 141: Multiplying fractions by whole numbers (Calc 15·5 & 15·6)

Practice

1 a $2\frac{1}{2}$ m²

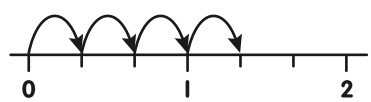
b Carrots $1\frac{1}{4}$ m², Peas $3\frac{3}{4}$ m²

c $25 - 7\frac{1}{2} = 17\frac{1}{2}$ m²

Going deeper

1 $\frac{3}{4} \times 4$

2



3 Answers will vary. Look for children who recognise the relationship with repeated addition when multiplying fractions by a whole number.

NPC Milestone 6

- Organize work independently and communicate ideas fluently
- Multiply proper fractions and mixed numbers by whole numbers
- Relate finding a fraction of a number or quantity to multiplying by a fraction

Page 142: Problem solving (Calc 16·1)

Practice

1 £1·05

2 £3·05

3 £53·20
Look for children who recognise they need to multiply each attraction cost by 4 or the total of all the attractions × 4.

Going deeper

1 **a** £51·85/child × 25 = £1296·25
b £8·15/child × 25 = £203·75

2 $\frac{1}{2}$ hour on trampolines and
$\frac{1}{2}$ hour on climbing wall

Page 143: Multiplying and adding (Calc 16·2)

Practice

1 £1·90

2 No, it should cost £2·05.

3 £44·85; £3·30 + £5·60 + £1·60 + £4·20 + £16·20 + £5·50 + £2·70 + £5·75

Going deeper

1 £41·35; £2·75 + £4·90 + £2·40 + £2·40 + £14·85 + £8·80 + £1·80 + £3·45

2 Answers will vary.

3 The total cost would also halve.

Page 144: Subtracting and dividing (Calc 16·3)

Practice

1 They spend £1·65 on apple juice leaving them with £3·45. This divided by 3 is £1·15 so they must have each bought a cake.

2 They spend £3·30 on drinks leaving them with £6·70 but they wanted to spend £6·75 meaning they wanted to buy an ice cream each because £6·75 divided by 5 is £1·35.

Going deeper

1 4 waters and 4 popcorn; 4 orange juice and 4 fruit packs

2 The possibilities are 5 pop-corns and 5 waters (50p change), 5 cakes and 5 waters (25p change) and fruit pack with any drink: apple juice (75p change), smoothie (no change), water (£1·50 change), orange juice (50p change).

Page 145: Adding and subtracting (Calc 16·4)

Practice

1 £2·25 × 50 = £112·50
£112·50 – £86·40 = £26·10

2 The trampolines

3 Yes, Tia is correct.
Each staff member costs £43·20/day.
Trampoline – £31·80
Bumper slide – £26·10
Go-karts – £15·30 LOSS
Climbing wall – £10·40
Boating lake – £36 LOSS

Going deeper

1 Answers will vary. Look for children who increase the price of those attractions making a loss or small profit.

NPC Milestone 6

- Solve a range of multi-step problems by choosing appropriate operations, strategies and methods
- Organize work independently and communicate ideas fluently

Page 146: Problem solving and time (Mea 7·2)

Practice

1 **a** Ravi – True. Tia – False. Ben – False. Molly – first part true, second part false.
b Tia – it should be 3 weeks and 4 days.
Ben – that takes him to New Year's Eve, not new year, so adding another 24 hours will make it true.
Molly – Although the first statement is true, April has only 30 days not 31. Correct answer is 3 months and one day, or 93 days.

Going deeper

1 **a** 60 × 60 × 24 = 86 400
b 365 × 24 = 8760
c 9 weeks = 63 days, so the answer is 9 and $\frac{4}{7}$ weeks.
d Answers will vary.

