Geometry, Measurement and Statistics 1

Implementation Guide

Written and developed by

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www.oxfordprimary.co.uk/numicon

About Numicon

Numicon is a distinctive multi-sensory approach to children's mathematical learning that emphasizes three key aspects of doing mathematics: communicating mathematically, exploring relationships, and generalizing.

Numicon was founded in the daily experience of intelligent children having real difficulty with maths, the frequent underestimation of the complexity of the ideas that young children are asked to face when doing maths and recognition of the importance of maths to them and to society as a whole.

Numicon aims to facilitate children's understanding and enjoyment of maths by using structured imagery that plays to children's strong sense of pattern. This is done through research-based, multi-sensory teaching activities.

Numicon takes into account the complexity of abstract number ideas and seeks to foster the self-belief necessary to achieve in the face of challenge or difficulty.

Through the combination of communicating mathematically (being active, talking and illustrating), exploring relationships and generalizing, children are given the support to structure their experiences: a vital skill for both their mathematical and their overall development.

A multi-sensory approach, particularly one that makes use of structured imagery, provides learners with the opportunity to play to their strengths, thereby releasing their potential to enjoy, understand and achieve in maths. By watching and listening to what children do and say, this enjoyment in achievement is also shared by teachers and parents.

Numicon strives to support teachers' subject knowledge and pedagogy by providing teaching materials, Professional Development and on-going support that will help develop a better understanding of how to encourage all learners in the vital early stages of their own mathematical journey.

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Welcome to Geometry, Measurement and Statistics 1

Before you start teaching, we recommend you take some time to familiarize yourself with the Numicon starter apparatus pack A, the teaching resources and the pupil materials, to help you and the children get the most out of using Numicon.

Use this Implementation Guide:

- to find out more about what Numicon is
- to find out how using Numicon might affect your teaching of geometry, measurement and statistics
- to learn about the key mathematical ideas children face in the Geometry and Measurement activity groups.

You will find more information, videos, ideas and free resources on the Numicon website: www.oxfordprimary.co.uk/numicon. Here you can sign up for our newsletter, which includes suggestions for topical mathematics and updates on Numicon.

What's in the Numicon starter apparatus pack A?

The following list of apparatus supports the teaching of Geometry, Measurement and Statistics 1. These resources should be used in conjunction with the focus and independent activities described in the activity groups.

Apparatus pack contents

- Numicon Shapes – box of 80 (× 2)
- Numicon Coloured Pegs – bag of 80 (× 2)
- Numicon Baseboard (× 6)
- Numicon Feely Bag (× 3)
- Numicon 10s Number Line (× 3)
- Numicon Spinner (× 4)
- Numicon 0–100 Numeral Cards (× 2)
- Numicon Post Box – set of 3
- Number rods – large set
- Numicon 1–100 cm Number Rod Track (× 3)
- Numicon 0–100 cm Number Line – set of 3 (× 2)
- Extra Numicon 10-Shapes – bag of 10 (× 2)
- Extra Numicon 1-Shapes – bag of 20

The following apparatus is not specifically listed for use in Geometry, Measurement and Statistics 1 activities, but should be used freely and as needed to support and extend children's work.

- Numicon Display Number Line
- Numicon 0–31 Number Line – set of 3 (× 2)
- Numicon Number Rod 1–10 and 20 Trays
- Numicon Dice – set of 4
- Numicon 1–100 Card Number Track (× 2)
- Magnetic strip

Numicon Shapes 1

These offer a tactile and visual illustration of number ideas, and are used to support work on transformations and money in Geometry, Measurement and Statistics 1.

Numicon Coloured Pegs 2

These red, blue, yellow and green Pegs are useful for making patterns and arrangements and work on position and movement on a Baseboard.

Numicon Baseboard 3

The square Baseboard has 100 raised studs to hold Numicon Shapes and Pegs. It is used in many activities, providing a defined 'field of action' for reflection, rotation, position and movement and money work.

Numicon Feely Bag 4

By feeling for geometrical shapes in the Feely Bag, children visualize their parts and properties, helping them to develop their own mental and tactile imagery of 2D and 3D shapes.

Numicon 10s Number Line 5

This number line shows Numicon 10-Shapes laid horizontally end to end and marked with multiples of 10 from 0 to 100. It helps children to develop a 'feel' for the cardinal value of numbers to 100 and connect this to place value.

Numicon Spinner 6

Different overlays (provided as photocopy masters) can be placed on the Numicon Spinner to generate a variety of numbers, coin values, shapes and movement instructions.

Numicon 0–100 Numeral Cards 7

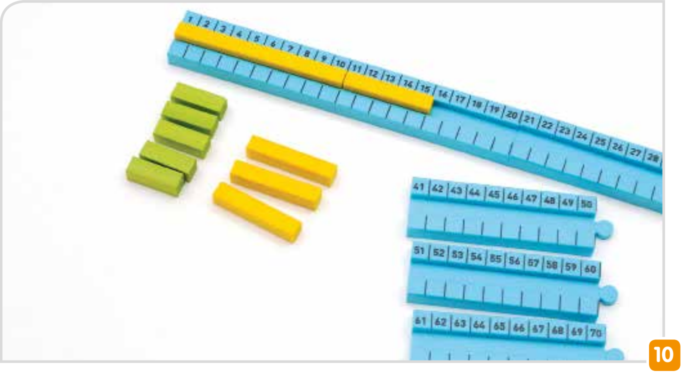
The pack of 0–100 Numeral Cards may be used in activities to generate numbers for children to work with. It is also used in some whole-class and independent practice activities.

Numicon Post Box 8

A card 'post box' through which Numicon Shapes, geometrical shapes and cards showing measurements (for example) can be posted.

Number rods 9

A set of ten coloured rods, 1 cm square in cross section. The shortest is 1 cm in length, the longest 10 cm. These offer another structured illustration for number. Being centimetre-scaled, they can also be placed along the Numicon 0–100 cm Number Line, and are used later in Geometry, Measurement and Statistics 2 to introduce centimetres as a unit of length.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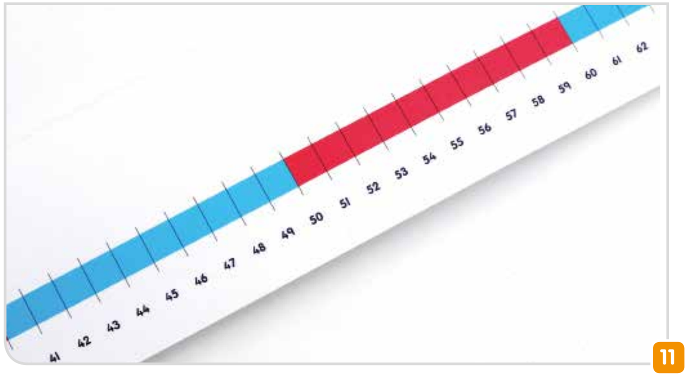
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Numicon 1–100 cm Number Rod Track 10

The 1–100 cm Number Rod Track supports children's use of number rods and measurement work. The decade sections click together into a metre-long track and can also be separated into sections to form an array.

Numicon 0–100 cm Number Line 11

The points on this number line are 1 cm apart and are labelled from 0 to 100. The number line is divided into decade sections, distinguished alternately in red and blue. It can also be used with number rods, as an alternative to the 1–100 cm Number Rod Track.

Numicon Display Number Line

The Numicon Display Number Line provides a visual reference for children, connecting Numicon Shapes, numerals and number words with the number line.

Numicon 0–31 Number Line

This number line shows the numerals 0–31, spaced so that children can place a counter on each numeral in independent counting activities. It can be used to support children's number work in any context, as needed.

Numicon Number Rod Trays 1–10 and 20

This set comprises a Number Rod Tray for each number up to 10, plus one for 20. They can be used for exploring and building patterns.

Numicon Dice

A set of four 22 mm dice, featuring Numicon Shape patterns alongside the numerals: two 0–5 Dice, one 5–10 Dice and one +/– Dice. They can be used to generate numbers, calculations and instructions.

Numicon 1–100 Card Number Track

This number track is divided into ten strips, numbered 1–10, 11–20, 21–30, and so on. The sections can be arranged end to end horizontally or as an array similar to a 100 square. It can be used to support number work in any context.

Magnetic strip

This self-adhesive magnetic strip can be cut into pieces and stuck onto Numicon Shapes or number rods so that they can be used on a magnetic whiteboard.

Available separately

Numicon Software for the Interactive Whiteboard 12

This rich interactive tool is designed for use with the whole class to introduce key mathematical ideas. It includes: number lines featuring the Numicon Shapes, the Numicon Pan Balance, shapes, coins, Numicon Spinners and much more.

Numicon Pan Balance 13

Using Numicon Shapes, number rods or other objects in this adjustable Pan Balance enables children to see equivalent combinations. Children can easily see what is in the transparent pans, making the Pan Balance especially useful for comparing quantities as part of measurement work. It can also be used, in the same way as any bucket balance, to explore the concepts of 'heavier' and 'lighter' in Geometry, Measurement and Statistics 1.

Individual sets of Numicon Shapes 1–10

In Geometry, Measurement and Statistics 1 children can use these to help them in handling numbers as part of measurement work, and they are especially useful for introducing coins, helping children to link each coin with its value in pounds or pence.

Other equipment recommended for the teaching of geometry and measurement

The activities in Geometry, Measurement and Statistics 1 also make use of other resources which are typically found in classrooms and widely available from suppliers. Resources which are particularly useful for geometry and measurement activities are described in more detail here. Other, more generally useful items, such as sorting equipment, interlocking cubes, string, squared paper and so on, are highlighted in the '**Have ready**' sections of the focus and practice activities.

Pattern blocks 14

Pattern blocks are a size-matched set of shapes which children can use flexibly to explore and understand the parts and properties of 2D shapes. They can be used to make pictures and patterns during play, as well as to investigate, for example, how shapes can be combined to build other shapes or tilings, or to make repeating or symmetrical patterns.

Photocopy masters 19–24 provide cut-out pattern blocks based on a side length of 2·5 cm.

Geo strips and connectors 15

By connecting geo strips – flexible, punched strips of various lengths – children can make, alter and remake shapes for themselves, helping them to link the physical, variable properties of their models to concepts of space, line, angle and shape.

Photocopy masters 10 and 11 provide a cut-out template for making geo strips, which can be laminated for durability and used with paper fasteners.

Geoboards and elastic bands 16

Stretching an elastic band around the pegs on a geoboard is a quick and simple way for children to make and alter shapes, allowing them to explore a variety of geometric concepts, including angles, symmetry, area and perimeter, transformations, and coordinates, among others. Children's ideas and results can also be recorded onto matching 'dot' paper, making the geoboard a very useful investigative tool.

A number of types of geoboard are available, the most common of which has a square grid of pegs. Others are available in an 'isometric' arrangement which can be used to illustrate equilateral triangles and a wide variety of 3D shapes in particular. Note that isometric 'dot' paper can be used for recording in this instance.

Geometric shapes 17

A variety of images and examples of geometric shapes should be available for children to look at, handle and talk about during their play and work. Both flat, 2D shapes – including squares, oblongs, triangles and circles – and solid, 3D shapes – including cubes, cuboids, pyramids, prisms, cylinders, spheres and cones – should be presented in a variety of sizes, forms and contexts. You can also provide images and examples in context, to encourage children to notice the shapes they come across in everyday objects.

Programmable robots

A number of different programmable floor robots for classroom use are available. These require the user to enter a series of instructions, typically for turns and straight-line movements, which they follow to complete a journey around the floor. They represent a practical and appealing context for demonstrating, thinking about and experimenting with position, direction and movement, angles and turns, and measurement, among other topics, as well as for problem solving and logical thinking more generally. They are also a means of encouraging children to begin making links between mathematics and ICT, and of approaching the idea of programming. Opportunities for their use are highlighted in the focus and practice activities.

Clocks

Clocks, clock faces and clock images are used to help children learn about time measurement and telling the time. In Geometry, Measurement and Statistics 1 children begin with clock faces with a moveable hour hand, illustrating the 12-hour cycle of 'o'clock' times. They later move on to showing these times on clock faces with both hands, and using the position of the minute hand on the dial to illustrate times 'half past' the hour. Geared clocks (or clock faces with geared hands) are needed to show the linked movement of the hour and minute hands.

Photocopy masters of clock faces are provided on pages 2 and 5 of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*, for children to use as needed to illustrate and record their learning.

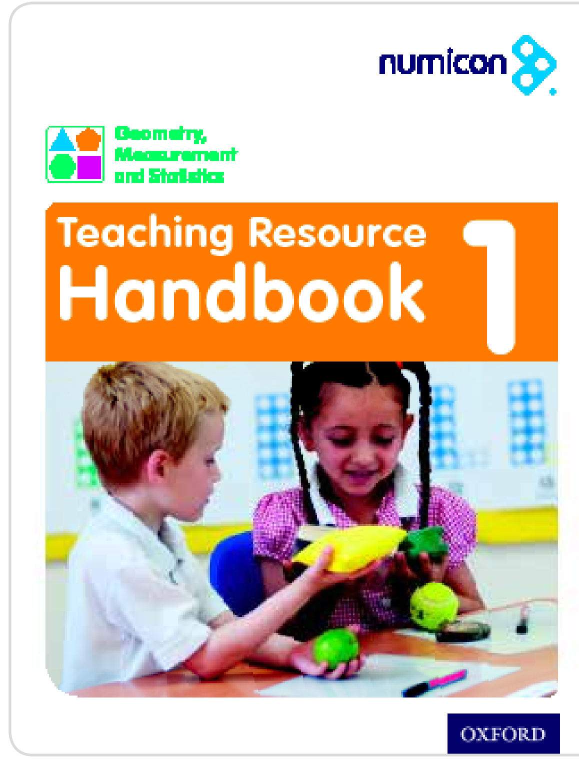
In addition, pictures and examples of a variety of different types and designs of clocks and watches are helpful in encouraging children to consider similarities and differences, and ultimately to generalize their skills in telling time. They are likely to be familiar with digital clocks from the time displays on many electronic devices and appliances, and where appropriate can begin to make links with the 12-hour cycle illustrated by the clock face. The digital 12-hour clock and simple Roman numerals are introduced in Geometry, Measurement and Statistics 3, and the digital 24-hour clock in Geometry, Measurement and Statistics 4.

Items and materials for measuring; measuring instruments

In Geometry, Measurement and Statistics 1, children choose and use their own units to explore length, heaviness and capacity; they might measure a distance in steps, for example, or a capacity in scoops.

For learning about mass/weight, real-life objects for weighing are needed, including some which challenge the idea that 'larger is heavier', for example a large box of popcorn (large but light) or a paperweight (small but heavy). Children use balance scales to begin comparing masses more precisely, then move on to using standard units of mass in Geometry, Measurement and Statistics 2. It is useful to provide a variety of examples and images of balance scales, if possible, to help broaden children's experience.

The activities exploring capacity make use of water, and children will therefore need access to a suitable space in which to work with water and a water supply, along with other water-handling equipment, as appropriate – for example buckets, water trays, funnels and protective clothing. Alternatively, dry materials which pour, such as sand or rice, could be used. A large variety of containers – jugs, bottles, pots, cups, scoops, bowls, beakers, and so on, in a wide range of sizes and shapes – is also needed.



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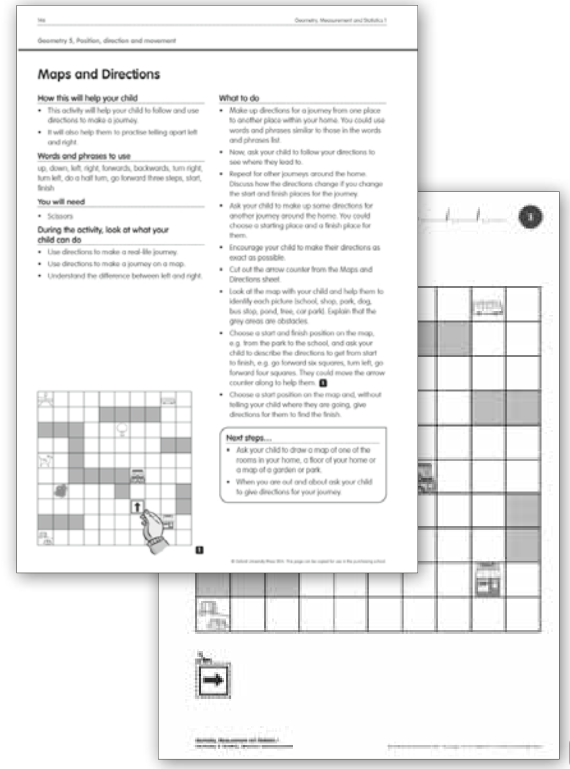

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What's in the Numicon teaching resources?

Geometry, Measurement and Statistics 1 Teaching Resource Handbook 18

This contains 11 activity groups clearly set out and supported by illustrations. Each activity group begins with the educational context, learning opportunities, assessment opportunities and important mathematical vocabulary. To support teachers' assessing of children, there are notes on what to 'look and listen' for as children work on the activities, as well as suggestions for whole-class and independent practice. Photocopy masters supporting the 11 activity groups are included at the back of the Teaching Resource Handbook.

Support for planning and assessment is included in the front of the handbook. There, you will find:

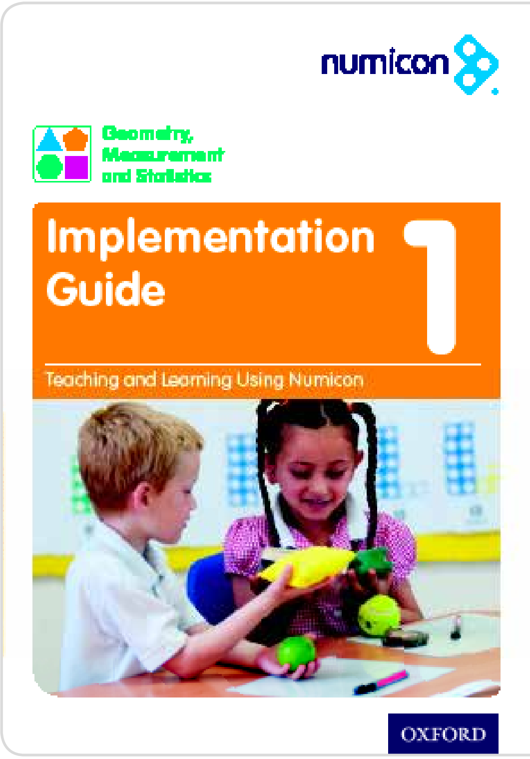
- information on how to use the Numicon teaching materials and the physical resources
- long- and medium-term planning charts that show the recommended progression through the activity groups
- an overview of the activity groups.

Geometry, Measurement and Statistics 1 Explore More Copymasters (provided in the Teaching Resource Handbook) 19

The Explore More Copymasters offer children the chance to practise and discuss mathematics at home with a parent or carer. An activity has been included for each activity group so that children have ongoing opportunities to practise their mathematics learning outside of the classroom.

Each activity is supported by information for the parent or carer on the mathematics that has been learned in class beforehand, and the learning point of the activity itself. Guidance on how to complete the activity is included, as well as a suggestion for how to make the activity more challenging or how to further develop the activity in a real-life situation.

The Explore More Copymasters can be given to an adult or child as a photocopied resource.



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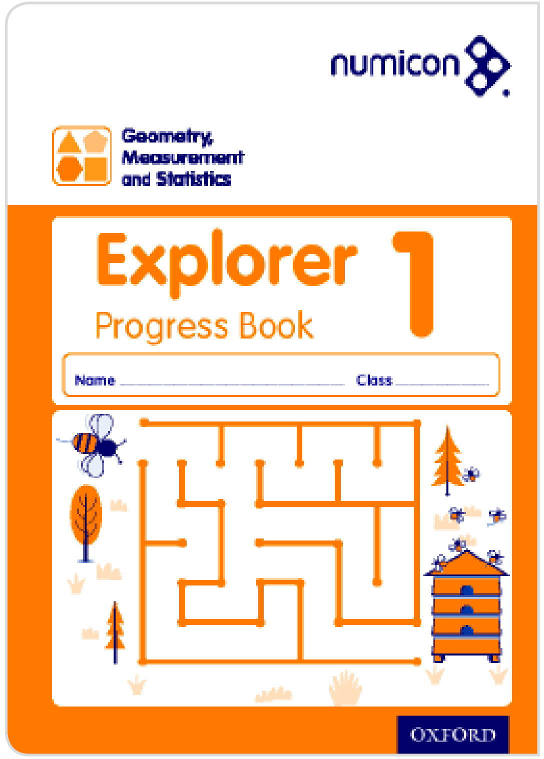

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***Geometry, Measurement and Statistics 1 Implementation Guide* 20**

This introduces what Numicon is and how Numicon helps children meet the demands of learning mathematics. It also includes some practical advice about what to do when preparing to teach with Numicon and answers some key questions about how to use Numicon in practice. The sections on 'Key mathematical ideas' provide useful explanations of the important concepts children will meet in 11 activity groups of the Teaching Resource Handbook and how these ideas develop when teaching with Numicon.

The different sections of the Implementation Guide can be accessed as and when necessary to help you with your teaching.

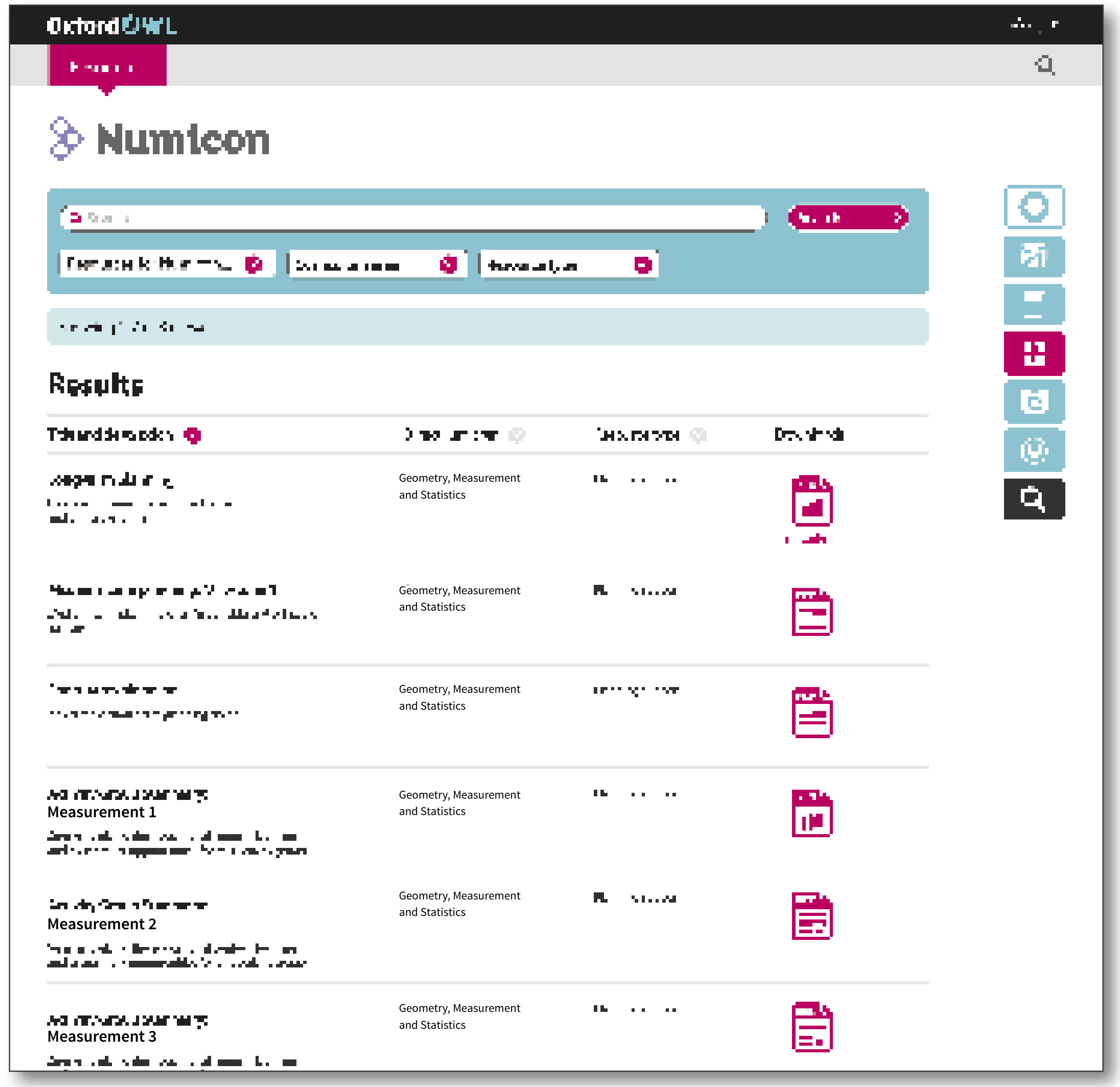
***Geometry, Measurement and Statistics 1 Explorer Progress Book* 21**

The Explorer Progress Book offers children the chance to try out the mathematics they have been learning in each activity group. In children's responses, teachers will be able to assess what progress individual children are making with the central ideas involved in every activity group. It should be stressed that the challenges in the Explorer Progress Book are not tests. There are no 'pass/fail' criteria; the challenges are simply designed to reveal how well children can use the mathematics they have been learning in a new situation.

Being able to use mathematics in an unfamiliar situation is a significant indicator of children's understanding. Many of the tasks set mathematics in a new or different context and, where possible, the challenges are set in an 'open' way, inviting children to show how they can reason with the ideas involved, rather than testing they have learned a routine solution to routine tasks.

As with children's classroom activity work, a range of apparatus should be freely available to children as they reason with the ideas in their Explorer Progress Book.

In addition, there is also scope for self-assessment. This can be used flexibly throughout a term, or to summarize learning at the end of a block of work.



Numicon Planning and Assessment Support

The Numicon Planning and Assessment Support is designed to be used flexibly within schools' own planning formats. Within the support you will find short videos introducing Geometry, Measurement and Statistics 1 and offering advice that will help you get started with teaching with Numicon.

There is also an editable summary of each activity group, including the title and number of the group, the educational context, learning opportunities, assessment opportunities and the mathematical words and terms to be used with children as they work on the activities. There are suggestions for how to use these activity group summaries in your planning, as well as editable short-, medium- and long-term planning frames.

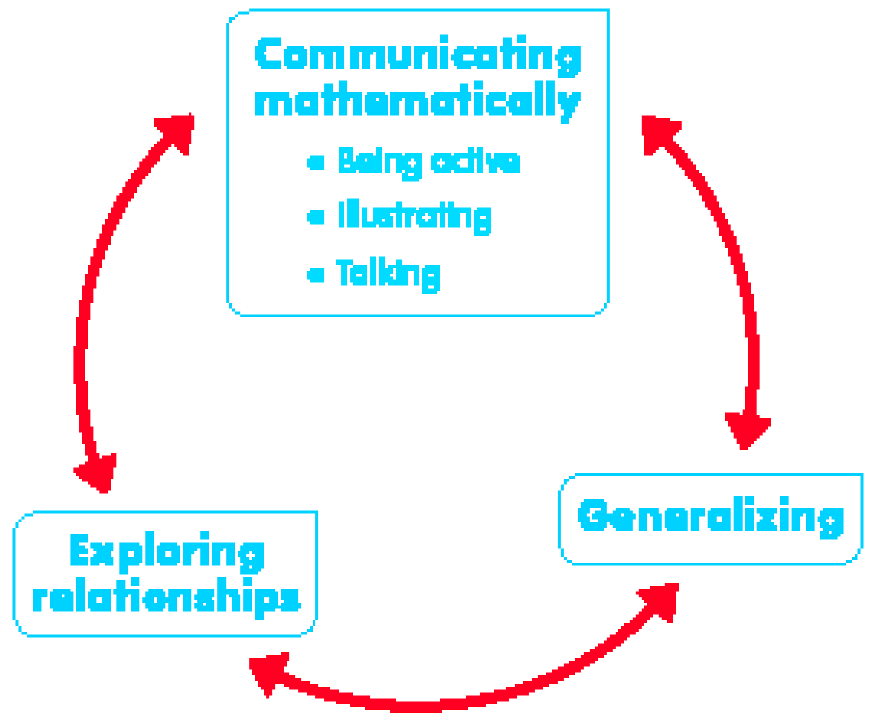
Assessment grids that support monitoring of children's work on the Explorer Progress Book and editable versions of the milestones for assessing children's progress are also available.

Charts that map Numicon to the English, Welsh, Scottish and Northern Irish curricula have been included in these resources, as have charts showing the progression of the Numicon teaching programme across Number, Pattern and Calculating and Geometry, Measurement and Statistics.

What is Numicon?

In order to illustrate how using Numicon in your teaching can help children learn mathematics, this section looks at:

- what Numicon is
- what doing mathematics demands of children
- how using Numicon helps children meet these unique demands.



What is Numicon?

Numicon is a distinctive approach to children's mathematical learning that emphasizes three key aspects of doing mathematics: communicating mathematically, exploring relationships, and generalizing.

Communicating mathematically

Doing mathematics involves communicating and thinking mathematically – and these are two sides of the same coin. We think in the same ways that we communicate, and communicate in the same ways that we think. As children learn to communicate mathematically, they learn to think mathematically. This involves them in the following:

Being active: Teaching and learning with Numicon requires children to be active. This does not only mean being physically active (e.g. fitting physical objects together, finding a number on a number line, drawing a shape), but is a requirement reflecting the understanding that mathematics itself is activity: mathematics is something children are learning to *do*.

What this means in practice is that it is always the children themselves who are to do the mathematics. Telling children (or showing, or explaining) 'what to do' can encourage children to be passive. Numicon asks that the children do the mathematics, that is, both the activity and the thinking. In other words, children actively learn to do mathematics for themselves.

Illustrating: Doing mathematics, that is, thinking and communicating mathematically, necessarily involves illustrating, because mathematics is about studying relationships between objects, actions and measures, and it is impossible to explore such relationships without some kind of spatial imagery being involved. Even when thinking about just two numbers, one of them 'comes after' the other one on a number line or is said to be 'bigger' or 'higher'; these relations all involve spatial imagery.

Numicon explicitly involves illustrating in every activity. This is how relationships in a wide variety of contexts are both explored and communicated.

Talking: Since doing mathematics involves communicating mathematically (both with others and with ourselves), doing mathematics also involves talking. Talking is an essential aspect of all Numicon activity.

Throughout Numicon, talking means dialogue in which points of view are exchanged between teachers and children, and between children and children. All partners in dialogue will be actively involved, not just passively hearing, or waiting to speak. Such exchanges encourage reflective thinking as children learn to discuss different points of view, both with others and with themselves.

Exploring relationships (in a variety of contexts)

Doing mathematics involves **exploring relationships**, that is, the structure, in any situation in order to develop some kind of control in that area. Relationships can be between amounts, between positions, between shapes, between things that are varying, between things that are constant, or between combinations of all of these things. Fully understanding the relationships in a situation makes it predictable, thus making it possible to find out and/or manage what is needed. The mathematical reasoning children are asked to do is their expression of relationships they perceive.

Numicon ensures that children explore the relationships within a wide range of contexts so that they learn not only *how* to do mathematics, but *when* the mathematics they are learning is useful.

Generalizing

In doing mathematics, exploring relationships and looking for patterns in various situations lead to **generalizing**. It is generalizing that allows us to make whole ranges of new situations predictable.

Numbers are generalizations that we all use to make predictions when calculating. For example, the '6', '2' and '8' in the number sentence '6 + 2 = 8' are generalizations; 6 of anything and 2 of anything, will together always make 8 things, whatever they are.

'The angles of a triangle add up to 180°' is a generalization that is often used when doing geometry; 'the area of a circle is 'πr^2' is another that is used when measuring.

In each of these cases, noticing patterns in relationships allows us to generalize about an infinite number of other, similar situations.

It is because we make and use generalizations continually as we do mathematics that mathematical thinking and communicating will appear to have an abstract character for children if they are not involved in doing the generalizing for themselves.

Communicating mathematically, **exploring relationships** and **generalizing** all come together when *doing* mathematics.

What doing mathematics demands of children

In learning to do mathematics at the age they are asked to, children face a unique challenge in their school curriculum: thinking and communicating about abstract objects.

Pure numbers, such as 6 and 254, are abstract objects and in no other subject are children asked to work with and reason about such things so early. It is not surprising that children can hesitate, be puzzled, get stuck, or take time to work things out for themselves.

Most mathematicians will say that doing mathematics is about looking for patterns in situations. It is, but there is more to it than that. When you find a pattern, it means you have noticed something regular, something that always seems to be happening, and this means that any pattern you see is a generalization of yours. Mathematics deals almost entirely in generalizations.

Which is what numbers are: as generalizations, they are abstract objects. Very early on in mathematics, young children are asked to do things with lots of abstract objects, like those we call '3', and 'ten'. Not '3 pens', or 'ten sweets', or '3 friends'. Just '3', or, even more curiously, the two-digit '10' by itself. Notice how quickly children are asked to add and subtract these abstract objects to and from each other. There are high expectations of every child from the very beginning in mathematics.

The central problem: communicating with, and about, abstract objects

How is it possible to communicate about abstract objects? And, since thinking is communicating with ourselves, how is it possible to *think* about abstract objects?

Notice that 'abstract' does not mean 'imaginary'. We can easily picture imaginary things, such as unicorns and beings from outer space, but abstract things are different. Abstract things include qualities like 'truth', 'intelligence', 'common sense', as well as generalizations such as '6 of anything'. The problem is, as soon as you try to picture '6 of anything', you find you are imagining '6 of something'.

The answer, as Bruner observed, is that we think about abstract objects with symbols; in the case of numbers, we do it with numerals.

The important thing about symbols is that they do not attempt to show literally what is being talked about; they are simply arbitrary, conventional marks on a page (or spoken words). When what we are communicating about with these symbols is abstract, that absence of a picture is inevitable; how could we possibly picture something that is abstract?

It is easy to picture '3 pens' or 'ten friends', but what might the abstract '3' look like? Or, how about the curious two-digit '10'? Since numerals do not look like the abstract things they 'stand for', how are children to learn to interpret such arbitrary symbols in their thinking and communicating? Doing mathematics – thinking and communicating about abstract things with symbols – is certainly not easy for young children.

How does Numicon help?

Essentially, Numicon does two things. Firstly, Numicon acknowledges that in order to understand what numbers are, children have to generalize. Secondly, Numicon follows Bruner's advice in using children's actions and imagery to prepare for their use of mathematical symbols in their thinking and communicating. In practice, these two things are done at the same time.

In Bruner's terms, *enactive* and *iconic* representations (action and imagery) are used to inform children's interpretation of the *symbolic* representation (e.g. numerals) that is necessary for communicating their pattern spotting (generalizing). To aid children's necessary generalizing, strong focus is placed upon the use of structured materials.

Generalizing and reasoning – an early years example of teaching with Numicon

Initially, in common with most teaching, Numicon involves a wide variety of everyday objects (such as beads, cubes, pegs and counters, see Fig 1) in order to help children develop their counting, before then introducing the challenges of calculating.

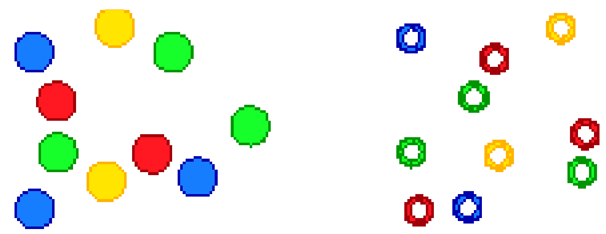
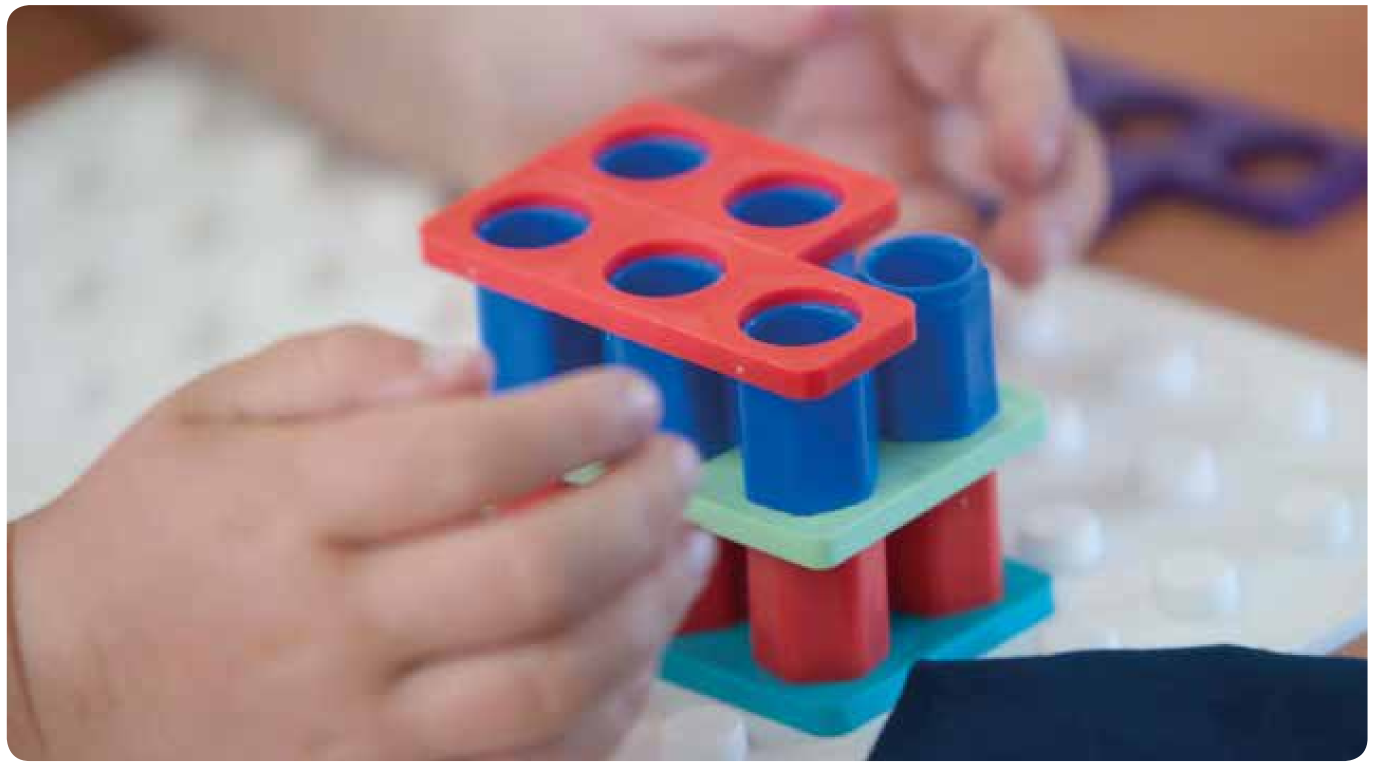


Fig 1



Importantly, Numicon also introduces sets of structured materials in which individual pieces have *regular* physical relationships with each other, for example, Numicon Shapes and number rods, see Fig 2. Children explore the physical relationships within these structured materials by, for example, ordering pieces, comparing them, combining them physically to make others.

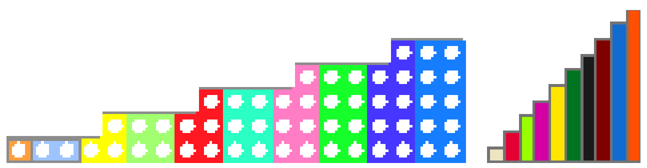


Fig 2

Thus, as children work with loose collections (beads, cubes) and with structured materials (Numicon Shapes and number rods), they are able to combine being active with physical objects and images as illustrations in their talking and thinking about numbers of things.

Numerals are introduced in association with loose collections of objects and with structured materials as children work; thus, symbolic representation appears alongside children's ongoing action and imagery with physical objects.

Number lines (e.g. Fig 3) are introduced to further illustrate the ordering that is evident within the structured materials, and to reinforce the associated ordering of numerals.

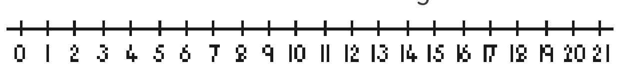


Fig 3

Importantly, loose collections of objects are arranged into the regular patterns of the Numicon Shapes; individual number rods are found to be equivalent in length to different multiples of the smallest 'unit' rod (e.g. Fig 4).

Fig 4

Through doing these activities, children learn that *any* collection of loose objects can be arranged into Numicon Shape patterns that can then be 'read'. Children learn to 'see' how many objects are in a collection without counting; numbers of things begin to 'take shape' visually, in organized ways.

It is now possible to prepare children for further generalizing about numbers through exploring relationships between numbers of things. Children's mathematical thinking and communicating continue to develop through their being active with the objects they are given, and in illustrating their talking about what they see and do.

Children generalize that any collection of loose objects can be arranged into Numicon Shape patterns. They also generalize that any number of 'unit' number rods may be exchanged for (will be 'as long as') one or more of the larger number rods. Thus *any* number of loose objects can be converted into (is equivalent to) one or more Numicon Shapes or number rods.

In realizing that any collection of loose objects can be arranged into Numicon Shape patterns, and that any number of unit

cubes is equivalent to one or more number rods, children become able to 'see the general' in any particular illustration; Numicon Shapes and number rods can be used to illustrate, in an organized way, any numbers of any kinds of things.

Numicon Shapes and number rods themselves may now be used to explore and to communicate about number relationships in general. They have become communication mediators in discussions about numbers and their relationships.

As a particular example, the Numicon Shape that has three holes fits together physically with the Shape that has five holes. The result 'forms the same shape as' (is equivalent to) the Shape that has eight holes, see Fig 5.

Similarly, the number rod worth three units, combined end-to-end with the rod worth five units, are together as long as the rod worth eight units, see Fig 6.

When laid end-to-end along a number line or number track, the '3 rod' and the '5 rod' together reach the position marked '8' on the line.

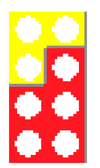


Fig 5

Fig 6

From these actions, and with these illustrations, children are able to generalize that: three *anythings* and five *anythings* together will *always* make eight things.

This generalization can be expressed using the numeric and verbal symbols: 3 and 5 together are equivalent to 8.

Later on, using further actions and further illustrations, children become able to interpret and use the further symbols '+' and '=' to express their generalization thus:

3 + 5 = 8

Importantly, at this stage children will have begun to use number words (one, two, three) as *nouns* instead of as adjectives (two sweets, three pencils) in their talking.

With their use of number words as nouns, numbers as *abstract objects* have now appeared in children's mathematical thinking and communicating, referred to with *symbols.*

Such generalizing and use of symbols can now be exploited further. If 'three of *anything*' and 'five of *anything*' together always make 'eight *things*', then:

3 tens + 5 tens = 8 tens
3 hundreds + 5 hundreds = 8 hundreds
3 millions + 5 millions = 8 millions

or

30 + 50 = 80
300 + 500 = 800
3,000,000 + 5,000,000 = 8,000,000

Such is the power of generalizations, and of the symbolic notation that children can by this stage use to think and communicate mathematically about them.

Progressing from such early beginnings

The foregoing example also illustrates how Numicon supports the teaching of children's subsequent mathematics.

In a wide variety of contexts, children are offered opportunities to be active and to illustrate their talking about any (and all) relationships they are exploring between both physical and abstract objects. They do this in order that they may generalize and, as a consequence, think and communicate about their generalizations with the conventional symbols of mathematics: in other words, that they may *do* mathematics.

Of course, there is much more beyond this first example. Children have many other relationships to explore, further generalizing to do, and thus many more conventional symbols to learn to interpret and use. However, wherever it is used, the approach is essentially the same. *Enactive* and *iconic* representation are used to inform children's interpretation of the *symbolic* representation that is necessary for thinking and communicating about their constantly developing pattern spotting (generalizing).

The going gets harder as children progress in their number work: place value, ratios, fractions, decimals and negative numbers will all challenge children's capacity to generalize onwards from these early beginnings. However, such advanced work is equally possible with the same approach of actions and illustrations building towards generalizing, and a consequent use of symbols to think and communicate about the generalized, abstract mathematical objects created. This is how the symbolism of mathematics becomes meaningful.

In their work on geometry, the generalizations that children reach in their mathematical thinking and communicating happen more gradually as they progress towards being able to reason about invented abstract objects such as 'any triangle' and (later) 'any polygon'.

It is impossible to draw the abstract object 'any triangle' in the same way that it is impossible to imagine 'six of anything'. As soon as you draw a triangle, however you have chosen to draw it, you have drawn a particular one; it does not matter whether you draw one that is right-angled, isosceles, equilateral, or scalene, what you have drawn is not a general triangle – it is a particular one, e.g. Fig 7.

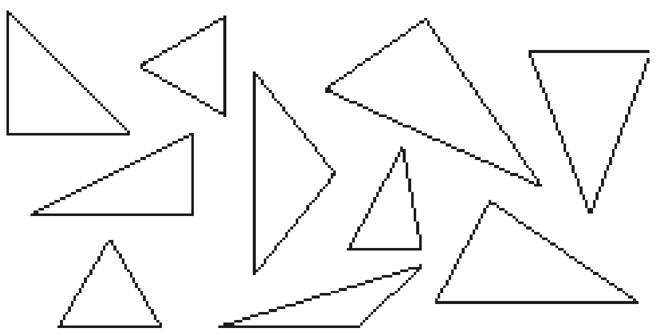
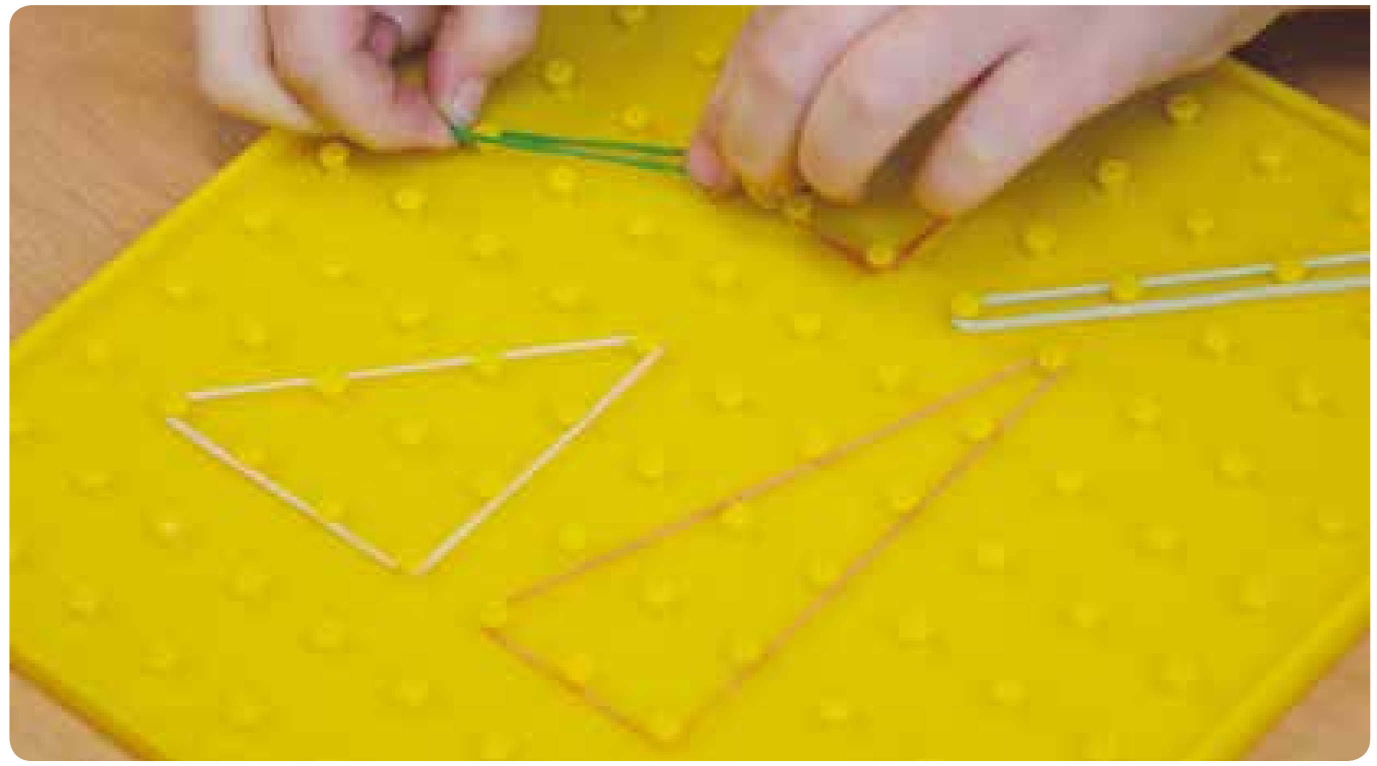


Fig 7

However, as with generalizing about numbers, in doing geometry, much exploring of relationships with action and imagery (enactive and iconic representation) prepares children for reasoning meaningfully about 'any triangle' with symbolic representation (words and symbols).



In this example, as children physically construct and transform many varieties of triangles with dynamic materials, what they see before their eyes is how lengths of sides may vary endlessly as angles change, how angles may vary endlessly as lengths change and yet, despite all the variety they see, the shapes they are creating all turn out to be triangles.

Dynamic mental imagery deriving from these physical experiences then allows children to speak and to think of 'any triangle' as they imagine an infinitely flexible closed 2D shape with three straight sides.

By stressing the straightness of the sides, that there are always exactly three sides and that the sides fit together to 'close' the shapes they make, and by ignoring the constantly changing lengths of sides and sizes of angles, children become able to imagine 'any triangle' and to reason about it with words and symbols.

Then, having generalized to a stage when their mental imagery allows them to imagine and speak meaningfully about 'any triangle', children are in a position to further generalize that, for example, 'the angles of any triangle add up to 180°' through further action, imagery and reasoning with symbols.

In the same way that generalizing about numbers is approached, exploring relationships with action and imagery in geometry prepares children for reasoning meaningfully about abstract mathematical objects with symbols.

We reach all the generalizations of mathematics, and the symbols we use to express them, through being active and illustrating our talking about the relationships we are exploring.

Doing mathematics in the world – solving problems

Of course, *doing* mathematics in the everyday world is not simply about making generalizations and using symbols. Crucially, it also involves making *use* of generalizations to solve problems in particular situations.

For example, the generalization '4 x 25 = 100' allows us to predict that the perimeter of a square of side 25 cm will be 100 cm, that the area of a field measuring 4 m by 25 m will be 100 m², and that if you save £25 a week for four weeks you will have £100. It can also be very useful to help calculate that:

$$36 \times 25 = (9 \times 4) \times 25 = 9 \times (4 \times 25) = 900$$

Children need to be able to readily connect the generalizing that they do with mathematical symbols with everyday world situations in which those generalizations are useful.