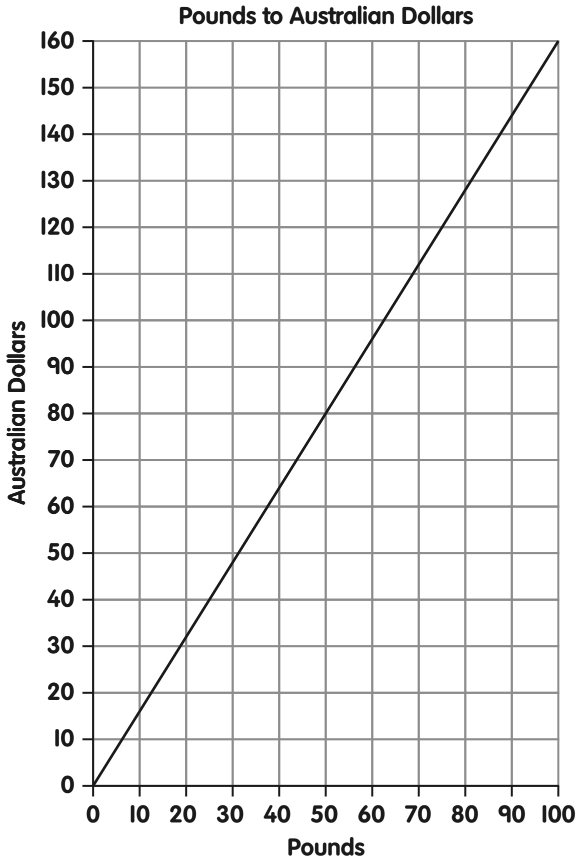
Page 147: Converting currencies (Mea 7·3)

Practice

1 **a** The rate at which it is possible to buy one currency with another.

b £1:\$1·60; £2:\$3·20; £100:\$160; £0:\$0

2



3 **a** \$40

b 'Half of half of 160 is one way', there are others, e.g. drawing a line on the graph up from £25 and then across to \$40.

c \$4000 – 100 times the previous answer

4 £62·50. Use the graph then estimate.

Going deeper

1 RM 5·20 is £1; RM is approximately £92·30, but \$160 is £100 so Becky will have more.

2 Gemma needs to take RM 5600, which is £1076·92. She has saved \$1000, which is £625. Therefore, she has not saved enough, as she still has to save £451·92 (£1076·92 – £625).

Also, RM 5200 would be £1000. Becky needs RM 5600 so she needs more than £1000, but we know that \$1000 is less than £1000 so this question is possible to answer without doing the conversion.

Page 148: Solving money problems (Mea 7·4)

Practice

1 **a** Lemonade **b** Orangeade **c** Lemonade **d** Cola

2 **a** 6 bottles of Cola are the cheapest.

b Largest discount is Orangeade (£4 discount).

Going deeper

1 **a** Calvin (£1·80 as opposed to £2)

b Tanya (£3·50 as opposed to £3·60)

c 10 miles (both will charge £3)

2 **a** £300 profit

b £50 profit

Page 149: Solving volume and capacity problems (Mea 7·5)

Practice

1 Cuboid – multiply the three dimensions to find the volume. In this case, $2 \times 3 \times 12 = 72\,cm^3$.

2 **a** $72 \times 20 = 1440\,cm^3$

b The 5 cm dimension does not divide exactly by any of the dimensions of the bar.

c 10 cm × 12 cm × 12 cm is the closest box shape to a cube, but there are other possible dimensions – 8 cm × 12 cm × 15 cm works, as does 6 cm × 12 cm × 15 cm and many more.

Going deeper

1 **a** $7200 \div 120 = 60$ so the volume of 1 bar must be $60\,cm^3$.

Here are all nine ways to make 60 in whole numbers: (some of them would look very odd shapes for strawberry bars though).

1 × 1 × 60

1 × 2 × 30

1 × 3 × 20

1 × 4 × 15

1 × 5 × 12

1 × 6 × 10

2 × 3 × 10

2 × 5 × 6

3 × 4 × 5

b Answers will depend on which dimensions are chosen but the dimensions must equal $7200\,cm^3$.