In their more general problem solving, children learning, for example, *how* to divide one number by another, need to learn *when* that operation is useful.

The Numicon teaching materials organize activities into groups based around mathematical themes. For example, 'length' or 'rotation'.

The Numicon teaching materials organize activities into groups based around mathematical themes. For example, 'Length' or 'Position, direction and movement'.

In the activity group titled 'Comparing, ordering and measuring length' (Measurement 1), children start their exploration by talking about and comparing lengths in the context of a story. They begin measuring lengths by devising appropriate non-standard units to suit the context and available resources. They then move on to make use of this approach to solve problems involving different types of measurements of length, including width and distance.

In the activity group titled ‘Position, direction and movement’ (Geometry 5), the activities involve giving and following directions, helping children to expand and refine their knowledge and use of positional concepts and terms. They are given opportunities to experiment with size and direction of turn, and discover connections and equivalences, for example that a full turn is the same as two half turns. Later they move on to using board games, and begin to develop an understanding of position in relation to objects and shapes, and visual representations of these.

In these ways, children’s mathematics is introduced, wherever possible, within contexts in which that mathematics is useful. Such contexts help children to ‘see the point’ of the mathematics they are learning, and prevent mathematics becoming a series of answers to problems they have never had.

Flexibility, fluency, and persistence

Being able to do mathematics effectively also includes being able to remember basic things such as those generalizations known as ‘times tables’ and number ‘facts’. There are many practice activities built into the Numicon written materials that encourage children to develop a regular familiarity with basic facts.

Even more important to effective functioning is flexibility in mathematical thinking and communicating. Three kinds of flexibility are especially useful.

Through being active, children are able to ‘invert’ their actions; put more simply, this is about ‘doing and undoing’. As children have found ‘how many’ objects there are in a collection by physically grouping them into tens (and hundreds), they are later readily able to ‘partition’ numbers (‘undo’ their groupings) when calculating with symbols.

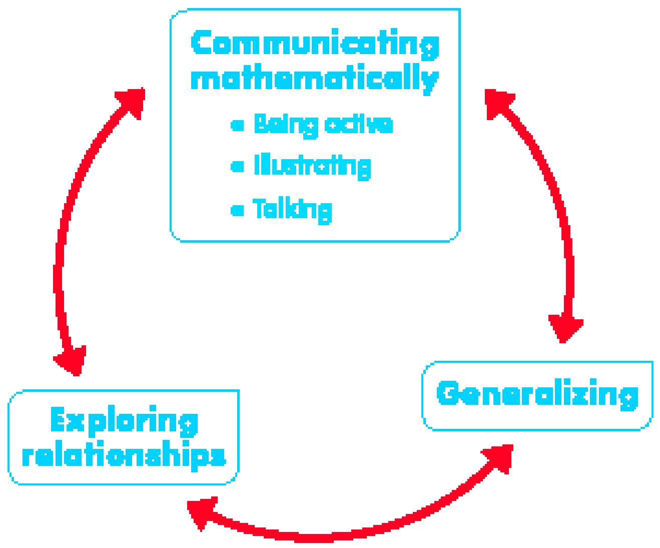
As children can combine and separate Numicon Shapes and number rods physically (‘do’ and ‘undo’ their actions), they are able to connect adding and subtracting as inverse operations and to check a subtracting calculation by adding.

The introducing and encouraging of a variety of ways of calculating means that children are able to choose methods of calculating that suit the particular numbers involved, rather than adopting a standard method for any calculation. Who would want to subtract 1998 from 4673 using a column method, just because the numbers are big?

As Numicon approaches children’s use of mathematical symbols with prior actions and imagery, if children become ‘stuck’ or hesitant while working with symbols, it is always possible for them to recall and return to the supporting activity and illustration from which their original generalizing arose. For example, if they are working on prime numbers, using symbols, they can immediately return to actively illustrating how factors ‘go into’ their various multiples physically with Numicon Shapes and with number rods. They can follow a similar pattern with any other kind of number relationship. This flexibility is one of movement backwards and forwards between Bruner’s *enactive*, *iconic* and *symbolic* forms of representation in their thinking and communicating.

Finally, because Numicon continuously emphasizes communicating mathematically, it is possible to stress to children that all is not lost when they feel ‘stuck’. The thing to do, always, when they ‘don’t know what to do’ in a situation is to communicate.

Being active, illustrating and talking about the relationships we are exploring *is* doing mathematics. Persistence – an invaluable quality when doing mathematics – comes from continuing to communicate, with yourself and/or with others, whenever you cannot see where you are going.



Preparing to teach with Numicon

This section is designed to support you with practical suggestions in response to questions about how to get started with Numicon in your daily mathematics teaching. It also contains useful suggestions on how to plan using the long- and medium-term planning charts as well as information on how to assess children's progress using the Numicon materials.

When getting started with Numicon, as well as reading this section, it can be helpful to refer to the suggested teaching progression in the programme of Numicon activities. This, along with the long- and medium- term planning charts, can be found in the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook.*

How might using Numicon affect my mathematics teaching?

There will be a continual emphasis on communicating mathematically. Once involved in this communication process, children become active, in the sense that they become engaged in 'doing' mathematics for themselves. They begin to reflect on different points of view and to develop the imagery required to communicate mathematically, as well as the ability to illustrate their ideas.

For geometry and measurement, this imagery and illustration has a particularly active, practical foundation. Children develop and refine their thinking about shape, space and size through physical exploration – through making, comparing and manipulating objects. In their early work on measurement, for example, they might line up one object with another to show clearly which is 'longer', while in geometry they might learn to distinguish a rectangle from a parallelogram through handling and comparing examples, showing which have 'square' corners. In this sense, increasing dexterity and control of movement is another element of their developing skill in mathematical communication. As this communication becomes established as part of the culture of the classroom, children will increasingly join in conversations with you and their classmates. Mathematics lessons will develop into dialogue with ready use of imagery to illustrate ideas. A sense of shared endeavour will emerge as children solve problems through communication and persistence. So, when children feel 'stuck', they will know that the thing to do is to talk about it, to try to explain what the difficulty seems to be and to use illustrations and actions to express the problem. Careful questioning can encourage children to really think about 'difficulty' and to arrive at solutions gradually.

As the focus on mathematical communicating grows, you may become more aware of the words and terms you use in your teaching. It is important to use words and terms consistently. Try to encourage other adults in your class – and throughout the school – to use mathematical language consistently. Listen out for children using the same words and imagery to explain their ideas, though to start with they may use them only hesitantly.

In their work on geometry, for example, children begin to classify solid 3D shapes through reasoning about the parts and properties different examples have in common, making use of ideas and terms such as 'surface', 'corner' and 'edge'. They are encouraged to name these shapes for themselves, and to further refine the names as their reasoning develops. The terms 'cube', 'cylinder', and so on are then introduced in the context of giving instructions for making models, where use of agreed names for these generalized, abstract objects makes practical sense.

Similarly, in their work on measurement, children are first introduced to the use of a 'unit' of measurement in the context of length, and then begin to generalize this through their experiences in the areas of mass/weight and capacity. They are given the opportunity to further refine the concept through arriving at an understanding of the usefulness of an agreed, standard unit.

This increased focus on mathematical communicating will make it easier, through watching, questioning and listening to children as they work, to judge whether they are facing a suitable level of challenge. Activities are structured so

that at each stage children encounter a new 'problem' to solve – Numicon aims to encourage children to relish this, to persevere in working through any difficulty, to gain a sense of achievement when it is overcome, and to be excited about and ready to progress to the next challenge.

How can I encourage communicating?

Children respond to the examples around them. As such, the ways in which you communicate mathematically provide a model for children's communicating. Engaging in dialogue with children, actively listening to what they are saying and responding sensitively with thoughtful questions will encourage children to listen to one another and respect each other's ideas.

Make sure that appropriate resources for illustrating ideas in the area of study are freely accessible. For example, when studying length, interlocking cubes, lengths of string and straight edges (e.g. counting sticks, rulers) can be made available to children, or, for geometry, pattern blocks, geo strips, construction blocks, 2D and 3D shapes, interesting packaging and modelling dough.

Observe how children use these objects, listen to what they are saying, watch what they are doing and respond with questions and qualified praise when you notice active listening and thoughtful questioning. What children do and say will help you to understand what they are thinking.

The activities in Geometry, Measurement and Statistics 1 are designed to build on a foundation of understanding, skills and knowledge in the areas of number, pattern and calculating. Within the Numicon teaching programme, the 'Securing Foundations' section of *Number, Pattern and Calculating 1* provides this basis for mathematical development, and helps children make the transition from play-based activities to a daily mathematics lesson. It will also ensure children's familiarity with structured apparatus such as Numicon Shapes and number rods. Consistency in the use of resources and language across children's mathematics learning will, as already mentioned, help them to develop a solid framework within which to share and develop their ideas.

Providing a range of examples and contexts for children's work will also encourage them to develop and refine the imagery they use to 'do' mathematics. Opportunities are highlighted throughout the activities; for example, real-life paths, pavements and walls might be used to illustrate the idea of tiling (tessellation), while wallpaper, wrapping paper and fabric can be used to encourage children to appreciate different aspects of shape, repetition and pattern. Both images and actual objects might be used.

Finally, the ways in which children are grouped or paired for working together has an impact on their communicating, so this needs careful consideration.

Using daily routines to encourage communicating

A 'morning maths meeting', perhaps 15 minutes long, has proven to be very successful in encouraging children's mathematical communicating.

These meetings are oral and practical and include a small selection of key routines in which children practise rapid recall and gain fluency with the ideas and facts that are the basis or focus of their current work. In addition to consolidating knowledge of number and calculation,

measures, geometry and data handling, they can be used as opportunities to provide variety and context. You might discuss with children observations about a mathematically-rich object, or solve a mathematical problem that has come up in the class, in the school, in the news, at home or in a story.

You can refer to the whole-class practice activities from the activity groups in the Teaching Resource Handbook for ideas. You could also select focus activities from the activity groups to use for class investigations or problem solving.

Children's mathematical thinking and reasoning can also be encouraged at other times during the day. Along with the 'morning maths meeting', this will help to ensure that they experience the full breadth of the maths curriculum, and that they do not see mathematics as something that only happens in their mathematics lessons.

The daily or weekly timetable and the school year provide a meaningful context for children to make predictions and to develop language for temporal relationships (such as 'next', 'before', 'after', 'in between' – note these terms are also used to describe the order of numbers) and for units of time (including the names of the days of the week and months of the year). Calendars, clocks, timers and stopwatches can be used to reinforce the idea of the order and passage of units of time – you might, for example, set an alarm clock or timer at the start of a particular lesson or activity, to ring at the end.

Mathematical thinking can be encouraged in a broad range of classroom tasks and activities. For example, if children are lining up you could ask them to do so in order of height or by birth month, when they are moving between places in the room or school you could ask them to estimate then measure the distance, in steps, or to describe the turns they need to take along the way.

What might the use of Numicon look like in my classroom?

Nearly all of us are acutely visually aware, and children are no exception. A mathematics-rich environment provides valuable learning opportunities; throughout the day, and particularly during mathematics activities, you will find children referring to the imagery and displays around them.

Number images and numerals can be incorporated into labels, notices and displays throughout the classroom, as well as being provided in the form of more specifically mathematical resources – number lines at children's eye level, for example.

You can also create displays to reflect children's current mathematical focus. As well as providing a space in which to celebrate children's mathematical work, this might include pictures, books, interesting objects (different types and designs of clocks, watches or timers, for instance, when children are learning about measuring or telling time) and relevant games and challenges (estimating how many repetitions of a particular skill they can complete in 30 seconds, for example, then finding out and comparing this with their actual result).

Alongside creating a visually-rich environment in which to think and communicate about mathematics, the organization of space and resources can encourage children's involvement in doing mathematics. For example,

it is useful to set up a mathematics table with an interactive display where children can freely explore Numicon Shapes, number rods, 2D and 3D shapes, measuring instruments and other familiar resources. When children are studying geometry and measurement, it is often also helpful to set up some classroom, play or outdoor space so that they can try out the equipment and practise the activities they meet in their mathematics lessons. For example, while working on capacity children might be given access to water or sand and a variety of containers; for position, direction and movement a maze could be marked on the floor using masking tape or a play-equipment obstacle course set up for children to negotiate by giving each other instructions.

Organizing classroom resources systematically, with storage containers numbered and stored in a logical order, ensures that mathematics equipment will be easily accessible for children to find and use themselves. They should be familiar with what is available to them, and encouraged to use it freely. The 'morning maths meeting' might be used, early in the year, to explore resources and how they can be used. This is also an aspect of creating a mathematics-rich environment, with the sorting and reasoning skills involved in putting away and finding resources contributing to children's mathematical thinking and communicating. Finally, geometry and measurement activities typically lend themselves to collaborative working, and some will require a suitable space to be set up in advance. As part of your preparations for a new activity group, you may wish to consider how working spaces or desks should be arranged to facilitate this. (See also 'How will I manage the class for mathematics lessons?' on page 29 and 'What about organizing resources and space?' on page 29).

What could using Numicon feel like for children in my class?

The activities in Geometry, Measurement and Statistics 1 are designed to make practical sense to children – they are introduced to coins in the context of trade and exchange, for example, and investigate real-life objects and packaging as a way of thinking about different types of 3D shapes. In this way, children are encouraged to recognize that mathematics is an ordinary, valuable and interesting part of everyday life, and of their ongoing learning about the world. As you engage them in solving mathematical problems, they also learn *when* to use mathematics.

Within the mathematics-rich environment of the classroom, you are likely to notice children glancing at displays and images to check an idea they are explaining. At other times, you will notice them simply looking thoughtfully at displays – they are likely to be noticing relationships and making connections as they assimilate new ideas.

The open-ended nature of the Numicon resources and activities invites children to experiment and explore, self-correcting as they seek solutions. Children begin to recognize self-correcting as part of the learning process; encouraging them to pursue opportunities to investigate, think, communicate and self-correct will increase their confidence.

Working in pairs within groups also supports children's confidence by encouraging them to share ideas and work things out together. Some children are more confident when working with a partner, setting challenges and exploring ideas more deeply than they do when working alone.

Children take their lead for listening to each other and sharing ideas from the ways in which the adults in the classroom converse with them and each other. They will need help with taking turns, showing respect for each other's ideas, listening to one another without interrupting, phrasing questions and expressing ideas. Over time, you will find that children become confident in sharing their thinking.

With Numicon, children will also know that there is nothing wrong with challenge: it is normal to get 'stuck' – what is important is continuing to communicate mathematically. Children come to relish challenge because they feel able to persist in the face of it, and gain a sense of achievement when a challenge is overcome. Part of this confidence comes from the ways in which their understanding is built cumulatively by following the suggested progression of teaching activities in the Teaching Resource Handbook.

Children also feel confident when tackling new ideas because they can use a variety of resources and imagery to illustrate problems. Their confidence will be further encouraged as you discuss their ideas with them, helping them to become increasingly aware of what they know and are learning. This discussion supports children's monitoring of their own learning.

What is the role of imagery in geometry and measurement?

In their work on number, children are asked from a very early point to 'handle' abstract objects, in the form of pure numbers. Numicon's focus on the use of structured materials helps them to approach this apparently contradictory task through actions and imagery, and in this way to gain access to the ideas we symbolize with numerals such as '3' or '10' (for more discussion, see the 'What is Numicon?' section).

In contrast, children's work on geometry and measurement is rooted in physical objects and how they occupy physical space – their shape, length, mass and so on. Children need to handle these objects directly and immediately: to look at the use of space around them; to draw or make, change, dismantle and remake shapes for themselves; to compare and judge size and quantity in concrete terms – according to how much of something they can fit in their own hand, for example.

Just as for number, though, learning about geometry and measurement requires children to identify relationships and develop symbolic representations (words and symbols) which enable them to spot patterns (generalize) – that is, to think and communicate mathematically. To identify any triangle successfully, for example, children must link the word 'triangle' with a generalized set of requirements which can apply in an infinite number of cases; that is, they must be able to distinguish any particular instance of a closed 2D shape with three straight sides. They can then refine their symbolic representation to identify other general types of triangle: scalene, isosceles, equilateral, right-angled, and so on.

For measurement, the reasoning is perhaps more straightforward, but the principles are the same. Identifying which measurement and unit to use requires a generalized understanding of dimensions of length (for instance), and relative sizes.

Children should be encouraged to experience and explore physical space and objects, investigating, constructing and transforming with a variety of resources and materials in a variety of contexts – but in doing this it is the dynamic mental imagery that they build up, rather than the 'raw' number of examples they encounter, that will enable them to generalize and think mathematically. It is important to support the development of children's imagery by encouraging a continual emphasis on visualizing, describing and predicting results in their work.

How much time should I plan to spend on mathematics teaching?

The time spent teaching mathematics during the school day can vary. In addition to a daily mathematics lesson lasting up to an hour and a 'morning maths meeting' lasting about 15 minutes, there are many opportunities for developing language of, e.g. comparing, position, movement, time and shape. There will also be many opportunities for estimating or making measurements as well as calculating. Taking advantage of these opportunities helps children realize that doing mathematics is normal and useful in all sorts of situations, and encourages them to recognize when and how to use the mathematics they have learned.

For planning teaching time, see the section 'How do I plan in the medium- and long-term using the Geometry, Measurement and Statistics 1 teaching programme?' on page 32.

What format might Numicon mathematics lessons take?

Look at the relevant pages of the Teaching Resource Handbook for the activity group you are working from. The first part of the mathematics lesson is a whole-class introduction. This is followed by a longer session of group work during which children are either working independently or as part of a focused teaching group. Finally, the class comes together for a concluding session.

During the lesson introduction, you are likely to use physical materials and other imagery in communicating and discussing ideas with children.

Children may join in this whole-class part of the lesson in lots of different ways: participating in a class conversation, talking with a partner or within a small group, jotting on an individual whiteboard, or using physical resources to explore and show ideas.

In the second part of the lesson, children will be arranged in groups working on, for example:

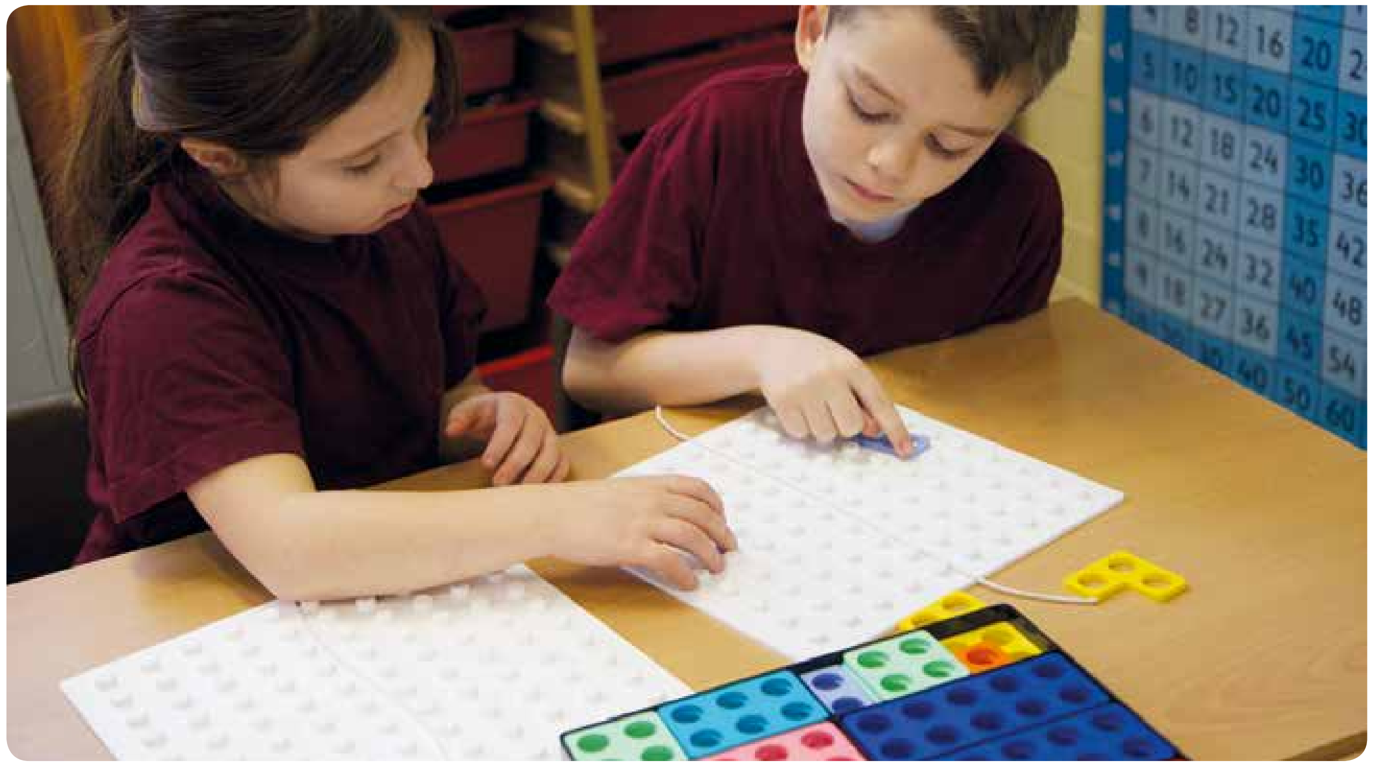
- a teacher-led focus activity
- a teaching assistant-led focus activity
- an independent practice activity or investigation, or further work on ideas introduced at the start of the lesson.

Groups will be exploring ideas within the same activity group but may not be working on the same activities. They may be using different apparatus – for example, for a lesson on length, some groups may be using lengths of string and others strips of paper, while others may be writing and drawing to record particular lengths.

Over the course of a week, the different groups may rotate through the various group-work activities, so that all children receive focus teaching, and explore the ideas using different imagery.

In the final part of the lesson, it is particularly important to encourage all children to reflect on their learning by asking questions of those working at different levels, depending on what you have noticed them doing and saying during the lesson. You may decide to ask the different groups to explain to the rest of the class what they have been doing and what they have noticed. You may have particular points you want to draw to children's attention. You could also suggest what might happen in the next lesson and anything children could think about before then. To help children reflect, you could ask them to think quietly for a few moments about what they have been doing, and guide their reflection with questions such as:

- Is there anything new you have learned?
- Is there anything you feel particularly pleased about?
- Is there anything you particularly enjoyed?
- Is there anything you found difficult?
- Is there anything you are still puzzling about?
- Is there anything you would like to do again?



How will I manage the class for mathematics lessons?

There are several important factors to consider for organizing and managing successful group work.

Firstly to engage children, it is important to plan differentiated activities that will provide appropriate levels of challenge for all children. Therefore, during the week, the group-work activities will need to be adjusted for different groups.

Secondly as children may be working on activities that have been adjusted, it is important that they are comfortable with how to do the activity and what is expected of them.

Thirdly the order in which the activities are introduced to different groups has an impact on how quickly children progress.

At the beginning of the week, the class may be taking in a lot of information, so give consideration to how you will introduce the activities, as this will impact on how quickly children can access and begin to make progress in the work. You will find ways that work well for you, but the following guidance may be of help:

- Explain the simplest independent work first.
- The first time you introduce a challenging practice activity, allocate it to a group of children who are able to follow instructions well.
- Groups which might need more support should begin the week with a focus activity; the adult working with the group can explain the activity, removing the need to spend time explaining it to the whole class.

Once the activities have been introduced, children will then go off to work in their groups. You may wish to focus on the one or two groups that need the most support at this point. If you have a teaching assistant, you may want them to work closely with the group who need the greatest assistance. Each class, and each lesson, will be different.

During this part of the lesson, it is quite likely that a child or children will put forward an idea that is worth everybody considering. You might choose to invite all children to take a moment to reflect on the idea, or make a note to discuss this in the final part of the lesson, when the class comes back together.

What about organizing resources and space?

The way in which resources are organized, used and presented and how the available space is set up sends children a strong message about how they are expected to work. (See also 'What might the use of Numicon look like in my classroom?' on page 25.)

When preparing the classroom for a mathematics lesson, consider how children will be grouped and how working spaces and desks should be arranged for particular activities and learning requirements. Set out the equipment where children are going to be working. Refer to the 'Have ready' sections of the activities you plan to teach. The photocopy masters needed can be printed from the *Geometry, Measurement and Statistics 1 Planning and Assessment Support* or photocopied from the Teaching Resource Handbook.

A number of activities in Geometry, Measurement and Statistics 1 require some open space (for example for pretend river crossings in Measurement 1, for practising a variety of skills for given lengths of time in Measurement 3, for working with water (or perhaps sand) in Measurement 5, and for an obstacle course, a maze and a short journey in Geometry 5). Again, refer to the '**Have ready**' sections of the activities in order to arrange for suitable space in advance.

It is also useful to consider collecting well in advance the real-life examples – both images and objects – which will be needed or provide context for children's work. These might include: items for sale; food items in packets, boxes and tins; patterned designs; a variety of containers; objects which are, or are made up of, common 3D shapes; and different types and designs of clock.

If you have planned for groups to take turns to do the activities over a week, it can be helpful to store the equipment for each activity in a separate container so it is ready for use each day. Encourage children to leave the resources as they found them, ready for the next group. Children can help themselves to any further equipment they need from the class mathematics resources. As children become used to working with Numicon and other mathematics apparatus, they can begin to collect the equipment they need for an activity by following a list (illustrated if necessary with pictures, symbols or container numbers), and thinking for themselves about any further items they may want to use.