2 There are several possibilities, e.g.

45 cm × 10 cm × 10 cm

25 cm × 10 cm × 18 cm

GMS Milestone $\boxed{4}$

- Find sensible solutions to multi-step problems involving a variety of measures, e.g. time, money or mass, choosing appropriate operations, strategies and methods
- Organize work independently and communicate ideas fluently

Page 150: Using equivalence (P&A 5·1 & 5·2)

Practice

1 **a** 9

b Since the RHS of the balance is higher, 76 – $\square$ must be lighter (i.e. less) than 76 – 8, and 9 is the smallest positive whole number that will make 76 – $\square$ less than 76 – 8.

2 **a** 234 – 76 > 240 – 83. 240 is 6 more than 234, and 83 is 7 more than 76 so the difference between the numbers on the RHS is 1 fewer than the difference on the LHS.

b 127 + 15 > 112 + 29. 112 is 15 fewer than 127, and 29 is only 14 bigger than 15, so the total on the LHS will be greater than the total on the RHS.

3 **a** 38 × 3 is smaller because 21 × 6 = 21 × (2 × 3) = (21 × 2) × 3 = 42 × 3

b 39 ÷ 3 is smaller because 84 ÷ 6 = 84 ÷ 2 ÷ 3 = 42 ÷ 3

4 Answers will vary, e.g.

a 347 + 25 > 349 + 22

b 67 – 18 < 65 – 15

c 29 × 4 > 9 × 13

d 32 ÷ 5 < 99 ÷ 15

Going deeper

1 Answers will vary.

2 **a** Assuming the answers are all positive whole numbers, if 77 – $\square$ > 79 – 7 then 0 < $\square$ < 5.

b 77 – $\square$ > 79 – 9

3 Answers will vary, e.g. assuming the answers are all positive whole numbers, then in

86 + $\square$ < 84 + 10, 0 < $\square$ < 7.

Page 151: Balancing number sentences (P&A 5·3, 5·4 & 5·5)

Practice

1 75 + 140 = 90 + 125 The two sides balance because although 75 is 15 fewer than 90, 140 is exactly 15 greater than 125.

2 Answers will vary, e.g. 250 – 30 = 265 – 45

300 – 30 = 315 – 45

275 – 35 = 290 – 50

280 – 35 = 295 – 50

3 Answers will vary, e.g. 6 + 8 + 8 = 11 × 2

Going deeper

1 Answers will vary, e.g. 23 × 8 = 46 × 4

23 × 8 = 92 × 2 23 × 8 = 184 × 1

(Or possibly, 23 × 8 = 11·5 × 16.) The method is to halve 23 or 8, and then double the other one, i.e. 8 or 23 respectively.

2 Answers will vary, e.g. 16 × 4 = 128 ÷ 2 6 × 4 = 48 ÷ 2

If $\blacktriangle$ × 4 = $\odot$ ÷ 2, then $\odot$ = 8 × $\blacktriangle$

3 Because $\frac{a}{b} = a \div b$, then 2 ÷ 3 = 4 ÷ 6 may be re-written as $\frac{2}{3} = \frac{4}{6}$.

Page 152: Using brackets (P&A 5·6)

Practice

1 8 + (3 × 5) and (8 + 3) × 5

2

3 **a** (6 – 2) × 8 = 32

b 3 × (4 + 2) × 5 = 90

4 Answers will vary, e.g. **a** (3 + 3) × 4 = 24
b (5 + 7) × 4 = 48 **c** 18 – (3 × 4) = 6

Going deeper

1 Answers will vary, e.g. **2a** 4 + (3 × 3) = 13. Molly bought a pack of pencils that contained four black pencils, and three pencils of each primary colour. In all, there were 13 pencils in the pack.

2 The number sentence should be of the form:
($\square$ × 2) + $\boxed{?}$ = £15·50, so

(£5 × 2) + £5·50 = £15·50 would meet the requirements.