Other ideas for organizing resources are:

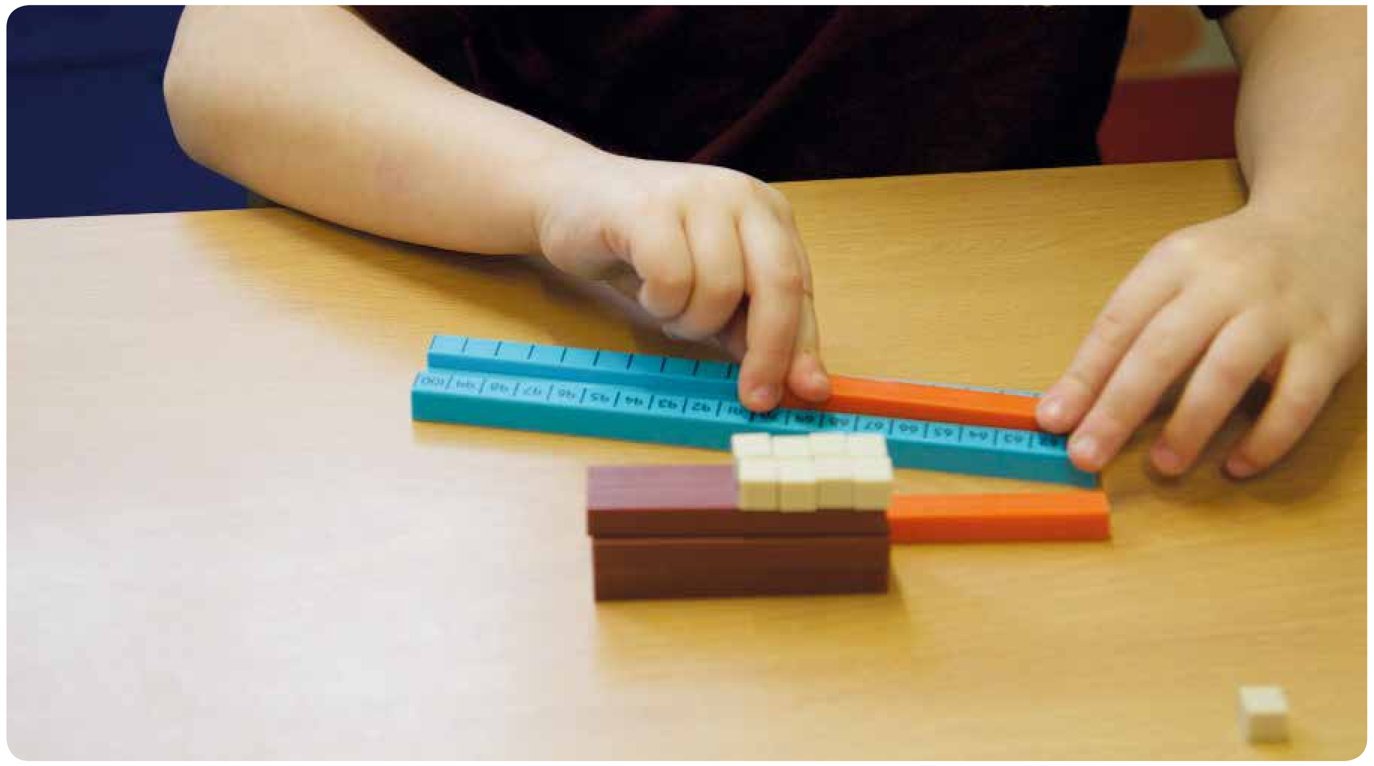
- providing smaller containers of equipment for children working individually or in pairs
- using sorting trays with separate compartments to allow children to access resources, such as numeral cards or pattern blocks, to save space and keep the resources organized
- storing paper resources which children may want to use, such as number lines and geo strips, by hanging them from hooks (punch a hole in the end, if necessary).

What writing or drawing might children do in mathematics if the activities are mainly practical?

Writing and drawing are aspects of communicating mathematically, and children's chosen forms of this communication may be varied and idiosyncratic. They might involve: drawing a measurement in pictures – for example the number of 'weights', e.g. pencils, equal to a particular mass, arranged in Numicon Shape patterns next to the object weighed; drawing a number story in pictures or by drawing around Numicon Shapes; drawing repeating patterns; or showing sorting of objects or shapes by making all those of a particular type the same colour.

Giving children the choice of how to communicate their ideas can therefore provide useful insights into how they are approaching problems, whether they are working systematically and how they are using conventional notation. It also helps children to formulate, clarify and develop their thinking, and to think of writing and drawing as part of doing mathematics (rather than in themselves *being* mathematics).

Accordingly, opportunities for children to communicate on paper are highlighted in the activities wherever this serves



a useful purpose. Some activity sheets are provided as photocopy masters, but in general it is recommended that children write and draw in their maths exercise books. This creates a useful bank of evidence to show how children's mathematical communicating develops over time.

The Explorer Progress Book (see page 10) also provides an extremely useful source of evidence for children's progress.

What about grouping children?

Children work well in pairs within larger groups of four or six. Working with a partner supports their confidence, giving them plenty of opportunities to discuss their work and solve problems together.

Schools vary in their criteria for grouping children. When grouping children of five and six years old, it is important to bear in mind that they will be at very different levels of development and will therefore require different levels of challenge. Some schools find an advantage in having mixed-ability groups comprising pairs of children working at slightly different levels; others group together children who are working at a similar level.

Whatever the policy is in your school, bear in mind that it is important to vary groups from time to time and to ensure that children do not always work with the same partner. Some children will work faster than others and have more developed ideas. It is important to make sure that they have opportunities to work with other children of similar ability. As you follow the suggested teaching programme for the first few weeks of term, you will discover which children work well together and their levels of understanding.

How do I prepare for teaching mathematics lessons using Numicon?

Understanding the mathematics yourself

Before teaching an activity group, read the relevant sections from the Key mathematical ideas sections (pages 44–68) to prepare for your teaching. If you want to do more research on an area of mathematics, you may wish to consult other sources, for example Derek Haylock's *Mathematics Explained for Primary Teachers* (4th edition, 2010).

Next, consider what generalizing there is in the activity group. For example, 'Is this where children could notice that a rectangle might be either a square or an oblong?' Or, 'Is this where children notice that the size of an object doesn't tell you if it is heavy or not?'

The children will be new to this, so another way to work on generalizations is to ask, 'What patterns in the work children are doing will they have to notice in order to progress?'

When children notice things, be prepared to keep asking: 'Will that always work?' 'What if those quantities were different?' 'Would that work with a different shape?' 'Will that ever work?' 'When does that work?' 'What never works?'

Appreciating the contexts

The educational context on the introductory page for each activity group will help you to see how the ideas involved fit into the continuum of children's learning about geometry, measurement and statistics.

After reading this educational context, consider when this learning may be useful. Children don't just need to know

how to do this mathematics, they need to know *when* to do it as well. How can you help them spot when this general mathematics applies to a particular situation?

Think about the kinds of contexts offered in the activity group, and ask yourself whether the mathematics is useful in particular kinds of real-world situations, or whether it helps with doing other mathematics. It can be helpful to think up one or two contexts of your own, so that it is clear what the point of doing this mathematics is.

Selecting and adapting activities

You know the children in your class and the materials and spaces which are available to you. You will be best placed to select which activities are most appropriate and adapt them creatively to suit the needs of individual children.

Read all of the activities in an activity group and identify what each activity contributes overall, as well as the resources and preparation involved. Then try the activities. Some might be 'revision' for your children, others might involve ideas, illustrations or techniques that are completely new. Some activities will be invaluable practice. You might think an activity will be too easy or too difficult for some children, so think about how you might adjust the level of challenge. Be flexible and adapt what is available for your children in the light of what they can already do.

It's also worth considering that challenge is normal; when children get 'stuck', they should be encouraged to communicate. Make sure they have available all the actions, imagery and language they need to communicate the challenge they are facing effectively.

How do I plan in the medium- and long-term using the Geometry, Measurement and Statistics 1 teaching programme?

The plan-teach-review cycle applies to Numicon, just as it applies to all effective mathematics teaching. Four important features of Numicon support this cycle:

Firstly the Numicon teaching programme – the recommended order of the activity groups – is structured progressively. A chart showing the programme appears in the long- and medium-term planning sections of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*; an editable version is also available in the Planning and Assessment Support.

Secondly practice and discussion activities are included in each activity group, for individual, paired and group work.

Thirdly accurate assessment is enabled through children's practical work with physical resources and imagery, and their mathematical communicating in conversation and on paper. This assessment will, in turn, help with planning.

Finally, 'using and applying' does not need to be planned separately. This is partly because all activity groups are based around problems to be solved, and partly because the cumulative nature of the programme means that children are using their earlier learning every time they face new ideas.

The teaching programme for Geometry, Measurement and Statistics 1 is arranged into two strands, Geometry and Measurement, with a series of activity groups within each. Both include some early coverage of Statistics, in the

form of simple data handling and recording in tables, and opportunities to create pictograms.

The long-term plan on page 17 of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook* shows the recommended order of the activity groups. It has been carefully designed to scaffold children's understanding, so that they are able to meet the challenges of each new idea. For example, children would not be expected to distinguish a 'clockwise' from an 'anticlockwise' turn before having recall of how a clock's hands move.

It should be noted that this structure has been designed together with the progression of the *Numicon Number, Pattern and Calculating 1* teaching programme. The long-term plan included on the *Numicon Planning and Assessment Support* contains suggestions for integrating the *Geometry, Measurement and Statistics 1* activity groups with those from the *Number, Pattern and Calculating 1 Teaching Resource Handbook*.

The medium-term plan on pages 18–22 of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook* gives expected coverage over the course of the year and also lists the activities and learning opportunities for each group.

You may decide to follow the long- and medium-term plans as they stand. You may also find that you need to split some of the larger activity groups and return to them later.

There are summary charts showing learning opportunities for each activity group in the long- and medium-term planning section of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook* and in the Planning and Assessment Support. You may find these useful for incorporating Numicon activities into your existing mathematics plans, should you decide not to follow the Numicon long-term plan for teaching the activity groups.

The parts and structure of each activity group are highlighted in the key to the activity groups on pages 8 and 9 of the Teaching Resource Handbook.

Each activity group begins with a 'low-threshold' focus activity designed to encourage and support confidence and ensure that all children are included. The remaining focus activities are designed to help children progressively develop their ideas around the mathematical theme of the activity group. The focus activities are designed for whole-class or group teaching. Some may be taught quite quickly to the whole class as an introduction to be explored later with a focus group. Opportunities for reasoning through challenging questions and problems are provided throughout.

Ensure that activities are differentiated where necessary so that all children who should be working independently can do so. Include activities that enable children to become more confident; encourage them to work more effectively and speedily through practising and celebrating what they are able to do. As you decide which activity to allocate to a particular group, remember to check that there is scope for children to take the activity further. You can increase the challenge by asking more challenging questions, either with specific groups or when the class comes together for the final part of the lesson. Your questions will depend on what you have noticed the children doing and saying as they work.

How long should I spend on each activity group?

The Numicon teaching activities for Geometry, Measurement and Statistics 1 primarily address the shape, position and measurement areas of the curriculum, along with some aspects of number.

For children who are new to Numicon, you may need to allow one or two weeks for making connections between Numicon Shapes and Shape patterns, number rods, number names and numerals. (For example, through the activities in the 'Securing Foundations' activity groups in the *Number, Pattern and Calculating 1 Teaching Resource Handbook*.)

There are 11 activity groups within the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*. Some of the activity groups contain more material than can be covered in one week: you might choose to work right through the activity group, have a break while you teach another topic, or even make selections from the activity group and integrate the material with work on other areas of mathematics. Returning to finish a group after a week or so has the advantage of reminding children about the ideas they have met previously, and gives you a useful opportunity to review what they have remembered. Alternatively, you may prefer to revisit or cover some ideas with whole-class practice at different times during the day, in particular during the 'morning maths meeting'. You will also find that children sometimes move very quickly through the work, and you will be able to combine two or sometimes three activities in one focus teaching session.

What about differentiating activities?

It is important for children of all abilities that activities provide appropriate levels of challenge.

The educational context and learning opportunities on the introductory page of each activity group give you an overview of the ideas children will meet and the learning to be built upon. If your assessments tell you that some children are not yet ready for the activity group, you can look back through earlier activity groups in the same strand to find appropriate activities. If the number content is too challenging, it may be appropriate to differentiate by, for example, adjusting the number range used in the activities; alternatively, consider whether it is best to revisit activities later, once children are more secure in the required knowledge.

Each activity group starts with a 'low-threshold' activity designed to be accessible to all children. In a mixed-age class you may need to modify the work for younger children and assess how they respond. You may decide some children are ready to go straight to more challenging activities, later in the activity group.

The open-ended nature of the activities and the emphasis on mathematical thinking means that there is always room for children to take activities further. For the highest-achieving children, you may decide to increase the challenge by planning specific questions that extend the reach of the activity. Encourage them to 'get stuck' and work through problems for themselves, by exploring and refining ways of communicating the ideas involved. You might, for example, challenge them to create their own problems for others to solve.

How can I support children to develop fluency and flexibility?

Each activity group includes suggestions for whole-class and independent practice and discussion to help children develop fluent understanding of the ideas they are meeting and to look at situations in different ways, considering a variety of approaches. You can select from these to give children appropriately differentiated opportunities that will help them to develop confidence.

The Explore More Copymasters provide further opportunities for children to practise and discuss at home the ideas they have been working on at school. The 'morning maths meeting' also provides an excellent opportunity for practice and discussion.

How can I support children to maintain fluency and flexibility?

Children's responses to questions and problems and ability to give examples or make up their own questions in the 'morning maths meeting' or other practice sessions will indicate whether they are maintaining fluency with past learning and thinking flexibly.

You might choose an activity from the practice section of a completed activity group, vary the context and present it to children without preparing them in advance – notice what they do and do not seem to remember, and plan accordingly for the next related activity group. You can also use questions and activities from previous activity groups to help keep children's past learning and creative thinking 'simmering'.

Using the activity groups

The first page of each activity group is clearly coloured according to the **strand** it appears in (Geometry – green, Measurement – purple; early statistics work is introduced within these strands through appropriate contexts). The title and the numbering of the activity group allow you to easily identify the content of the activity group and how far through the strand you are.

The **key mathematical ideas** clearly highlight the important ideas children will be meeting within each activity group.

The **assessment opportunities** signal key information to 'look and listen for' that indicate how much of the focus activities children have understood.

The **educational context** gives a clear outline of the content covered in the activity group, for example how it builds on children's prior learning; how it connects with other activity groups; the foundation it establishes for children's future learning.

All activity groups have been extensively trialled in the classroom, so the **learning opportunities** come from real classroom experiences. They are designed to help children develop their understanding of the key ideas of an activity group.

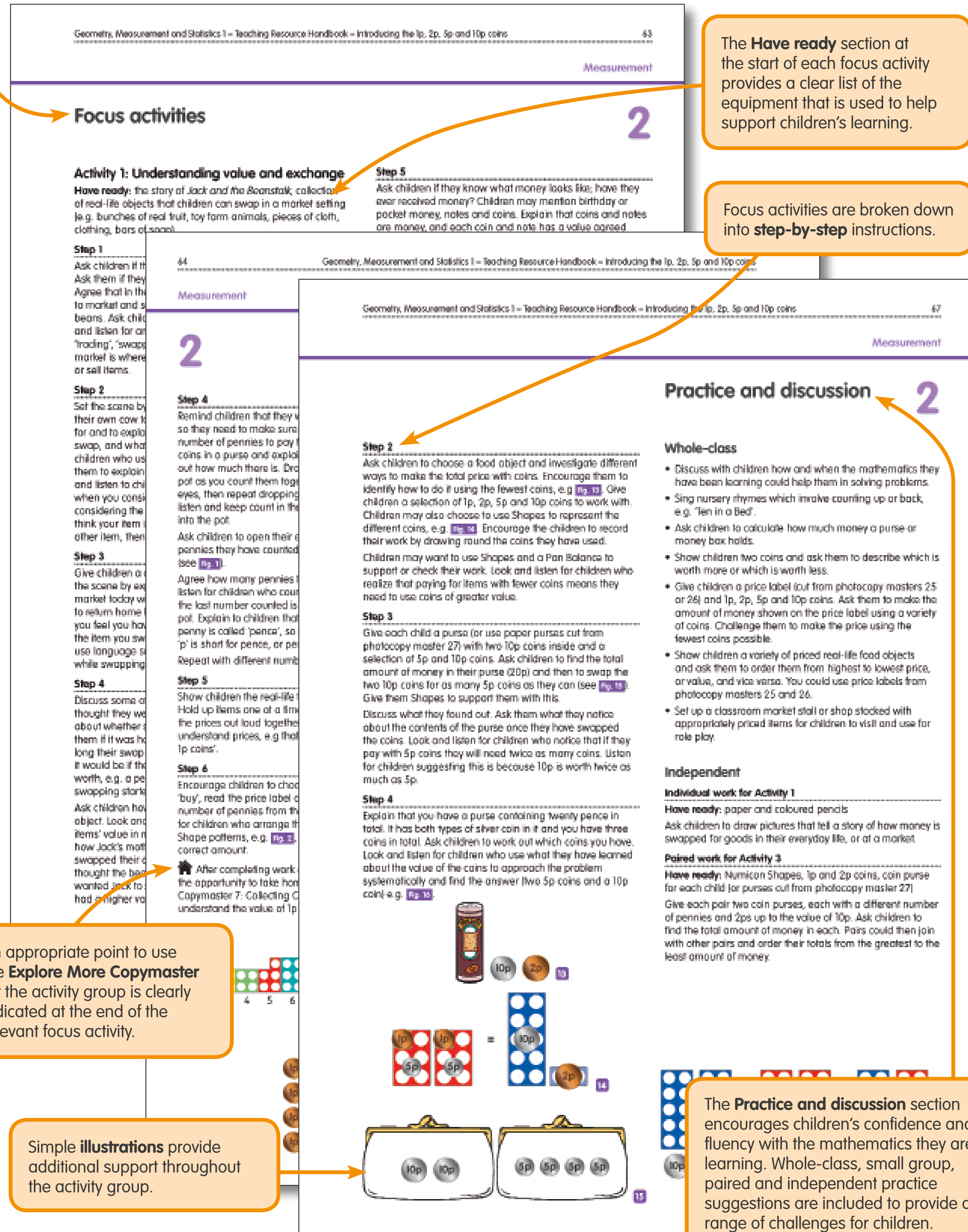
Explore More Copymasters provide an opportunity for children to practise the mathematics from the activity group outside the classroom through fun, engaging activities.

Clear links are made to the **Explorer Progress Book**. This book provides an invaluable chance to see children's thinking, monitor their progress and assess how much of the activity group they have understood.

Each activity group includes several **focus activities**, each clearly titled to show the specific learning points addressed by the activity. The first focus activity is a 'low threshold' activity allowing all children to engage with the work. Focus activities then build progressively to a 'high ceiling' that provides challenge and allows for differentiation within the same activity group.

The **Have ready** section at the start of each focus activity provides a clear list of the equipment that is used to help support children's learning.

Focus activities are broken down into **step-by-step** instructions.



An appropriate point to use the **Explore More Copymaster** for the activity group is clearly indicated at the end of the relevant focus activity.

Simple **illustrations** provide additional support throughout the activity group.

The **Practice and discussion** section encourages children's confidence and fluency with the mathematics they are learning. Whole-class, small group, paired and independent practice suggestions are included to provide a range of challenges for children.

Planning and assessment cycle

This summary shows how planning is informed by your assessments of children's understanding:

1. Choose an activity group	Before embarking on the activity group, review the educational context, consider your initial assessments of children and decide whether there is a need to revise any ideas children have met earlier.
2. Choose a focus activity	If this is the first lesson using the activity group, start with an early 'low-threshold' activity to include all children. Before the lesson, check the learning and assessment opportunities, brief teaching assistants and prepare resources.
3. Choose the practice activities	**Independent groups:** Refer to your assessment notes and allocate suitable practice activities (found at the end of the activity group). **Focus teaching groups:** Refer to your assessment notes and the learning and assessment opportunities from the activity group and allocate focus activities.
4. Plenary session (normally during and at the end of lessons)	Think about the important ideas that children meet in the lesson, particularly any generalizations that you want children to make. Plan questions to prompt discussion and encourage children to reflect on ideas they may have learned. Refer to the practice section of the activity group to find suggestions for whole-class practice.
5. After the lesson	Reflect on how children have responded in the lesson and note any significant steps on your assessment records. Use what you notice to determine the plan for the next lesson. The whole-class practice suggestions will also help children develop the ideas they have learned in the lesson. At some point after children have completed the activity group, ask them to complete the relevant pages of their Explorer Progress Book. This will allow you to assess how well they have retained the information they have been learning. It will also give you a chance to see how well they are able to apply this knowledge when faced with a 'non-routine' problem.

Creating short-term plans

	Warm-up	Main teaching focus	Focused group work with the class teacher or teaching assistant	Independent work	Plenary
Activity number/title	Select activities from the **Whole-class practice** section. This could be from a previous activity group to review and refresh children's previous learning.	Select one of the focus activities from the activity group, matched to the needs of the children. Place the **activity number/title** of the chosen focus activity in your short-term plan.	Decide whether to: • select the next **activity number/ title** from the focus activities in the activity group. Place this in your short-term plan; or • consolidate the activity covered in the main teaching focus.	Decide whether to: • choose activities from the **Independent practice** section for groups, pairs or individual children. Make notes on your plan or work from the Teaching Resource Handbook; or • select a focus activity for groups to work on independently. Place the relevant **activity number/ title** in your short-term plan.	Encourage children to have a reflective conversation to draw together what has been learned in the lesson. Select activities from the **Whole-class practice** section.
Learning opportunities	Place the selected learning opportunities, from the chosen activity group summary in your short-term plan.				
Notes and educational context	Decide whether to: • use the activity directly from your Numicon *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*; or • draw on the Teaching Resource Handbook to make your own notes for teaching the activity.	Decide whether to: • use the focus activity from your Numicon *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*; or • draw on the Teaching Resource Handbook to make your own notes for teaching the focus activity.	Decide whether to: • use the focus activity from your Numicon *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*; or • draw on the Teaching Resource Handbook to make your own notes for teaching the focus activity. If working with a teaching assistant, you may want to select the relevant **Educational context,** from the chosen activity group.	Decide whether to: • use the practice or focus activity from your Numicon *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*; or • draw on the Teaching Resource Handbook to make your own notes for teaching the focus activity.	
Words and terms	Decide which **words and terms** you will use in **conversation**. Place these in your short-term plan.				
Resources	Prepare any resources you may need for the activity. Use the **have ready** section at the beginning of the focus and practice activities.				
Assessment opportunities	Select from the chosen activity group summary the **assessment opportunities** that you and the teaching assistant will be looking and listening for in the different parts of the lesson. Place these in your short-term plan. Remember to note whether children know when to use their understanding.				

How can I assess children's progress?

Assessing mathematics using Numicon involves making judgements about developments in children's mathematical communicating – both receptive and expressive. You need to know which are the key developments to look for: check the assessment opportunities given on the introductory page of each activity group and consider how the achievements listed would show up in children's mathematical communicating. Look for developments in children's actions (what they do and notice), the imagery they use and respond to, and listen for their use of and responses to words and symbols in their conversation.

It is also important to notice children's fluency; when their communicating is stilted, when it is punctuated by gaps and hesitations, and when it flows consistently and well, suggesting a strong understanding of well-established ideas and the connections between them.

Assessment should be as open as possible so that children can communicate as much as possible. It is through their mathematical communicating that you will gain a real insight into how children are thinking. This will enable you to make the most accurate assessment of their progress.

Specific challenges for the purposes of assessing are provided in the form of the Explorer Progress Book (see page 10). Children cannot pass or fail these assessment tasks – they simply respond in their own way. How they approach the tasks informs you about their mathematical communicating and gives you an opportunity to 'see' their thinking through the imagery they use. This insight makes it easier to gather meaningful and accurate evidence of where children are. Preparing for formal test situations is something different, and is addressed on page 42.

Specific indications of children's progress

Each activity group lists several assessment opportunities that point to key achievements to look and listen for as children work on the activities. All of these achievements will be observable in children's actions, imagery and conversation as they progress.