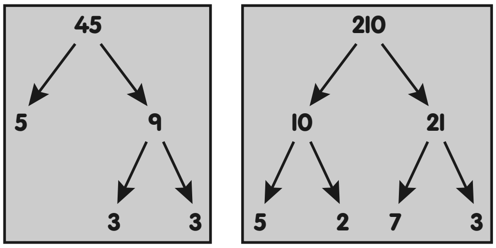
3 Do the calculations inside the brackets first. (5 × 15) – (12 × 4) = 27.

4 Either (7 × 12) – (10 × 4) = 44 or (7 × 8) – (3 × 4) = 44.

Page 153: Factor trees (P&A 5·7)

Practice

1



2 **a** $3 \times 3 \times 5 = 45$

b $3^2 \times 5 = 45$

3 Answers will vary, e.g. $6 \times 3 \times 5 = 2 \times 3 \times 3 \times 5 = (3 \times 3) \times (2 \times 5) = 9 \times 10 = 90$

4 **a** $2 \times 2 \times 3 \times 3 \times 5 = 180$

b $(2 \times 5) \times 2 \times (3 \times 3) = 10 \times 2 \times 9 = 180$

Going deeper

1 The prime factors of 60 are: $2^2 \times 3 \times 5$, so factor pairs will be (1, 60) and then:

$2 \times (2 \times 3 \times 5)$ giving factor pair (2, 30)

$3 \times (2 \times 2 \times 5)$ giving factor pair (3, 20)

$(2 \times 2) \times (3 \times 5)$ giving factor pair (4×15)

$5 \times (2 \times 2 \times 3)$ giving factor pair (5, 12)

$(2 \times 3) \times (2 \times 5)$ giving factor pair (6, 10)

2 $16 \times 25 = (4 \times 4) \times 25 = 4 \times (4 \times 25) = 400$

3 There are at least two ways of tackling this question: positively, systematically setting out all the possibilities of multiplying sequences of 2s, 3s and 5s together and listing all those results that come to 150 or less, or negatively, systematically listing all those numbers between 1 and 150 that cannot be made only from multiples of 2, 3, and 5 and ruling these out.

Since the numbers we are looking for can only be made using multiples of 2, 3, and 5, any number that has a factor of any other prime number (up to 79) cannot be made with a combination of 2, 3, and 5, e.g. because 14 has a prime factor of 7, so it cannot be made from any collection of 2s, 3s, and 5s. Similarly, any number that has a prime factor of 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, or 79 cannot be made.

Investigating these possibilities is made easier by using a 1–100 square.

NPC Milestone $\boxed{6}$

- Explain how to solve missing number problems involving both equivalence and inequality
- Approach problems involving number relationships systematically, logically and effectively

Page 154: Logic and reasoning (P&A 6·1)

Practice

1 35. Two methods could be by simply adding individual shape values, and by reasoning that the total will be 7 times the central value (5). Such reasoning will follow from Activity 1, and recognising that the shapes can be 'paired' around the central value of 5 thus, '8 is 3 more than 5, but 2 is 3 fewer than 5, so 2, 5, and 8 added together are equivalent to 3×5', and similarly for the 3- and 7-, and 4- and 6-shapes. This leads to the total being the number of shapes (7) multiplied by the central value (5) = 35.

2 Simply by adding fifths, i.e. $\frac{1}{5} + \frac{3}{5} + \frac{5}{5} = \frac{9}{5} = 1\frac{4}{5}$. Or by reasoning that $\frac{1}{5}$ is $\frac{2}{5}$ less than $\frac{3}{5}$ and 1 is $\frac{2}{5}$ more than $\frac{3}{5}$, so the total will be equivalent to $3 \times \frac{3}{5} = \frac{9}{5}$.

3 Multiply $4{\cdot}4 \times 5$. Same reasoning as above, i.e. 4·4 is the central value in a sequence of five equally spaced values.