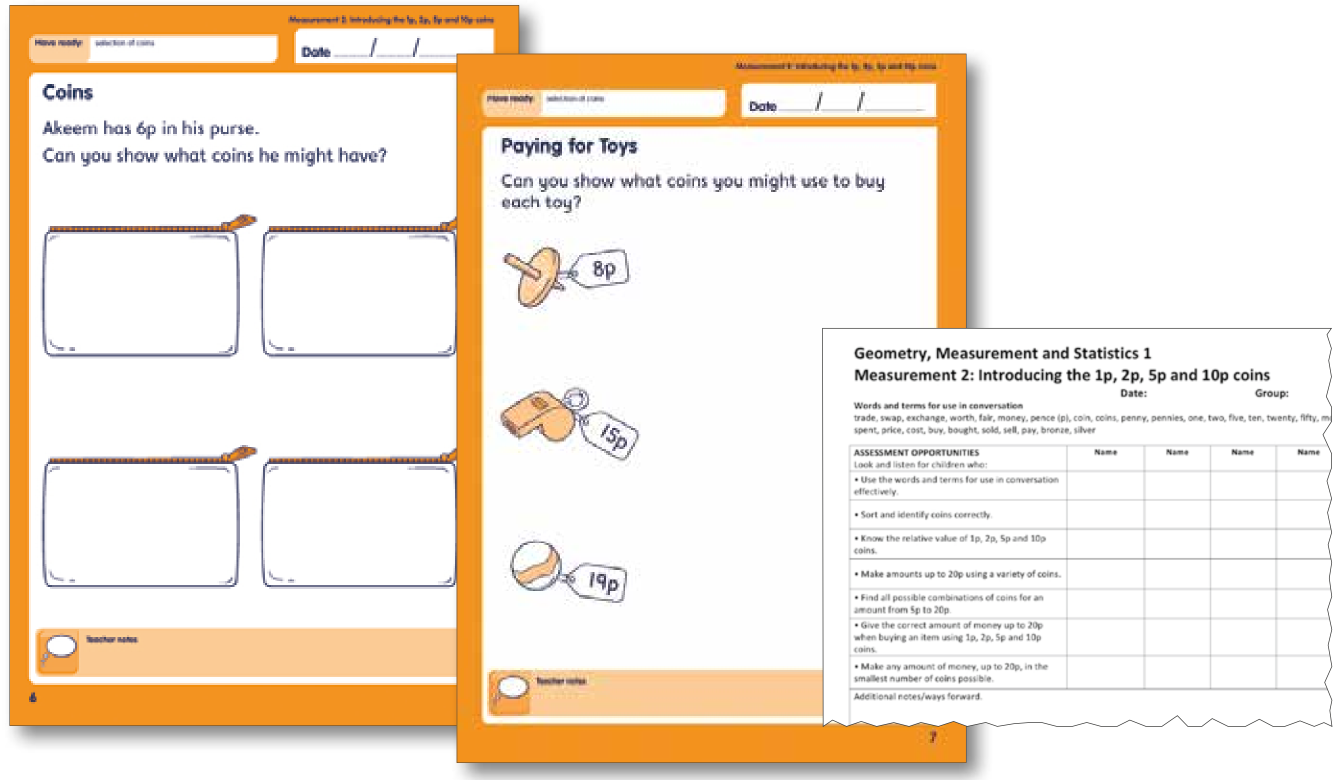
Familiarize yourself with the assessment opportunities before you begin teaching an activity group. Use them to help guide your interactions with children, and also as indicators of progress and sources of information to help you group children and plan your teaching as you move on to further activity groups.

Suggestions for what to 'look and listen' for are given within each activity. Focus on children's communicating and ask yourself whether they know both how to do the mathematics they are learning and when to use it.

You will also find that how children use physical resources gives an insight into their thinking. If a child is using pattern blocks or weights by trial and error and giving a muddled explanation, this would suggest they don't yet understand the activity. Plan to revisit it, focusing on careful use of mathematical language and imagery as well as their actions.

Children self-correcting – that is, working by trial and improvement rather than simply by trial and error – suggests their understanding is developing. Give them time to experiment and practise the activity and encourage them to discuss their ideas.

Children communicating clearly about what they have done, whether with apparatus, in conversation or on paper, suggests they have gained a solid understanding. Plan, then, to move them on.



What about summative assessing?

Assessment milestones and tracking children's progress

The medium-term plan in the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook* includes milestones – summary statements of specific points that children need to have a good understanding of before they move on to the next set of activity groups.

The milestones are based on the assessment opportunities in the preceding activity groups and are also aligned to the national curriculum in England (2014). Your ongoing assessment of each child will build up over the preceding period and you can keep a record of attainment and track progress using the photocopy master of the milestones for the year on pages 95 and 96 of the Teaching Resource Handbook.

Each milestone represents a point at which to reflect on each child's achievement and decide whether you need to plan further support and practice for them, giving them time to consolidate their understanding, or whether they are ready to move on. If children are moved on before they are ready their difficulties are likely to be compounded, because they will not be adequately prepared for the new ideas they meet.

Explorer Progress Book

Each activity group has two corresponding pages in the Explorer Progress Book. The first page presents children with opportunities to use the mathematics they have been learning in the activity group. The second provides more open tasks, allowing children to further apply their learning.

The Explorer Progress Book is designed to be tackled in focus groups, so that you can administer and monitor each child's responses. In this way, you are able to build up a cumulative idea of a child's progress. The tasks enable you to assess children's ability to think mathematically and persist in their work, as well as whether they understand when to use particular mathematics techniques. They are as open as possible, inviting a full range of responses. They are not pass or fail tests, rather they are there to support you in assessing as accurately as possible children's current understanding, so that you know what needs addressing. It may be useful to keep notes on children's responses and what you see as their significance for future work.

Consider carefully when to give children each Explorer Progress Book task. You might ask them to complete one page at the end of their work on the activity group and then the other two weeks later, to check how much they may have retained. Alternatively, you might give children both pages after they have completed the next activity group, or just before they face the next related activity group. The aim is to gather information about children's understanding at a point when this information is useful for their learning – decide which is the most useful point, in each case.

Children should have available to them all the materials and imagery that have been available during the teaching of the activity group, and should be invited to express what they are doing as they do it. It is best to avoid affirming or denying anything a child says or does as they work. Look and listen for what children can do without your guidance.

What about formal testing for national authorities?

Formal tests and examinations are important hurdles for children and teachers, parents and carers, schools, universities, professionals, employers and governments. They also tend to be artificial settings in which to 'do' mathematics. In this sense a formal test does not correspond to children's encounters with mathematics in their learning or everyday lives, and this means devoting time to preparing them for the uniqueness of the experience.

In a formal mathematics test, communicating is almost always restricted to 'on-paper' forms; this allows for some imagery, but not usually for action with physical materials. Also, the language used in test papers can be very formal. Thus children will need plenty of practice at interpreting such language and 'internalizing' their use of action and imagery. The development of mental imagery is a key aspect of Numicon, and children should be encouraged to 'imagine' actions, objects, movements and shapes as often as possible.

Children will also need to prepare for encountering 'difficulty' during formal tests. In their mathematics lessons, they are encouraged to express difficulty – to explain why something is challenging and to use action and imagery to illustrate their thinking. Under exam conditions they will need to respond positively to being 'stuck' by communicating mathematically with themselves, working silently to express what the trouble is and using mental imagery to explore possible solutions.

Tests and examinations should not become the paradigm for 'doing mathematics', however. Children need to learn to function mathematically in a very wide range of situations. For ongoing assessment of children's understanding, allow them the full range of actions, imagery and conversation, and encourage them to communicate mathematically in their own ways.

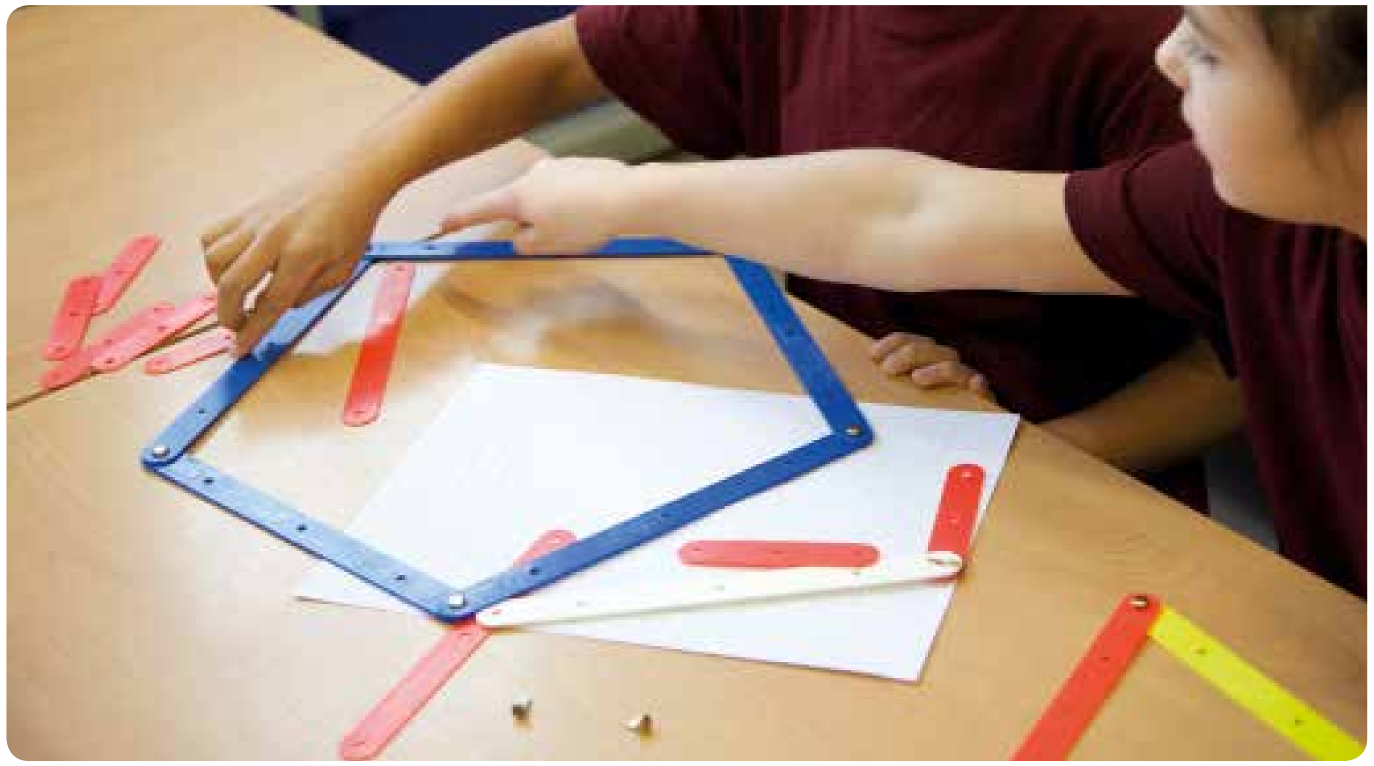
Key mathematical ideas: Geometry in the primary years

Underlying the activities in Geometry, Measurement and Statistics 1 are many key mathematical ideas that children will be developing and extending, as well as some conventions they may be meeting for the first time.

In order to teach these ideas effectively, those who are working on the activities with children will need to be clear themselves about the mathematical ideas and about which activities address which ideas.

The following section includes a brief outline of the key mathematical ideas that children will be encountering in the Geometry activity groups of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*. The introductory page for each activity group lists the key mathematical ideas associated with each activity. As you prepare for your teaching, you may find it helpful to remind yourself about the key ideas behind each activity by referring to this section.

The mathematics coordinator may also find it useful to work on the key mathematical ideas in professional development sessions with the class teachers and the wider school staff.



Introduction

It is important to remember that doing geometry, measurement, and using statistics are significantly related aspects of mathematics, and that each of these areas of activity has strong connections with the other two. It is important to remember too that all three also have strong connections with number, pattern, and calculating activity.

'Geometry' in ancient Greek means 'earth measurement'; many physical measures involve geometrical ideas (e.g. pressure is usually measured per 'square' unit), and patterns in data (themselves usually records of measuring activity) are commonly represented visually – as are numbers themselves, of course, in the form of number lines.

Each of these areas of mathematical activity also has its own distinctive concerns: shapes are commonly classified with little reference to their size; we measure time, force, and temperature as well as distance, area, and volume, and in statistics we are often concerned to interpret and illustrate a 'significance' in identified relationships. In what follows we treat these three broad areas of mathematical activity relatively separately for ease of reference to their key ideas, but when you are teaching each area be sure to exploit inter-connections at every opportunity.

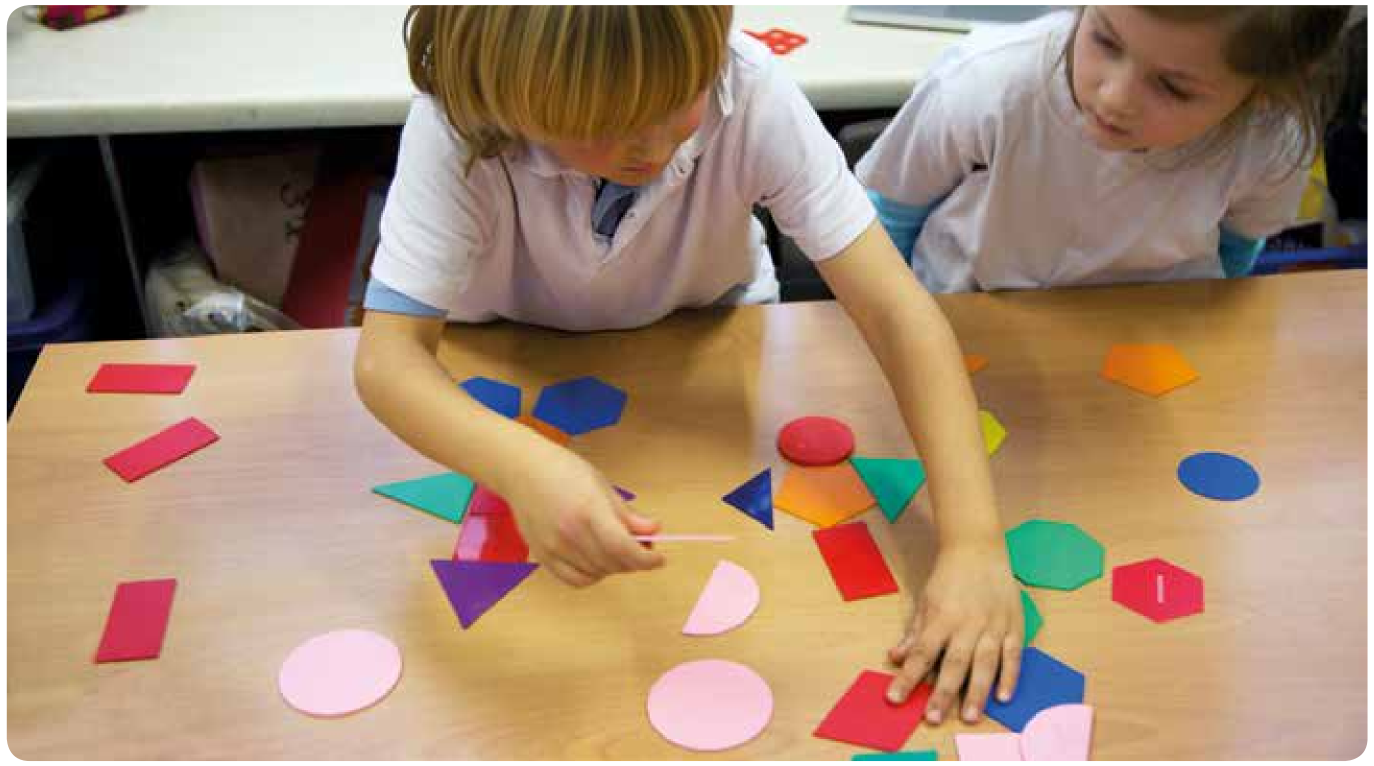
Geometry in the primary years

Children begin exploring shape and space literally as soon as they are born. In fact it was Piaget's view that young children's first experiences of the physical world are closely related to the central ideas of a relatively recent geometry called topology. (In topology, key ideas include 'proximity', 'enclosure', and 'inter-connectedness', and it is easy to understand why such things are so important to infants from their earliest days[1].)

In order to engage children in doing geometry at school, they need first-hand experiences with shapes and with position, direction, and movement, within which shared opportunities arise both to describe actions, movements and observations freely and to introduce conventional mathematical names and terms. The point of using mathematically conventional language is to help develop 'a common way of seeing' a world with children, a common way of *communicating* about a geometrical world, so that we can share our individual perceptions and experiences with each other more easily. In the early years, children spend much of their time learning to join in with established geometrical conventions (ways of seeing and describing), as well as developing their individual 'seeing eye'; these two aspects need to proceed together, in close relation to each other.

The importance of first-hand practical experiences for children is two-fold: young children do not yet think about shape and space in the same ways that adults do, and for communicating to become effective in any context there

1 If you want an example of topology in adult use, think about a map of the London Underground and about why such ideas are the important ones in this context.



has to be a shared underlying experience that all involved can build their communicating upon. (It is very difficult to communicate effectively about, say, a film that only one of you has seen; similarly, it is very difficult to *discuss* with children things that only you have experienced). Children should be physically active and thoughtfully reflective in their geometry activities, and fully involved in *discussion* in all activities. It is only children's *agreement* to join in with doing mathematics and its established social conventions that will open up its possibilities to them; children asked to use terms they can't relate to their personal experience – however important we might think those terms are – will find themselves easily forgetting those words.

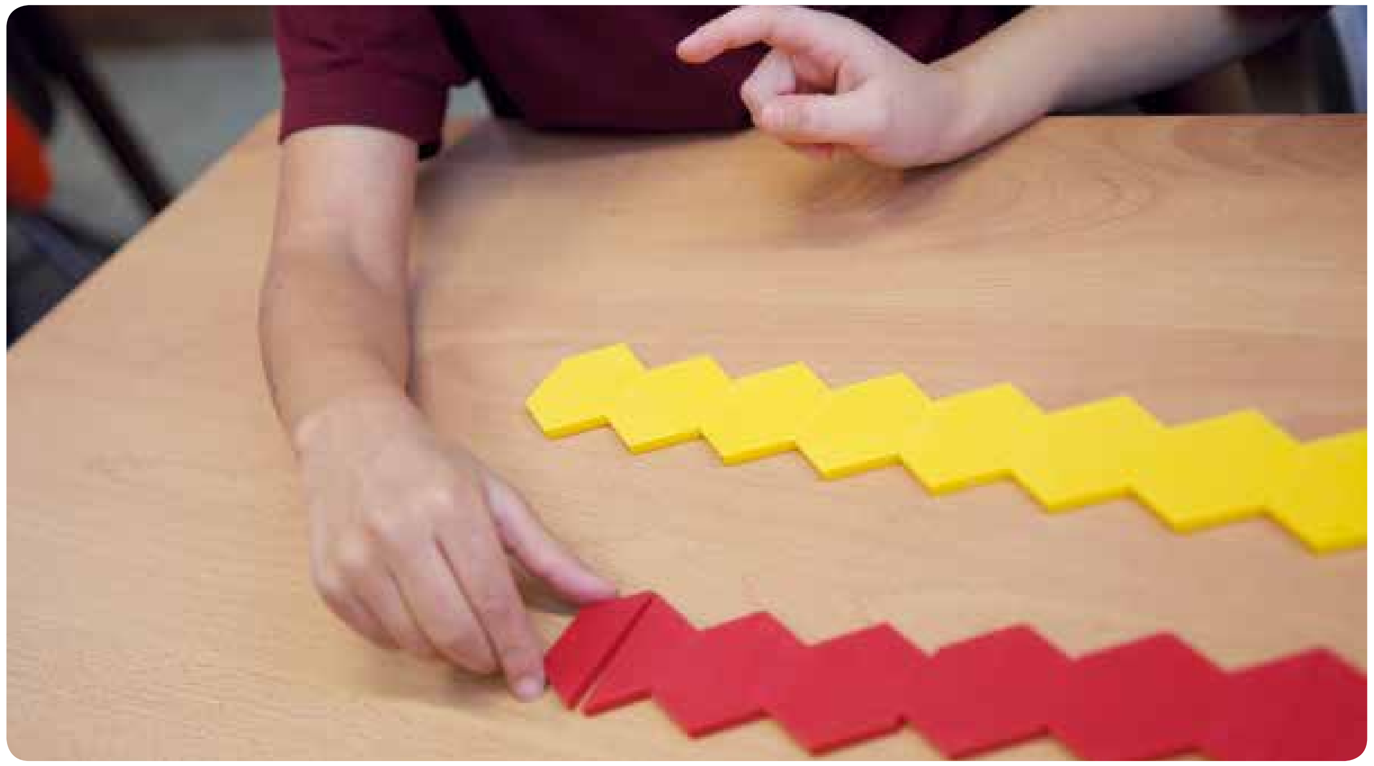
Bear in mind also that in learning to do geometry in school, children are encountering some very old conventions traditionally used in classifying 'shapes' and describing 'position', the origins of which often lie quite literally thousands of years ago. It is quite easy for children sometimes to feel that they have to learn a strange foreign language to talk about abstract shapes in school, the point of which (at the time) can seem obscure. As with medicine and the law though, many of the terms we use today in geometry have Greek and/or Latin origins and make good descriptive sense in those ancient languages, e.g. 'tri-angle' meaning three angles. Today, using words like 'polygon' and 'quadrilateral' can seem a bit odd and rather fussy, but if children are introduced to the Latin and Greek roots of these terms (e.g. 'poly' meaning 'many', and 'quad' meaning 'four') they quickly get used to connecting the sense of these old words with what they are used to describe, and this deeper level of reading can help develop their geometrical thinking and communicating significantly.

Doing geometry – transformations, invariants and equivalence

An important general point concerns the history of geometry and consequently what we tend to emphasize today in schooling. For many hundreds of years after the Greeks and Romans there was effectively only one kind of geometry used in western civilization – that of Euclid (fl. 300 BCE). But during the 19th century CE, mathematicians' attention began to focus on connecting new and ever more varied kinds of geometry with each other.

One key connecting idea turned out to be no longer thinking of geometry as simply 'earth measurement', but instead as the study of 'invariants under transformations': people began to think of geometry as studying what changes, and what stays the same, as various transformations (e.g. 'rotating', 'reflecting' or 'translating') are performed on shapes and space.

With the introduction of this idea, Euclid's traditional geometry has today become a concern with studying what stays the same (invariant) as we transform shapes. For example, a square will still have equal sides and four right-angles, equal diagonals, the same area and so on, however it is rotated, reflected, or moved about. Whereas topology is concerned with what stays the same under what are called 'rubber-sheet' transformations – imagine what happens to shapes drawn on a rubber sheet as the sheet is pulled and stretched in any ways that you like; a 'square' when stretched about on rubber will still form a continuous boundary (thus keep an 'inside' and an 'outside') so we would say that 'enclosure' is invariant, and points that are close together originally will stay relatively close together, so we say that 'proximity' is also an invariant under topological transformations. Actual



measured distances and angles all change when shapes are stretched and pulled, and so they are not invariants.

Those topological invariants are the only ones that matter to a traveller on an underground system, and also in young infants' spatial worlds. Studying 'invariants' under different kinds of transformations turns out to be a very useful way of understanding shape and space, and this informs children's geometrical activity in school today: essentially we want children to *explore* 'What happens if we *do* this ...?'

This in itself requires a necessarily *active* approach to doing geometry. We want children to be dynamically making shapes, and moving shapes (and themselves), and noticing and discussing what changes and what stays the same. 'Transforming' involves action, as does moving between and among 'positions', and *discussing* all this action is what allows children gradually to join in with the language and conventions of doing the geometry that we use in mathematics today. Gradually too, children will begin to learn how we *reason* about aspects of shapes and space in doing mathematics.

Equivalence

Just as children need to learn about equivalence in number work and algebra (e.g. $2 \times 3 = 6$, and $a + b = b + a$), so they will meet important equivalence relations in geometry. In primary school geometry, equivalence judgements are typically made in relation to specified transformations. Most commonly, children will learn that two shapes are 'the same as' each other if they could be rotated, reflected, and/or translated onto each other; two such shapes are called **congruent** to each other. If two shapes could be rotated, reflected, translated, and/or *scaled up or down* onto each other they are said to be **similar**.

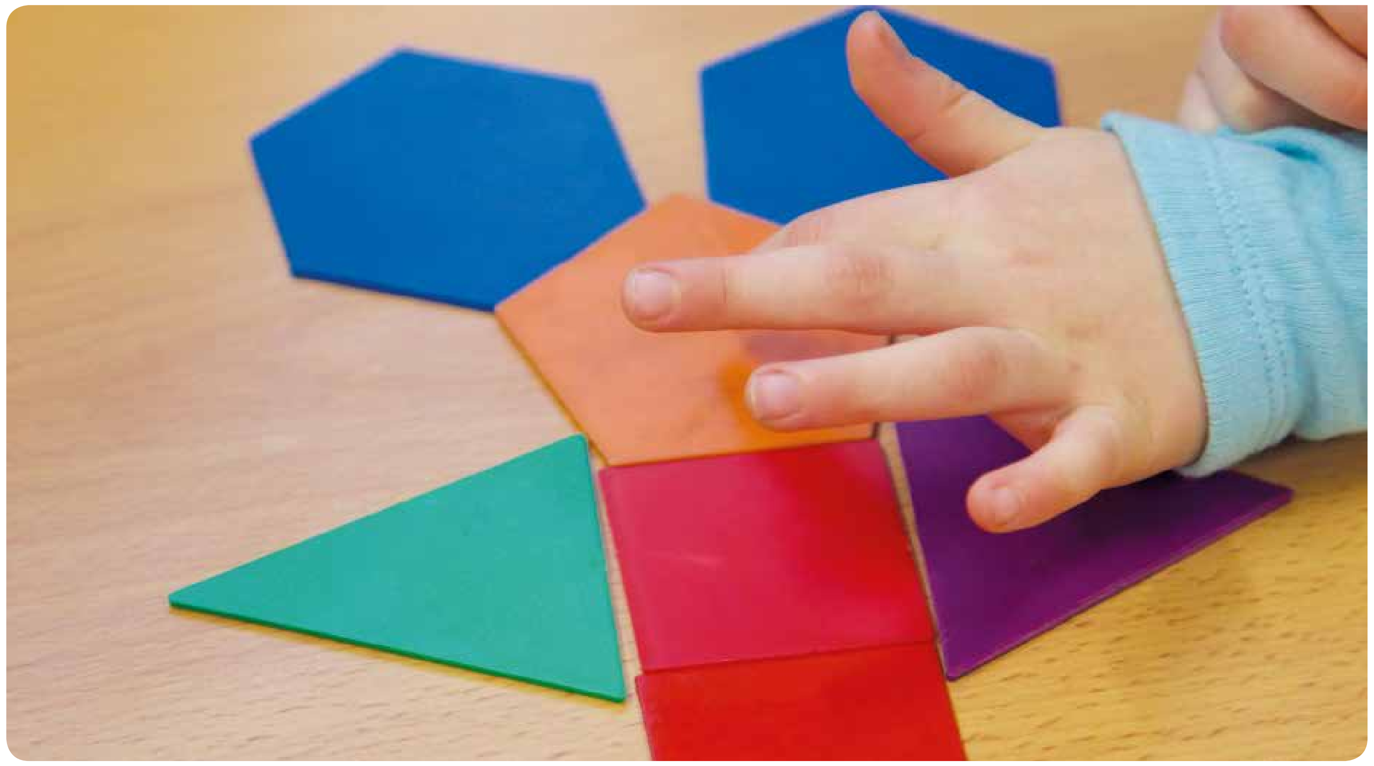
Importantly, transformations can be performed one after the other, and some sequences of transformations are *equivalent* to each other in that shapes 'end up in the same state/place' after them. For example, a rotation of 180° about a point, followed by another rotation of 180° about that point, is *equivalent to* one single rotation of 360° about the point. Investigating sequences of transformations and their equivalences explicitly is a significant part of school geometry, though children will have already begun to explore this in their very early pre-school play with their simple handling of shapes (e.g. fitting different shapes into differently-shaped holes).

Doing geometry – being logical

As in doing any kind of mathematics, when we do geometry we reason by making and using **generalizations**. Indeed, this is one key aspect that distinguishes doing geometry from measuring; when we measure, we are always measuring something specific, something particular.

Being logical in doing mathematics usually involves using what is called deductive logic[2]; and *deducing* something involves moving from a general statement to a particular one. For example, knowing that 'the exterior angles of any polygon add up to 360°' allows us to deduce that the exterior angles of a triangle will add up to 360° because a triangle is a polygon. By using generalizations in various logical ways in mathematics, we can be sure that our reasoning is reliable.

2 There is a form of reasoning used in mathematics called 'proof by induction' because it moves from particular relations to a general conclusion about a whole sequence of relations. This form of proof nevertheless also relies upon at least one initial generalization about the sequence involved, for its validity.



As another example, we know that we could not try to tile a flat surface with regular pentagonal tiles because each interior angle of a regular pentagon is 108°, and there is no way you can fit a whole number of 108° angles together to make a 360° complete intersection (that is, leaving no gaps), because 360 ÷ 108 = 3·33, which is not a whole number. This is a **logical** argument: if the exterior angles of a regular pentagon add up to 360° (because it also is a polygon), then each exterior angle must be 72° (because 360 ÷ 5 = 72). If each exterior angle is 72°, then each interior angle must be 108° (because an interior and an exterior angle added together make a straight line, that is, 180°, and 180 – 72 = 108).

We have no choice about accepting reasoning like this; if our generalizations are valid our conclusion is *necessarily* correct. Notice in the preceding paragraph just how much this reasoning depends upon using our *generalizations* (including numbers themselves), *logic* and ***definitions*** (e.g. of exterior angles). Doing geometry, that is, reasoning about shape and space relationships, depends crucially upon such logic, our definitions and generalizations.

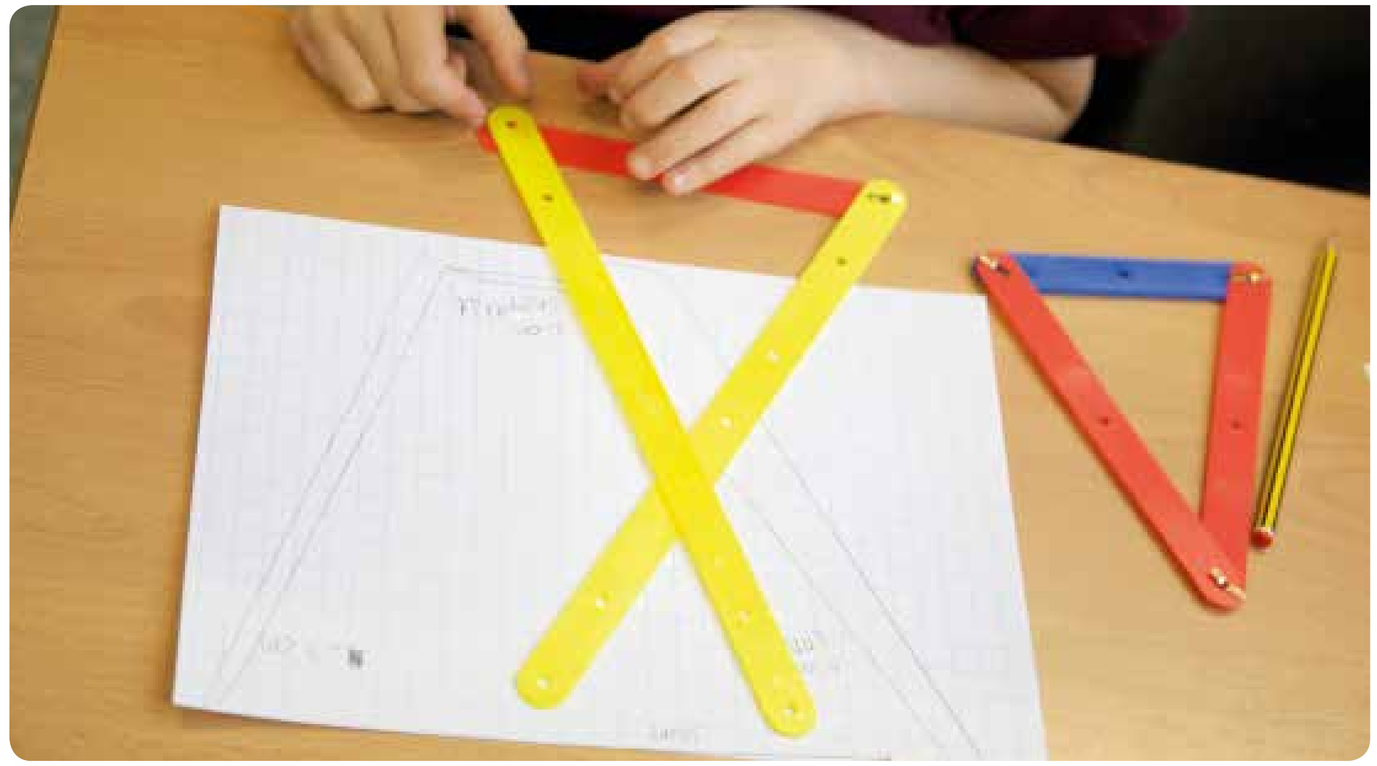
It is worth noting too, that though we may make, move and draw physical shapes on paper to help us to think, our logical reasoning is about shapes *in general*. Importantly, we can only contemplate such general shapes in our heads, that is, we can draw a particular triangle of particular dimensions physically but not a 'general' triangle. The general 'triangle' we reason about in geometry is *imagined*, and hence no measuring is involved.

Of course, as Piaget noted, young children are generally not yet capable of reasoning for themselves with the kind of 'formal operational' logic described above, but in primary school we can do much to prepare the way for children's mature geometrical thinking, and for their understanding of what 'doing geometry' involves. We can encourage children to *imagine* shapes and movements, we can encourage them to *generalize*, and we can encourage them to notice that there are significantly different ways of being 'correct' in mathematics, through ourselves always being careful how we answer the question, '***Why*** …?' We can also teach children what doing geometry is *not*.

What geometrical reasoning is *not*

It is quite common for children to want to test a generalization such as 'the angles of any (flat) triangle add up to 180°', by drawing or choosing a particular triangle and measuring the angles with a protractor (or tearing off the corners of a particular paper triangle and re-arranging them in a line). There's possibly a feeling that lots of people could have done this in the past, and that every time anyone does it, they always come up with 180° – roughly. And so, if we could measure accurately enough, measuring lots of different kinds of particular triangles would eventually *prove* it.

This kind of activity is misleading for children in an important way. Working like this is how scientists work – by dealing with series of individual cases rather than reasoning about a *general* triangle, as mathematicians do. If all we did in geometry was measure actual particular triangles, however many triangles we measured practically – even assuming we could measure anything perfectly accurately (which we can't) – there would always be the possibility that in a triangle we haven't looked at yet, the angles might come to more or less than 180° – because we have no special *reason* for knowing it to be impossible.



In mathematics we reason *logically* about our generalizations using our imagination, and children will find that it is possible to reason logically that the angles of a flat triangle *must* add up to 180°, without doing any measuring at all[3]. And further, that if by measuring we come up with 181°, then the extra 1° is a measure of the *inaccuracy* inherent in our measuring; it is our measuring that is wrong.

As teachers, we don't help children understand what doing geometry is about if we encourage them to believe that the angles of a triangle really do add up to 180° because they've measured some triangles, or because they've torn the corners off one and arranged all three angles physically along a straight line. *Convincing them* is not the aim; *finding logical reasons* is.

Being logical is not to be confused with being conventional

Just as children need to learn the difference between measuring and being logical, they need to learn the difference between being conventionally 'correct' and being logically 'correct', as well. Being conventional is a matter of social agreement; being logical leads to a *necessary* acceptance of truth (it's not a matter of any kind of agreement).

The fact that three-sided polygons are called 'triangles' is a *social* agreement (they might have been called 'trilaterals'); that the exterior angles of any polygon add up to 360° (that is, one whole revolution) is *necessarily* true – there is no choice about it.

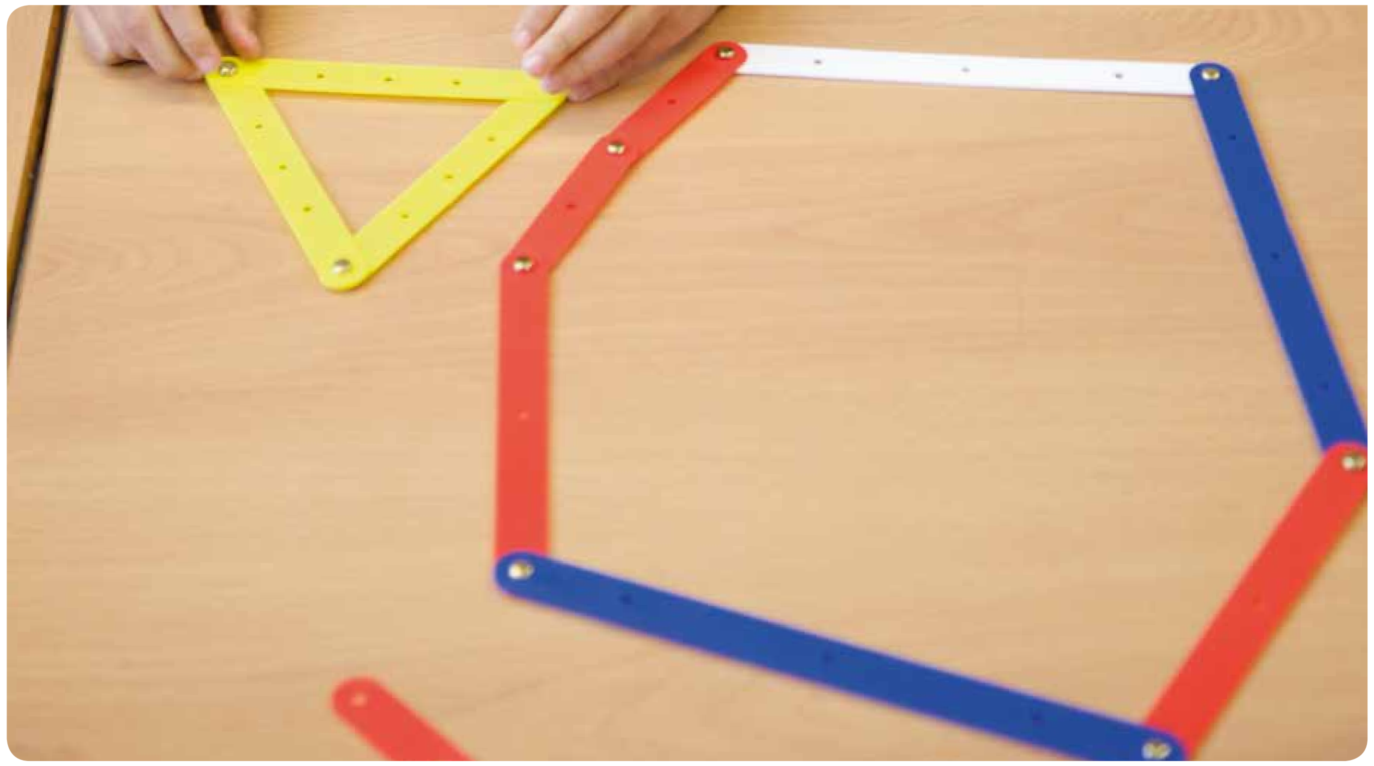
Have you tried drawing 'any polygon' yet, by the way? It is a two-dimensional closed shape, consisting of straight sides…[4]

We learn conventional names for things in geometry in the same way that we learn conventional names for things in any other walk of life, simply by agreeing to go along with what everyone else seems to call things. This makes our communicating 'correct' in a social sense, but not in a logical or necessary sense.

We can help children learn this important distinction through the ways that we answer their questions. This in turn will help children learn that there are different *kinds* of 'facts' in geometry, and therefore importantly different ways of being 'correct'.

3 The exterior angles of any triangle add up to 360° (because it is a polygon). Also, the combined interior and exterior angles at each vertex of a triangle add up to 180° (they make a straight line), so the total of all six exterior and interior angles together for any triangle will be 540° (i.e. 3 × 180°). Since we know that the exterior angles together account for 360° of this total, then the total for the interior angles together must be 180° (i.e. 540° – 360°).

4 Trying to draw a *general* polygon illustrates very well how mathematical objects are works of the imagination. We can define it, describe several of its visual properties, and develop valid theories about it, but *draw* it we cannot.



Because …

When children ask 'Why …?', teachers need to make sure they give the right kind of answer. If a child asks, 'Why are there 360° in a whole turn?' we need to say something like, 'Well, the Babylonians used to love numbers like 60 that you can divide up exactly without using fractions, so they thought 360 (6 x 60) was a really helpful number for dividing up a circle'. (Or something like that.) In other words, it's a convention and one can point to a social history …

If a child asks, 'Why do the angles of a quadrilateral add up to 360°?' we need to invite them to *reason* it through. 'Because by drawing a diagonal you can divide 'any quadrilateral' (a generalization again, in your head) into two triangles, and we know that the angles of a triangle always add up to 180°, so 2 × 180° = 360°.' It's *logical*; we have no choice.

If a child asks (pointing to a shape), 'Why is that a triangle?' we need to be careful to give a full answer. 'It's called a triangle because we've agreed to put *all* shapes like that one (all those flat, closed shapes with three straight sides) together into one group, and call them all 'tri-angles' – probably because they all have three angles as well.' It's an *agreement* that could have been otherwise.

And so as children work with and explore shapes and movements, and speculate and try things out, they will find themselves both 'correct' and 'wrong' in significantly different kinds of ways. The ways we that use the word 'because …' will help them develop important distinctions. Children need to know *in which way* they are 'right' or 'wrong', and that in the world of shapes, as with people, 'a rose by any other name would smell as sweet'.

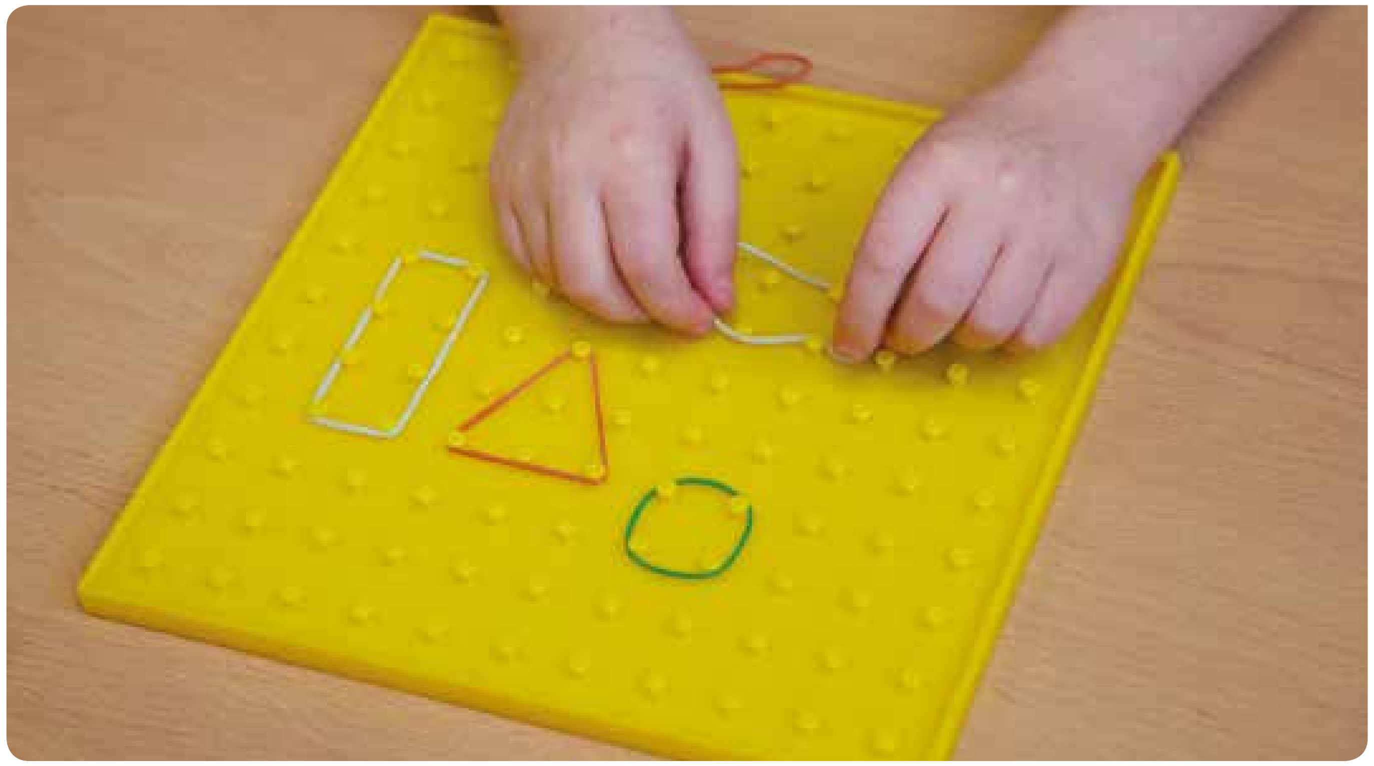
Parts, properties, movements and definitions – the social basis of geometry

Definitions are social agreements in the same way as the names of the categories they define. We first *agree to define* categories in a particular way, and then later use those categories in our classifying and reasoning. Doing mathematics is the shared occupation of a social community, and within mathematics definitions are *chosen* (and hence allowed to change).

We tend to use parts and properties that we distinguish within shapes and movements to define our conventional categories, and so early work with children involves inviting them first to agree to our *distinctions between* certain parts, properties and movements before doing any categorizing based on these. None of this yet involves logical necessity; it is still only about inviting social agreement.

Initially children seem to distinguish between shapes that are named for them in a *holistic* way, simply learning, '*That's* (called) a triangle' and '*That's* (called) a square' and so on, without distinguishing any parts[5]. Gradually their discrimination becomes finer and they are able to distinguish **parts** such as 'corners' and 'lines' that they can later agree to call 'vertices' and 'sides' and so on. Later still children will begin to notice how parts relate to each other to give shapes **properties**, e.g. a trapezium has just two sides that are parallel to each other, or adjacent sides of a rectangle meet at 90°. Children also learn to distinguish **movements** from one another, e.g. between turning a shape around (rotating), and turning it over (reflecting).

5 See Van Hiele, P. M. (1985) *Structure and Insight. A Theory of Mathematics Education.* Orlando: Orlando Academic Press



So as young children make and handle shapes, fit them together, and move them (and themselves) around, they gradually learn to *join in* with the conventional language we offer them to describe the distinctions they see and notice with us. We invite their agreement to joining in with the language that reflects the distinctions and definitions the rest of us currently use; all of this is social activity – and the agreements could always be otherwise[6].

Using transformations and invariants to name and define

By playing with, handling, and fitting together collections of physical shapes children will be using the Euclidean transformations of rotation, reflection, and translation. This is a world in which lengths, angles, and areas all stay the same (invariant, constant), and are therefore features *constantly available to be noticed* as shapes are moved around and fitted together.

By including similar physical shapes as well as congruent ones in the collection, the transformation of **scaling** can be imagined. (Note the transformation of scaling affects a previous invariant: area.)

Under the transformations of rotation, reflection, translation, and scaling together, the angles of a square stay the same – they are invariant – as are the **ratios** of lengths to each other. Paying attention to just these invariants leads to a 'square' being defined in our geometry as a polygon with *four right-angles* and *four equal sides*. These invariants can be used to define the shape. As the actual lengths of the equal sides and the area all change under scaling, we don't use the changing attributes of individual lengths or area, which vary, in defining a square.

Triangles are a different challenge to organize – there are many kinds of triangle, whereas there is only one kind of square. Children need to make and meet many different kinds of triangle before they can realize[7] again that what stays invariant in any kind of triangle under rotation, reflection, translation, and scaling are the angles and the ratios of sides. They will notice that no one seems to care how long individual sides are, or how big the angles: as long as there are three of those things making this closed, flat shape everyone calls a 'triangle'.

Significantly, noticing what stays the same and what changes as we transform something leads naturally to generalizing. In effect, children are learning to use terms such as 'triangle', 'square', 'oblong', 'circle' and so on in ways that acknowledge that, for example, 'a triangle' (that is, *any* triangle) is a closed shape with three straight sides forming three angles. These are the *invariants* of 'a triangle' in the geometry we begin with at school; any other properties

6 To help children realize this, one very nice activity is to invite them to make as many different pentagonal shapes as they can, and then to sort these into different kinds of pentagon themselves - also inventing names for the categories they make. Since there is no conventional tradition for this (as there are for triangles and for quadrilaterals), the experience can illustrate very well to children the fact that names, categories and definitions of shapes are all simply social agreements – which could always be otherwise. (Thank you David Fielker, (1981) *Removing the Shackles of Euclid* Derby: ATM.)

7 They are unlikely to be realizing this consciously, or be capable of expressing what they see in these terms. What they are *noticing* – informally – is that the 'corners' don't change however you move the shape about, and that it still *looks* as if it's the same shape (i.e. it's what we call 'similar') whenever we do those things with it.

that you might notice when you look at a particular triangle (e.g. this one's got a right angle) could *change*, yet the shape would still be a triangle. Similarly, it doesn't matter 'which way up' a triangle is (that is, how it is rotated), it will still be a triangle. Orientation can *change*; 'having a point at the top' is not an invariant property of triangles in the geometry we use.

It is clear how physical experiences with a wide range of *dynamic* materials are essential for children doing geometry. The active making, moving, describing, transforming and combining of physical shapes that children do with physical and IT materials forms the vital basis of their flexible *imagining* and forming of categories.

Gradually, through meeting, making and moving shapes in their activities and discussing what they and we see changing and staying the same, children learn to use words (category names) in the same ways that we do. These distinctions and agreed categories then become formalized into definitions.

Definitions essentially spell out explicitly the agreed boundaries around a category; they are a kind of *verbal contract* underlying discussion that is always open to revision and scrutiny. 'Triangles' become defined as, 'closed, flat shapes having exactly three straight sides' for as long as it suits us to look at them that way[8].

In our **Glossary** we list the definitions that are usually agreed for work in primary schools. Notice how important your visual imagination is to you as you read these agreements, and remind yourself just how important children's physical experiences with actions and movements are to them, as you discuss and invite them to agree with the conventions you suggest.

From defining towards classifying and relating

It is one thing to be able to distinguish and name shapes (and parts and properties of shapes), but it is quite another to be able to relate such categories to each other and to reason with such relations. Being clear about categories is fundamental to thinking logically with them.

In schools in some countries much early discussion with children focuses explicitly on 'different *kinds* of things' (or categories), relations between categories, and hierarchies of categories, so that children's attention is consciously drawn to the importance of category distinctions for logical thinking. This becomes important with children doing geometry as we introduce them to relationships between categories of shapes and the ways in which inclusive hierarchies of categories lead to the possibilities of reasoning with ever broader generalizations[9]. Putting things into categories and relating those categories to each other is called *classifying*, and classifying shapes is one key way in which we put a conventional order and structure onto the many possible worlds of geometries.

Sometimes category distinctions are very subtle. *Logically* we can't add different *kinds* of things together; '3 m 15 cm' is not 18 anythings, it's *either* 315 cm or 3.15 m. Children will encounter this logical feature again when they try adding

8 If we move to another kind of geometry, perhaps with shapes on curved surfaces, we might want to change our definition of 'triangle'.