Going deeper

1 **a** By adding, or $3 \times 42 = 126$

b By adding, or $3 \times 24 = 72$

c By adding, or $5 \times 24 = 120$

2 In each series there is an odd number of equally spaced values.

3 By adding, or $4 \times 31{\cdot}5 = 126$. 31·5 is the 'halfway' or mid-point in the series, so the numbers on the left of 31·5 are less than 31·5 by the same amount as the numbers to the right of 31·5. So the total of the series is equivalent to $4 \times 31{\cdot}5$.

Page 155: Testing general statements (P&A 6·2)

Practice

1 Sometimes. $2 + 2 = 2 \times 2$, but $2 + 3 \neq 2 \times 3$.

2 Answers will vary, e.g. if you add two numbers together, you will always get the same result as when you multiply them. Counter-example: $2 + 3 \neq 2 \times 3$.

3 Sometimes. 3482 > 986, but 0·3482 < 236.

4 Answers will vary, e.g. the more digits a number has to the left of the decimal point, the larger it is in value.

Going deeper

1 Answers will vary, e.g. (76, 99, 77, 44); 76 is the odd one out because the others are all multiples of 11. Or (76, 51, 9, 3); 9 is the only square number.

2 Sometimes. $1 + 2 + 3 = 1 \times 2 \times 3$ but $2 + 4 + 6 \neq 2 \times 4 \times 6$.

3 Answers will vary, e.g. "All multiples of 4 are also multiples of 2."

4 Answers will vary, e.g. "Quadrilaterals have parallel sides."

5 Answers will vary, e.g. "All multiples of 3 are also multiples of 6." Counter-example: 15.

Page 156: Trial and improvement (P&A 6·4)

Practice

1 The sets of Shapes in the bag could be either 1, 4, and 5 or 2, 3, and 5. Since two Shape values add up to the third value, and the total of all three values is 10, the third Shape must be the 5-shape, i.e. the Shape having half the value of the total.

2 Using the same reasoning as above, the third Shape must be the 7-shape. Hence there are three possible solutions: (1, 6, 7), (2, 5, 7) and (3, 4, 7).

3 Yes. This clue told us immediately that one of the Shapes had to have the value of half the total.

4 Answers will vary.

Going deeper

1 Reading from the left, the numbers are: 8, 3, 7, 1, 6, 4, 5, 2, 9.

2 Reading from the left, the numbers are: 7, 6, 5, 2, 3, 8, 1, 4, 9.

Page 157: Reasoning about numbers (P&A 6·5)

Practice

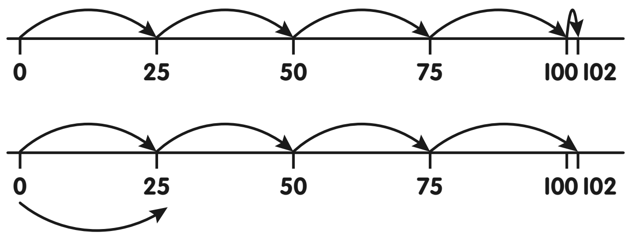
1 Holly's brother is now 4 years old In two years' time Holly will be 12, and her brother 6.

2 Chi buys 4 trucks and 3 cars. $(4 \times 9) + (3 \times 4) = 36 + 12 = 48$.

3 Assuming Jeremy is not an adult, he is 12 years old, 12 + 3 = 15. No other multiple of 6 (lower than 42) + 3 gives a multiple of 5.

Going deeper

1 25



2 a All three lights will be off after 15 seconds. The blue light is off between 14 and 16 seconds, and the red and green lights both switch off at 15 seconds.

b After 12 seconds. The green light is on between 10 and 15 seconds, and the blue and the red lights both switch on at 12 seconds.

This problem is most easily solved by drawing a 'triple' number line, i.e. by drawing one parallel line for each of the lights.

NPC Milestone 6

- Explain how to solve missing number problems involving both equivalence and inequality
- Approach problems involving number relationships systematically, logically and effectively

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