9 E.g. polygon → quadrilateral → rectangle → square, is one such inclusive hierarchy of categories, enabling us to *deduce* logically that the sum of the exterior angles of a square will add up to 360°, for example, because those of *all* polygons do, and a square is one kind of polygon.

together, for example, $\frac{1}{4}$ and $\frac{2}{3}$. Conventionally however, we often say '3 metres 15 centimetres' as if we've 'added' them together, whereas we've simply put them *next to* each other, one after the other, not combined them.

And so to generalizing in geometry …

In doing geometry, having agreed categories and definitions we then reason about mathematical objects and their relationships *in general*, such as 'a polygon', 'a quadrilateral' or 'a cylinder' or 'a sphere', and these generalized objects are similar kinds of things to those other mathematical objects we ask children to make – pure numbers, such as '6', or '23', or '−0·5'. As in number and algebra, in doing geometry we *reason* with our *generalizations* (whereas in measuring we're always measuring something specific, something particular).

What children importantly become better at as they work and develop through their primary years is the *virtual* action essential to generalizing – they gradually become able to reason about '*any* quadrilateral' and so on, and it is only possible to do this in our *imaginations*. Any quadrilateral that we actually make or draw is always a particular one; the important *general* quadrilateral that we want children to be able to reason about can only be constructed in our heads, and the same goes for all the other geometrical generalizations we want children to make. In this important sense, geometry can only be done in our heads.

Notice that it's more difficult to imagine a general triangle, or quadrilateral, or polygon, or prism, than it is to imagine a general square; that's because there are many different *kinds* of triangle, quadrilateral, and so on, but only one kind of square (in our conventional classification). Again in this, the variety and range of children's physical actions, and the maximum use of *dynamic* materials (such as geo strips) is essential; the more *transforming* children can do physically as they notice and discuss what changes and what stays the same, the better.

When doing practical geometry with children always encourage them towards generalizing with questions like, 'Will that *always* work …?', and 'What if …?'. And encourage their reasoning with questions like, 'Is that because …?', and 'Why is that, I wonder …?' And don't feel that some children are too young to be able to contemplate such questions; even before children are capable of answering them, the *habit of asking* such questions in one's teaching is what is important[10]. It is for the children to do the generalizing, the reasoning and the imagining.

Working in 2D and 3D

Essentially, working with shapes and space in three dimensions rather than two involves no change in approach. Work in 3D begins with children handling, rotating, reflecting, and translating shapes in various ways, and also noting that scaling only changes lengths, surface areas and volumes. A cube is a cube is a cube, however big or small it is. Angles are invariant under these four kinds of transformation, as are ratios of lengths. Fitting 3D shapes together is also a key explorative activity.

10 'The palace of reason has to be entered by the courtyard of habit.' See Peters, R. S. (1966) *Ethics and Education*. London: George, Allen & Unwin p314. For Peters, this was what he called 'the paradox of education'.

Some agreements on names are changed in a shift from 2D to 3D work, and some interpretations of transformations shift up a dimension. So for example, 'sides' become 'edges' in 3D, a 'rotation' would be around a line (instead of a point) and a 'reflection' would be around a plane (instead of around a line). Curved surfaces become notable in 3D, as opposed to curved lines in 2D. These are all conventional agreements we invite children to join in with, as their experiences in 3D invite new *discriminations between* parts and properties, prior to agreeing to use these discriminations in new definitions (see **Glossary**).

Once 3D categories are agreed (e.g. 'polyhedra', 'prisms'), children can begin to relate categories to each other, to classify, to *generalize* and to reason *logically* about aspects of three-dimensional space. Reasoning about the possibilities of tiling in two dimensions, for example, shifts up to reasoning about the possibilities of packing in three dimensions.

Activities directly connecting 2D and 3D parts, properties, transformations and shapes are invaluable. So in the early years, printing with 3D shapes offers important connections between 2D shapes and 3D 'faces', and later on work on the 'nets' of 3D shapes does the same.

Essentially, work in 3D involves the same developmental sequence of activities as work in 2D: *discriminating between* parts, properties, and transformations in order to agree conventional *definitions*; *generalizing* from such definitions; *imagining* 'a (general) cylinder', 'a prism' and so on, and thence to *reasoning logically* about such mathematical objects.

Walking the line … a difference of perspective

Finally, much geometry activity is done from the point of view of 'looking *at*' shapes, and moving them around in front of us and so on. It is quite often very helpful however to shift perspective and to imagine walking around, along, and inside shapes. Young children do this readily and actually in climbing frames and playground equipment, but often, in school, activities are done on tables with materials in their hands, which almost exclusively involves 'looking *at*'.

Programmable robots and the programming language 'Logo', however, allow us to explore shape properties, and position and direction in ways that invoke the 'walking along and inside shapes' perspective to good effect. And a shift in perspective often makes different things clear, and familiar things look helpfully different.

When children are 'looking *at*' polygons, for example, the only angles that are obvious are the interior angles. When they try making a polygon with a Logo turtle however, all of a sudden it's the unwritten exterior angles that become important – and the 'total turtle trip' experience travelling around such a shape allows children to appreciate how *all* the exterior angles *must* add up to 360°, in a very different and immediate way.

Encourage children to shift perspectives frequently, and thus to unite experience of 'position' and 'direction' closely with what are often thought of as the more 'static' aspects of shape.

Doing geometry – using Cartesian coordinates

René Descartes (1596–1650) is generally credited with inventing a system for describing a position in space using axes and a set of numbers, called coordinates. This revolutionary idea subsequently allowed algebra and geometry to become united so that equations could be interpreted visually, and shapes could be defined and explored with algebraic equations.

Doing geometry with coordinates - uniting geometry with algebra – is usually called 'analytic geometry', with René Descartes credited as its father. Analytic geometry is developed systematically in secondary school mathematics, but children begin to meet this approach with us as they are introduced to Cartesian coordinates in Geometry, Measurement and Statistics 4.

Geometry in Geometry, Measurement and Statistics 1

Both artists and mathematicians pay attention to shapes, and lines, and forms in the physical world. When teaching geometry though, it helps to remember that artists and mathematicians pay attention in different ways. Mathematicians' primary concern in geometry is to render a physical world predictable, and so the shapes and lines and forms selected for early mathematical attention are those that are potentially *predictable*.

Straight lines are predictable, as are regular curves; we know where we are going if we 'walk along' them. Although when teaching art we might discuss the varied forms and outlines of clouds with children, mathematicians save the complexity of weather forecasting for post-graduate study (and still have much to do on that). Geometry in school is restricted to the study of **parts** and **properties** of only those shapes that are formed with straight lines and/or regular curves.

So in Geometry, Measurement and Statistics 1 children are invited to be active in various ways with simple, predictable and common 2D and 3D shapes. Three aspects are important at any primary level though: the **transformations** involved, the **invariants**, and the associated **communicating** that is developed in discussion.

The transformations

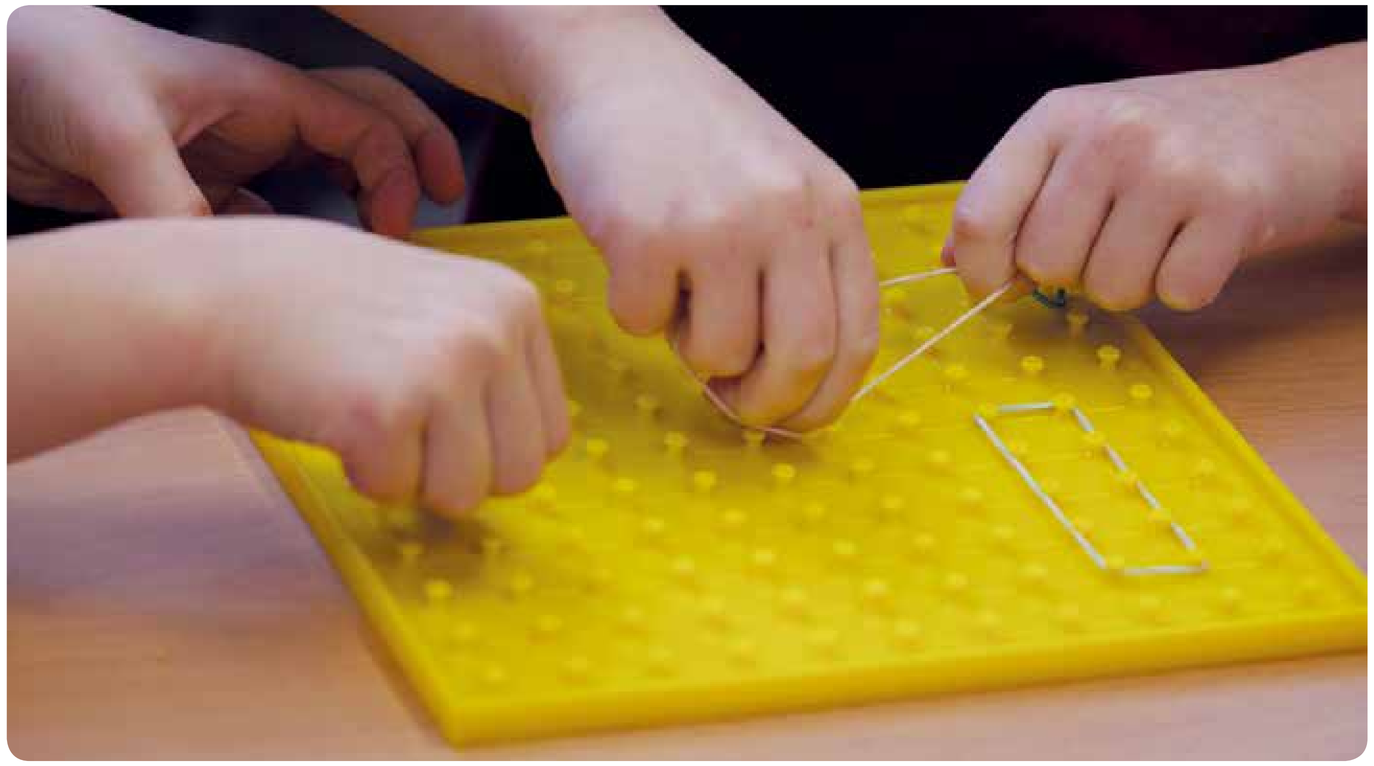
The four kinds of transformation children use in activities at this level are: rotations, translations, reflections and scaling. At this stage these are discussed intuitively and informally with children as they make and handle a variety of regular and irregular shapes.

Rotation: children are invited to turn shapes 'around', 'upside down' and so on – and to turn themselves around – to experience what changes, and what stays the same, as *orientation* alters.

Translation: children are invited to move both shapes and themselves to, from, and between various positions, and in various directions.

Reflection: although children are not invited to think about and explore line symmetry explicitly, they may well 'turn shapes over' in the course of making patterns and so on and thus 'reflect' them in practice.

Scaling: children work with shapes in various sizes (that is, shapes that are similar to each other) and thus have opportunities to notice what changes, and what stays the same, as only 'size' changes.



The invariants

The parts and properties of shapes that we invite children to notice particularly at this level are *invariants* under the above four transformations.

At this stage children are not expected to be able to say things like 'the angles stay the same' under all these transformations, but they would be expected to be able to *sort shapes successfully* into categories such as 'triangles', 'squares', 'oblongs', 'cubes', 'pyramids' and so on, regardless of their size and orientation. They are very unlikely to be able to explain *why* a pyramid is still a pyramid if it is upside down (or twice as big as this other one), but they can in a practical, intuitive way show *in their sorting* that they are able to discriminate between different 'shapes' on the basis of those parts and properties that 'stay the same' as we scale up or down, rotate, translate, or reflect in a broad, holistic way.

Distances, angles (and consequently areas and volumes) are invariant under rotations, reflections and translations. Under scaling, angles and ratios of lines (and consequently things like parallel lines) stay the same.

The communicating

Discussion needs to be relatively open and informal as children work at this early level, introducing conventional terms as and when children seek to *discriminate* between, describe, explain and name what they see and do. Children shouldn't have 'correct' (that is, conventional) names and terms imposed on them at this stage; invite children to agree to use conventional vocabulary as it becomes useful to them in describing what they see.

In particular, agreeing conventional *categories* of shapes is important, and you will need to remember that, e.g. the category of 'triangles' is much more complicated to distinguish for children than that of 'squares' (there are many different kinds of triangle, yet only one kind of square).

The conventional category 'rectangles' is also especially problematic, since technically it includes the category of 'squares'. Young children often have difficulties with 'inclusion' relations between categories. It is perfectly acceptable for children to distinguish the category of 'oblongs'. **The combining of 'oblongs' with 'squares' to agree the inclusive category 'rectangles'** needs to be discussed fully with children, since for some the idea that a shape can have two different names is confusing ('How can that be both a square *and* a rectangle?'). Also, it is very common in everyday discussion for people to speak as if the term 'rectangle' meant only 'oblong'. Allow children plenty of time to absorb a technically-correct *inclusive* definition of 'rectangle'.

Please see individual activity groups (and **Glossary**) for lists of the particular conventional terms likely to be introduced during particular Geometry, Measurement and Statistics 1 activities.

Key mathematical ideas: Measurement and statistics in the primary years

Underlying the activities in Geometry, Measurement and Statistics 1 are many key mathematical ideas that children will be developing and extending, as well as some conventions they may be meeting for the first time.

In order to teach these ideas effectively, those who are working on the activities with children will need to be clear themselves about the mathematical ideas and about which activities address which ideas.

The following section includes a brief outline of the key mathematical ideas that children will be encountering in the Measurement and statistics activity groups of the *Geometry, Measurement and Statistics 1 Teaching Resource Handbook*. The introductory page for each activity group lists the key mathematical ideas associated with each activity. As you prepare for your teaching, you may find it helpful to remind yourself about the key ideas behind each activity by referring to this section.

The mathematics coordinator may also find it useful to work on the key mathematical ideas in professional development sessions with the class teachers and the wider school staff.

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Introduction

It is important to remember that doing geometry, measurement, and using statistics are significantly related aspects of mathematics, and that each of these areas of activity has strong connections with the other two. It is important to remember too that all three also have strong connections with number, pattern, and calculating activity.

'Geometry' in ancient Greek means 'earth measurement'; many physical measures involve geometrical ideas (e.g. pressure is usually measured per 'square' unit), and patterns in data (themselves usually records of measuring activity) are commonly represented visually – as are numbers themselves, of course, in the form of a number line.

Each of these areas of mathematical activity also has its own distinctive concerns: shapes are commonly classified with little reference to their size; we measure time, force, and temperature as well as distance, area, and volume, and in statistics we are often concerned to interpret and illustrate a 'significance' in identified relationships. In what follows we treat these three broad areas of mathematical activity relatively separately for ease of reference to their key ideas, but when you are teaching each area be sure to exploit inter-connections at every opportunity.

Measurement and statistics in the primary years

We always measure for a *purpose*, that is, measuring something is never an end in itself. Because of this, children's experiences with measuring are most effective when set within purposeful contexts, for example when reviewing an arrangement of classroom furniture we could ask, 'I wonder if that bookcase would fit in that gap over there?' Similarly, measuring mass/weight and volume or capacity has a clear purpose when cooking, or in any other situation where we might want to be able to repeat something. Timing is important for organizing deadlines and appointments. Cross-curricular links generally are invaluable for offering great varieties of purpose to children's measuring activity.

There is another consequence of measuring being purposeful that can be overlooked in primary school activities: our measuring in life is always only as formal, or informal, as accurate, or approximate, as suits the particular measuring *purpose*, on any occasion. As adults, we do not usually measure our drink of coffee at home in millilitres, nor do teenagers measure how far apart to put their coats down in metres when making a 'goal' for a game in the park. Standard units do not become appropriate to measuring tasks simply because children are of a certain age, or because they appear from some point onwards in a curriculum; standard units are not somehow 'grown-up' or 'proper' measuring and should not be introduced as such.

Standard units are used only on particular occasions *for particular reasons*, usually to ensure clear communication, and/or when there is a lack of personal trust. When trying out a new recipe we usually measure the named amounts precisely, because we don't yet trust our own judgement in the new venture; standard measures are very important agreements in global trade, and in scientific communication – both contexts in which 'trust' depends crucially upon careful use of agreed units. We might not use standard measures for our drinks at home, but we certainly expect them in shops.

If children are to fully understand measuring, the aspect of *purpose* – and its consequences - needs to feature clearly in all work and discussions. Standard units are not the 'best' units, nor are they the 'most accurate'; they are simply the units most appropriate to particular occasions. Probably we do more measuring in everyday life without standard units; children need to recognize why they use the types of units they do when they are measuring in all types of situations. Standard units are necessary only for *communicating* and for supporting *trust*; children need to learn *when* standard units are appropriate, in context.

Measuring – contrasting and comparing

There is a sense in which all measuring can be understood as comparing, in that an underlying purpose in our measuring action is always to compare **qualities** or **quantities**. Interestingly, not all comparing involves measuring – sometimes we count, and sometimes we order, to compare (the number of votes in an election, or the winners of a race for example).

It is worth noting that we can't do any comparing until we have first distinguished whatever quality is to be compared. Thus contrasting qualities (distinguishing between them) is an essential prerequisite to comparing, and so sorting, and distinguishing between, in topics such as length and capacity, different kinds of 'big' and 'small', are crucial preparatory activities with young children.

Measuring and counting – continuous and discrete quantities

There is an important difference between the two questions, 'How much?' and 'How many?' To answer the first, we measure; to answer the second, we count. **We measure continuous amounts, but we count discrete objects.** We measure time, length, area, volume and so on, but we count votes, cars, and the number of items in our basket.

This distinction is important – it leads to the consequence that where measuring involves a continuous quantity, the outcome is always an *approximate* figure. And this observation itself has important implications for our use of continuous measuring scales and instruments, and for always having to decide on an appropriate level of accuracy in any situation. It also has very important connections with the work we do with children on fractions and interpreting the spaces between whole numbers on a number line.

There are a variety of questions that ask about 'how much' of something we have, and thus lead us to measure: 'how far' and 'how long' ask about length, or distance, or time; 'how heavy' asks about weight/mass. 'Where is …?' is a question that can invite a combination of measures, e.g. an angle and a distance. Volume, capacity and mass all tend to be asked about with the broad question itself, 'How much?' (e.g. 'How much sugar would you like in that?').

Children thus need to learn to use a wide variety of terms and language in association with purposeful measuring and comparing, including the usual grammatical distinction between having 'less' of something continuous, or having 'fewer' discrete things.

Measuring and types of scales

There are important distinctions between different types of measuring scales that have implications for the kinds of calculating we can do with measurements we have made, and thus for the types of statistics we can do with measures and counts of various kinds.

As a handy reference, in 1946[11] S. S. Stevens proposed a classification of measures that has proved very productive in stimulating debate, and that also connects usefully with our approaches to data handling and measuring. It is worth considering not only because of the connection with statistics, but because it explains why although 20 cm is 'twice as long' as 10 cm, 20 °C is not 'twice as hot' as 10 °C (that is, we can't do the same calculations with temperature readings on the Celsius scale that we can do with lengths). Stevens' classification is not put forward here as a 'correct' view of measuring, but simply as one that has an interesting and helpful relevance to the development of both measuring and statistics work with children.

As a psychologist, Stevens was much concerned with statistics, and his classification is thus concerned with both measuring and handling data in the contexts of physical and social science. In statistics, both measuring and counting are used in contrasting (that is, distinguishing between) and comparing.

The Stevens (1946) classification distinguishes four types of 'measuring' scale:

11 Stevens, S. S. (1946) *On the Theory of Scales of Measurement*, in Science, **103** (2684): 677–680

- **Nominal**: this involves simply distinguishing between (contrasting) and putting items into different categories, according to names or specific qualities, e.g. distinguishing between male/female, or distinguishing between English/French/Spanish, and so on, as different modern European languages. Distinguishing between team players by giving them different numbers on their shirts is a kind of nominal scale. Importantly, no ordering or value is involved or implied.
- **Ordinal**: an ordinal scale distinguishes between and also *ranks* items qualitatively, but without attending to any degree of difference between ranks, that is, such a scale simply compares qualities of objects or events by position in an order. The Beaufort scale of wind strength is one example, as is the Mohs scale of mineral hardness. Social surveys often use ordinal scales when asking people if they 'agree strongly', 'agree', 'don't mind', 'disagree', 'disagree strongly', and so on.
- **Interval**: an interval scale discriminates between values, and orders them, but also focuses on constant *degrees* of difference between values on the scale. Examples are temperatures in °C, or dates on an infinite timeline. The intervals on the scale – e.g. in degrees or years – are all the same as each other, so *differences between* temperatures or dates can be computed and added or subtracted (and ratios *of differences* make sense), but we can't sensibly 'add' or 'multiply' with dates or temperatures themselves. On 1 January 2010 we could say that 1 January 2000 was 'twice *as long ago*' as 1 January 2005 (that is, a ratio of differences between dates), but we cannot compare two dates (or temperatures) directly themselves with each other in any other way apart from simply ordering them and saying how far apart they are[12].
- **Ratio**: a ratio scale distinguishes 'difference', 'order', and 'degrees of difference', but also includes an important, non-negotiable 'zero' boundary that actually leads to different possibilities of calculating[13]. The classic examples of ratio scales include our physical measures of length, area, volume and mass wherein it makes no sense to think of 'negative' lengths, areas, and so on[14]. Importantly, this also means that differences between values on a ratio scale are also values themselves, and we can thus make sense of *ratios* of particular *values* to each other, e.g. a difference between two lengths is itself a length (whereas a difference between two dates on an interval scale, is not itself a 'date'). We can compare two different values on a ratio scale in *two* ways: we can say that 6 cm is 4 cm longer than 2 cm (their difference), but we can also say that 6 cm is three times as long as 2 cm (their ratio).

The above classification might seem a rather complicated background to work in primary schools, but it can actually help draw attention to several key basic ideas in measuring and comparing, and is highly relevant to the kinds of questions and data handling with which many primary children can engage.

12 This is actually because in practice there is no necessary, fixed 'zero' point on an interval scale, and both positive and negative values can potentially be thought of as 'going on forever' either side of an arbitrary 'zero'. In later physics, children will meet the Kelvin temperature scale with its 'absolute zero' and also the idea of a 'Big Bang' and 'zero time'.

13 Interestingly, it is the importance of 'starting from zero', e.g. when we measure length with a ruler, that children often do not appreciate immediately.

14 We will also leave discussion of negative values within quantum physics and the notion of 'anti-matter' for another occasion; we're just doing primary mathematics at the moment!

Notice that Stevens' classification is cumulative; first we pay attention only to *differences* between items, and we simply *distinguish between* various qualities. We can thus make categories. This 'nominal' scale relates closely to children's early sorting activities, to their early distinguishing of qualities such as 'heaviness', and 'volume' as refinements from their early global use of terms such as 'big' and 'small'.

Secondly, by using an 'ordinal' scale we still distinguish between items and qualities but we now *order* these categories or qualities as well. This relates to children's early comparing and ordering of lengths, heaviness, volumes, and so on.

Thirdly, the idea of repeated equal *units* and specified *values* on an 'interval' scale is introduced, and the combining of (adding) and finding differences between (subtracting) values becomes possible. This relates to children meeting the idea of measuring 'units', adding and comparing lengths and so on, and introduces the possibility of meaningful 'negative' values.

And finally, once scales that have all the previous properties *and* a fixed 'zero' boundary are introduced, *ratios* of values become meaningful comparisons. This relates to children progressing from 'additive' thinking to the crucial further possibilities of 'multiplicative' thinking in relation to measures.

Thus there is a close correlation between Stevens' proposed classification and the development of measuring and statistics with children. First we invite children simply to ***discriminate*** *between* various qualities (general sorting;

noting qualities such as heaviness, extension, heat and so on). Then we introduce the idea of ***ordering*** various degrees or levels of those qualities (***comparing***: e.g. longer than, shorter than and so on). Next we introduce *equal* ordered degrees of difference in qualities (that is, intervals or ***units***), and thus the possibility of adding and subtracting (comparing) differences, and *naming* individual amounts (***values*** on a scale). Finally, with ratio scales we introduce the full possibilities of comparing (adding and subtracting) values themselves and ***ratios*** of values with multiplicative thinking.

Together these activities and objects (in bold, above) constitute the key aspects of measuring in the primary years, and can be used as key focuses for attention both in teaching and in judging children's progress. These are, as it were, the *roots* of measuring activity. Notice how in teaching, these key focuses are approached in the same order as Stevens' cumulative classification of scales.

A special note about money

Stevens' classification has been valuable for the amount of productive debate it has engendered, but a weakness in it is possibly revealed when we ask which type of scale applies to money. In terms of *cash*, money is measured with a ratio scale; you can't have 'negative cash'. But of course you can *owe* money, and that makes money seem more like an interval scale; 'zero' money becomes a fairly arbitrary point when you're a student, and both debt and riches appear to stretch potentially endlessly in either direction. Some *ratio* comparisons still work: £10 is still 'twice as much as' £5, and owing £10 means owing 'twice as much as' owing £5, but 'how many times' as much money have you got if you *have* £10 as opposed to when you *owe* £10?

It is worth noting as well that money is not 'continuous' in the same sense that time, length, area, volume, mass and so on are considered to be. Thus *actual* amounts of money (like cash) are exact (not approximate), even though we see around us currency exchange rates quoted such as £1 = $1·51715.

Money is included as a 'measure' (of economic worth) in the curriculum, but the ways in which it is used and calculated are significantly different to other physical measures.

The physical measures

The physical measures introduced during primary schooling in England are: length, mass and weight, volume and capacity, time, temperature, area, and speed.

Length and distance: Technically, when we measure 'length' we measure what would perhaps be better called 'linear extension', and confusingly for children, in everyday life linear extension gets called different things in different contexts. Height, width, depth, length, and distance are all different ways of referring to the same quality of linear extension, and so children need to connect references to their 'height' and how 'tall' they are, with the 'depth' of a swimming pool, the 'width' of their bedroom, the 'length' of a football pitch, and with how 'far' it is to the shops, as all measures of 'the same thing'. Much discussion is needed around this great variety of language use, and also around the wide variety of instruments used to measure different 'lengths' and 'distances' in different contexts.

Gradually, children will learn that there is also an important distinction between 'distance' and 'displacement' when measuring 'how far' it is from A to B. 'Distance' is simply an

amount (a magnitude, e.g. how far you actually have to travel), whereas 'displacement' is both a magnitude *and* a direction (called a vector generally, and a 'translation' in geometry). In everyday life we describe the displacement between two places as the linear distance between them 'as the crow flies'; we assume crows fly along the shortest (straight) path between two points, whereas, e.g. the distance from our home to school will be further than 'the crow flies' because we won't be able to travel in a straight line. Because displacement is a straight-line path, we are able to specify it as movement in a constant direction. This distinction is obviously crucial in answering, 'How far is it from A to B?'.

There are interesting later developments in measuring 'distance' on a global scale; the shortest distance between, say, London and Los Angeles lies along what's called a 'great circle'[15] (a section, or arc of a circle drawn around the earth with its centre at the centre of the earth). Aircraft generally navigate along great circles, but typically if we are asked to say how far away Los Angeles is from London we are more likely to say it's '11 hours' away, than 5,452 miles. This also connects with the measuring of astronomical distances in 'light years'; when distances are large, the 'distance' from A to B becomes more meaningfully expressed in lengths of time than in units of 'linear extension'.

The standard SI unit of linear extension in all contexts is the metre (m). Length is measured with ratio scales (metric or imperial), since 'zero length' is an absolute. Consequently, ratios of lengths to each other make good sense, and are used frequently in both everyday life and in science.

Mass and weight: The 'mass' of a physical object is the pleasingly simple idea of 'how much of it' (that is, the material stuff) you've got; measuring this directly however, is not so simple. In practice, as Isaac Newton (1643–1727) pointed out, we take advantage of the fact that under the effect of gravity *weight* and *mass* are directly proportional to each other, that is, if you double the 'amount of stuff' you've got you will find that it now weighs twice as much as it did.

What this means is that (unless we are out in space) we can compare the masses of two objects with each other by comparing their weights; if a proud new father weighs 22 times as much as his newly born daughter, then we can be sure that his 'mass' is 22 times greater than hers (there is 22 times 'as much' of him as there is of her).

The standard SI unit of mass is the kilogram (kg) and in order to measure mass we do in practice compare objects with this standard unit by 'weighing' them – that is, by comparing their weights with the *weight* of a 1 kg mass. 'Weight' however is a force; it is the gravitational force acting upon any object, and in imperial units it is measured in 'pounds' (lb) and 'ounces' (oz). In the metric system the force due to gravity, that is, 'weight', is measured in 'newtons' (N).

The designers of space stations have to do their calculations based on the knowledge that in orbit the masses of everyone involved will not have changed, even though their weights will have. The force of a collision between two astronauts in space does not change however (it hurts just as much), because their masses do not change; even though each is weightless, two astronauts' bodies still have as much momentum when moving in space as they did on earth (and therefore will take just as much stopping).

15 Or it would be if the earth were actually a sphere; for all practical purposes we treat it as if it is.

In school, using the correct language for the metric system can sound odd because it is not the everyday language children meet outside school. In the everyday world we do compare masses by 'weighing' them, but technically we should not go on to say that the 'weight' of something is so many kilograms – that's its *mass*.

So teachers have something of a problem in deciding whether to use the scientific language of physics as they talk about the SI units of mass (kg), or whether to carry on talking about the 'weights' of objects in kilograms and – in everyday language - pretend that kilograms are units of weight. Many teachers talk of 'heaviness' to avoid using the word 'weight', and do indeed ask children to 'compare masses'. In Geometry, Measurement and Statistics 1 we recommend that you follow common everyday language use. In the subsequent Geometry, Measurement and Statistics teaching resource Handbooks, the term 'mass' is used throughout in relation to its related units of measure: grams and kilograms.

Both mass and weight are measured with ratio scales, since their 'zeros' are absolute. Ratios of masses and weights to each other make good sense, and are used frequently in both everyday life and in science.

Volume and capacity: 'Volume' is the amount of space something occupies, whereas 'capacity' is how much space there is inside a vessel or container of some sort, or how much volume it could 'hold'. In the metric system, the volumes of liquids are usually measured in litres (e.g. drinks, petrol), and the volumes of solid objects in cubic metres (m^3); capacities are typically measured in m^3, but can also be expressed in litres (e.g. a 1 ℓ bottle). 1 litre (ℓ) is equivalent to 0·001 m^3 (one thousandth of a cubic metre). Measuring either volume or capacity in m^3 introduces children to what is called a 'derived' measure; the unit of volume (or capacity) is *derived* from the so-called 'base measure' for length (m)[16].

Interestingly, 1 ℓ of pure water has a mass of 1 kg. Both 'volume' and 'mass' are in a sense measures of 'how much' of something you've got, but 'mass' is the basic scientific SI unit for how much 'matter' there is, whereas 'volume' judges 'how much you've got' by measuring the amount of space something takes up. In everyday life we tend to use 'weights' and 'volumes' for all practical purposes, e.g. buying and selling, cooking and so on.

In science, where 'how much matter' (or material substance) you are dealing with is generally much more important than the space it is occupying, we use kilograms to measure how much 'material' there is of an object. When we want a drink, we are generally much more interested in how much volume we will be given.

The capacities of containers and vessels are usually measured by calculating using the three dimensions of space (e.g. length, width, and depth), or by filling them with a known volume of liquid. Capacities are then typically quoted in both m^3 and in litres.

The volumes of liquids are usually measured inside graduated containers (or through pumps), or in the case of solid objects of awkward dimensions, volumes of objects can be measured by using a displacement vessel.

16 For this reason it is usually more effective to introduce children to litres before 'cubic' measures for volume and capacity; calculating with *derived* measures (e.g. m^3) depends upon a good facility with multiplying if the effort of the arithmetic involved is not to prove a distraction for children.

Both volume and capacity are measured with ratio scales, since their 'zeros' are absolute. Consequently, ratios of volumes (and capacities) to each other make good sense, and are used frequently in both everyday life and in science – particularly in the repetition of mixtures in specified proportions.

Time: There are several aspects to the topic of 'time': telling the time, duration, succession, and speed. As a compound measure, 'speed' is addressed formally in the final year of primary schooling, although it will certainly come up in children's everyday conversations from an early age as they are asked to 'go faster' or more 'slowly' or 'quickly', and as they experience 'racing' in a variety of forms.

There are two central ideas that children need to connect together for effective development of their understanding of how we measure time: the idea of *linear* time, and the idea of a repeating *cycle*. Linear time is the sense that 'time' stretches both backwards into history and pre-history forever, and endlessly forwards into an infinite future. The key visual illustration of this is the 'timeline' – which bears an uncanny resemblance to the number line, even down to the fact that there is an identified 'zero' point for most cultures, 'before' and 'after' which 'things were different'. In the activity goups we refer to these times as BCE and CE: before common era and common era.

Work with timelines connects well with children's developing understanding of numbers as distances along a number line, and with their appreciation of different kinds of numbers; the ideas of 'order' and 'succession' are crucial in this.

Since any timeline includes an 'arbitrary' zero point (it could have been chosen differently), the line becomes an *interval* scale stretching potentially infinitely either side of zero. As such, ratios of dates to each other don't make sense, only ratios of *lengths* of time (durations) to each other; 4 hours is twice as long as 2 hours, but 4 July is not 'twice as' anything in relation to 2 July.

The idea of repeating 'cycles' is closely related to our experiences in a constantly-changing natural world of moon cycles, daily (sun) cycles, and seasons. We use such natural cycles in our everyday measuring of time as we plan 'summer' holidays, prepare for important dates with our calendar, sleep 'at night' and so on. The hands of an analogue clock illustrate to children constantly how we measure time in repeating cycles.

Since both linear time and repeating cycles are crucial to how we deal with succession, duration, and telling the time, children need *both* kinds of illustration prominently as we work with them on measuring time.

Since time is also sensed rather more indirectly (unlike heaviness, volume, and sheer physical size), because the scale of time ranges so widely from instants to eons, and because our awareness of how fast 'time is passing' changes so much depending on context and mood, children are also particularly dependent on discussion, illustrations and active experiences with instruments to develop their personal understanding of time.

The standard SI unit of time is the 'second' (s).

Temperature: Temperature is a measure of how 'hot' or 'cold' something (or someone) is[17], and is measured in degrees on the Celsius (°C), Fahrenheit (°F), or Kelvin (K) scales. In Europe and in much of the world the Celsius scale is generally used in everyday life; in the USA the Fahrenheit scale is commonly preferred. In much of science, the SI unit of K (Kelvin) is used. Degrees Kelvin are equivalent in size to degrees Celsius, but the zero of the Kelvin scale is a theoretical 'absolute' zero meaning there are no 'minus' temperature values possible in K.

The Celsius scale is based around the freezing point (0°C) of pure water (at one atmosphere of pressure, or sea level) and the corresponding boiling point (100°C). Thus children will probably first experience the idea of 'negative' values (or 'minus' numbers) in the context of 'below zero' winter temperatures in either °C or (much more rarely) °F, and/or ice in everyday life, and this is a very helpful context to use when introducing negative numbers.

Since the Celsius scale includes an 'arbitrary' zero point (that is, it could have been chosen differently), it is an *interval* scale stretching potentially[18] infinitely either side of zero. As such, ratios of individual temperature values in °C or °F to each other don't make sense, only ratios of temperature *differences* on these scales; a drop of 6°C is twice as big a drop as one of 3°C, but $^{-}6$°C is not 'twice as cold' as $^{-}3$°C.

Children are very well aware of temperature through their senses from an early age, and so work in primary school is mainly directed to introducing the Celsius scale, and experiencing various types of thermometer. Many children will be familiar with mercury or alcohol-in-glass, digital, and liquid-crystal thermometers simply from their own experiences of illness, but it is helpful for them to meet and discuss these different instruments in school as well.

Area: Area is usually the second 'derived' measure that children meet, after volume. The SI unit for area is the 'square metre' (m^2), derived from the base unit for length (m).

The intuitive idea underlying our conception of area is that of 'surface', and children need plenty of active experience with surfaces before attempting to measure them. Covering surfaces is helpful activity, so painting, tiling, jigsaw puzzles and so on are all beneficial early experiences.

Since area is measured with a derived unit there are strong connections between understanding multiplying as an arithmetic operation and work on measuring area; the two dimensions of any flat surface correspond helpfully to the two numbers involved in the binary operation of multiplying. Visual areas are a most effective illustration for many properties of multiplying, and multiplying is essential to the measuring of area.

Since there is such a close connection between multiplying and measuring area, formal work on measuring area is usually timed to coincide with a suitable stage in the development of children's calculating.

There are many interesting facets of our natural world to explore that relate surface areas with volumes and the significance of their relationships to biological life, e.g. babies and small creatures have *proportionally* much more surface area to their bodies in relation to their volume than adults and larger creatures. This is one key reason why babies and small creatures are much more vulnerable to extremes of heat and cold than adults and larger creatures, and also why small creatures have to eat so much more everyday (proportionally) than larger forms of life.

Measuring in Geometry, Measurement and Statistics 1

In these early activity groups attention is paid to discussing and discriminating between different lengths, capacities (and volumes), weights, and amounts of UK money; also to the succession and duration of events, and to telling the time.

This work in all areas is developed according to Stevens' classification of measuring scales in that the various attributes to be measured (length, capacity and so on) are firstly distinguished and named (*nominal*), then objects are compared according to those attributes and ordered (*ordinal*) before units (or regular *intervals*) are introduced. Finally, the standard *interval* scale for time is introduced.

Length and distance: Children are introduced to the wide variety of language used to refer to linear extension (e.g. 'length', 'width', 'height' and so on), and learn to compare and order lengths with each other, noting that comparisons depend upon both a common starting point and a 'straight' distance between two ends. Non-standard units are also introduced.

Weight: Children are introduced to the idea of weight as 'heaviness', and learn to compare and order the weights of objects using a pan balance. Non-standard units are introduced in order to describe the heaviness of a single object.

Capacity and volume: Children compare and order the capacities of a variety of containers, and are introduced to the idea of volume through capacity, that is, through the question, 'How much does this hold?' Non-standard units of capacity are introduced in the form of the capacities of relatively smaller vessels, e.g. the capacity of a large bottle measured by the number of yoghurt pots taken to fill it.

Time: Children are introduced to the ideas of duration in the form of days, weeks, hours, minutes and seconds and the succession of these units of time. They learn about our standard and everyday measures. Children also begin to learn to tell the time to the hour and half hour on an analogue clock. At this stage, succession is introduced through the regular cycles of everyday life, rather than use of a timeline.

17 Technically, it is a measure of the thermal energy per particle of matter or radiation; it seems the 'hotter' something gets, the more those little things get agitated.

18 This means that as far as the Celsius scale is concerned, temperatures could go on getting ever hotter or colder 'forever'. Only the Kelvin scale acknowledges explicitly that in our universe it is presently thought impossible for temperatures to become any colder than $^{-}$273·15°C.

Money: Children are introduced to the idea of economic 'worth' implicitly in the context of trade and markets, and quickly meet the idea of money as a collective measure of worth. Standard UK monies are introduced and relative values explored up to the value of £1. The *equivalence* of coins that so often puzzles children initially is explored thoroughly, and thus the equivalence of different amounts of cash.

Statistics during the primary years

As with measuring, no one ever uses statistics in real life without a specific *purpose* to it; in practice there is always some question or questions that people are trying to answer. Thus in work with children, any work on statistics should always be led by a key question (or questions) that shapes and directs the work itself.

The simplest of beginnings to statistics can be made with collections of real objects (perhaps things children bring back from a nature trail walk), about which a teacher could ask, 'So, what have we got here then?', and children can be encouraged to sift through, discriminate between and identify various categories into which objects can be sorted. Answers then become something like, 'We've got some of these, a few of those, only one of those' and so on. The objects themselves can then be displayed, grouped within their categories and suitably labelled.

Note how closely this activity relates to the use of 'nominal' scales in measuring; there is no ordering of objects or categories involved, simply a *discriminating between* objects. Such work is at the beginning of handling **categorical** data.

Thus sorting through a collection of real objects, noting similarities and differences and organizing the objects into categories in order to respond to a particular question, precedes similar activities with collections of data. The important thing is that the statistical activity is purposefully directed towards answering an initial specific question.

The data-handling cycle[19]

Purposeful statistics begins with a specific question (A) being posed. At the next stage (B) one identifies the kind(s) of data that would help answer the question. The following stage (C) is when the chosen data is collected, and then organized (including any visual representation) so as to reveal any patterns of significance relevant to answering the question posed. At stage (D) one interprets any patterns found at (C) in order to return and try to answer the question at (A).

It is always best to choose an initial question within an area of topical interest to the children. So for instance during some work on road safety children might be being advised to 'walk straight across' roads, rather than to cross them diagonally. Discussion could raise the point that it takes longer to cross a road diagonally than it does to go straight across, and therefore crossing diagonally puts a pedestrian in potential danger for longer. A relevant question (A) could be, 'How much more dangerous is it to cross a road diagonally, than it is to walk straight across?' Children could then (B) discuss what kinds of

19 See Graham, A. (1990) *Supporting Primary Mathematics: Handling Data*, Milton Keynes: The Open University, for a similar version.

data might help them to answer the question, and come up with the idea of marking out a 'road' on the playground and timing each child in the class both walking straight across and walking diagonally. Collecting and organizing the data so that comparisons can be made (perhaps involving the averaging of times) follows at stage (C), and then discussion (D) could interpret whether the time differences found at (C) help to answer question (A) well enough.

It may be that children feel the first run-through of the cycle answers the original question satisfactorily, in which case the work is over. Quite often however, a first run-through reveals that the initial question was not quite 'fit for purpose'; the question is then refined, and the whole cycle run through again on the basis of a different question at (A). It may be that in the road safety example, children decide it actually matters more *where* you cross a road, than whether you go exactly straight across – in which case a different question can be raised at point (A), perhaps timing how much warning you have of impending traffic if you try to cross a road near a bend.

Thus gradually one gets to understand the purpose of the statistical activity by increasing refinement of the task as repeated cycles are undertaken. Children get to appreciate how important it is to ask the right question in the first place, a range of data-collection techniques, data-organizing techniques, forms of pictorial and graphical representation, and approaches to interpretation, as different cycles are pursued.

In this important way, children come to realize that data handling and statistics are always undertaken for a specific purpose. That is, we do not construct graphs from data and then decide what questions those graphs could answer. We begin with a question, collect data, and then the construction a graph to show that data can help us to find the answer to the question.

Connections with measuring

Early measuring and early work on statistics are virtually indistinguishable from each other; in both types of activity children are learning to *distinguish between*, to *discriminate*, and to identify qualities that may be measured and related. Of course how various qualities may be related to each other will eventually take children to algebra and the study of relationships *in general*, and any calculating involved in statistics automatically relates back to their work in number. Everything is connected, in doing mathematics.

As children collect and analyse different kinds of data, in different kinds of ways, to answer a variety of questions, they will learn to *distinguish between* qualities, between *ordered* and un-ordered data, they will need *units* and utilize interval scales, and *compare* amounts multiplicatively using *ratio* scales. In all these ways the development of children's measuring is woven within the development of their work in statistics.

Statistics is a key context within which measuring demonstrates its own clear purposes.

There are no explicit statistics activities written for Geometry, Measurement and Statistics 1, although the sorting of shapes and of money that occur within geometry and measurement work respectively have clear relevance for the sorting of 'data' that will occur later.

Glossary

Most mathematical terms used in the *Geometry, Measurement and Statistics 1 Implementation Guide* and *Teaching Resource Handbook* can be found in a good mathematics dictionary such as the *Oxford Primary Illustrated Maths Dictionary*.

Other terms you might not be familiar with or which may provide particular challenges for children are explained in this glossary.

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Bruner

Jerome Bruner (1915–2016) was a distinguished psychologist. His distinctions between enactive, iconic and symbolic representation have been particularly influential in the field of education.

communication mediator

A communication mediator is an object, image or action that helps communication in some way. These communication mediators need to be carefully introduced to children, for example Numicon Shapes and number rods become communication mediators when they help to illustrate discussions about the relationships between numbers, coins and lengths of time. However, there's no 'magic in the plastic' – a physical object or image is just a physical object or image unless it is actually supporting communication.

corner

Used in Geometry, Measurement and Statistics 1 to refer to the vertex of 2D and 3D shapes. The term 'vertex' is introduced in Geometry, Measurement and Statistics 2 to introduce the more formal geometrical name for a 'corner' or a point at which lines meet (see **edge**).

edge

Used to refer to a ridge on a 3D shape at which two faces or surfaces meet. Initially children may talk about both straight and curved edges; later, 'edge' refers only to boundaries of shapes which are straight lines. (See also **side**, **surface**.)

enactive, iconic and symbolic representation

Jerome Bruner (1966) distinguished three key ways in which we humans represent experience to ourselves: through enactive (internalized action), iconic (sensory) and symbolic (e.g. language-based) representations. In the Numicon approach we seek to combine all three forms of representation so that children experience number ideas through action, imagery and conversation.

flat shape

A two-dimensional (2D) shape, such as a rectangle, hexagon or oval. 'Flat' provides a physical and visual description which helps children distinguish or sort shapes of this type, and hence to develop mental imagery and generalize about them (see also **solid shape**).

generalization

A statement or observation (not necessarily correct) about a whole class of objects, situations or phenomena. Generalizations are essential and everywhere in mathematics – numbers, for example, are generalizations, as are shape names. For this reason children need to generalize and to work with generalizations constantly.

length

Length may refer to a dimension of an object, and in this sense be used to distinguish from width or breadth, height, thickness or distance. Alternatively, and perhaps confusingly for children, it can be used as a general term to describe all these measures of dimension.

mass

The amount of 'matter' in an object, which gives it heaviness or **weight** under gravity. The mass of an object is found by weighing it.

non-standard unit

In Geometry, Measurement and Statistics 1, children choose and use their own units to explore length, mass and capacity; they might measure a distance in steps, for example, or a capacity in scoops. These are non-standard units both in the sense that their size is not widely agreed (and may not even be fixed), and in the sense that they are not commonly used for making exact measurements. Rather they are practical, everyday-world measures which enable children to access and investigate ideas of measurement as they work towards understanding and using 'standard' units such as metres, kilograms or litres.

number fact

The term 'number fact' usually refers to an operation on two or more numbers, together with its outcome. So '6 + 3 = 9' is a number 'fact', as is '256 ÷ 16 = 16'. In UK schools simple addition and subtraction facts are often referred to as 'number bonds'.

number names/objects/words

Adults often use number words such as 'four' or 'twenty-three' as nouns, and ask children questions such as 'What is seven and three?' In our language, nouns name objects, so we commonly (and unconsciously) assume that number words must be being used to name number objects – thus numbers are often treated as if they are objects.

It is important to remember that we don't always use number words as nouns; quite often we use those same words as adjectives, as in 'Can you get me three spoons?' One of the key puzzles for children to solve is when to use number words as adjectives, and when as nouns.

number sentence

The metaphor of a 'sentence' – in the sense of a unit of language – is sometimes used to describe a number fact written horizontally, left to right, as in '4 + 23 = 27'.

numerals

Numerals function as shorthand for number words. The numeral '5' is shorthand for the word 'five'. When used as a noun, the numeral '5' is often said to represent or stand for the number object we call 'five'.

Numicon Shape pattern

The system of arranging objects or images (up to ten in number) in pairs alongside each other; this is also sometimes called 'the pair-wise tens frame'. Fig. 1 shows the Numicon 7-pattern.

Numicon Shape

Numicon Shapes are pieces of coloured plastic with from one to ten holes, arranged in the pattern of a pair-wise tens frame (see Fig. 2).

oblong

A **rectangle** which is not a square – that is, a 2D shape with four straight sides and four right angles in which one pair of sides are longer that the other pair (see Fig. 3).

parts and properties

The **properties** of a shape are defined by how its **parts**, e.g. 'vertices' and 'edges', relate to each other. For example, one property of a trapezium is that it has just two sides that are parallel to each other. A property of a rectangle is adjacent sides meeting at 90°.

Piaget

Jean Piaget (1896–1980) was a philosopher and psychologist who spent years studying young children and how they learn. He believed that the development of knowledge is based on developing cognitive structures and that children should be actively involved in their own learning.

rectangle

A 2D shape with four straight sides and four right angles. This means that a square is also a rectangle – a special case in which the four sides are also of equal length – and all other rectangles are **oblongs** (see Fig. 3).

side

Used to refer to a boundary or edge of a 2D shape which joins two corners. Initially children may talk about both straight and curved sides; later, 'side' refers only to boundaries of shapes which are straight lines. (See also **corner**, **edge**.)

solid shape

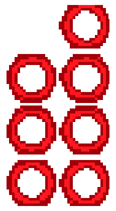
A three-dimensional (3D) shape, such as a cube, sphere or cone. 'Solid' provides a physical and visual description which helps children distinguish or sort shapes of this type, and hence to develop mental imagery and generalize about them (see also **flat shape**).

surface

Used in Geometry, Measurement and Statistics 1 to refer to the surface of a 3D shape; it may be flat, as in a cube, or curved, as in a sphere. The term 'face', which describes a flat surface only, is introduced in Geometry, Measurement and Statistics 2.

weight

The heaviness of an object. By weighing an object to find out how heavy it is, we also find out how much 'matter' it contains – that is, what its **mass** is. The standard unit used to measure weight is Newtons; the standard units for measuring mass are kilograms and grams.



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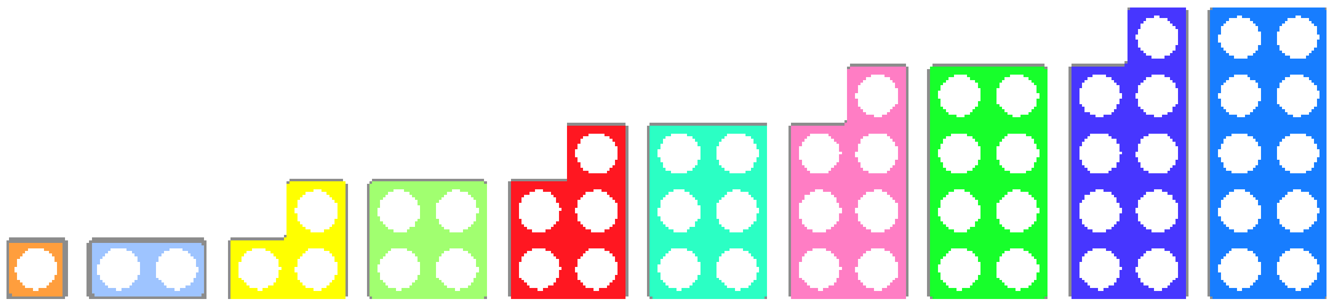

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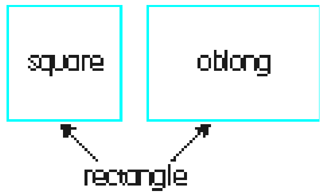

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