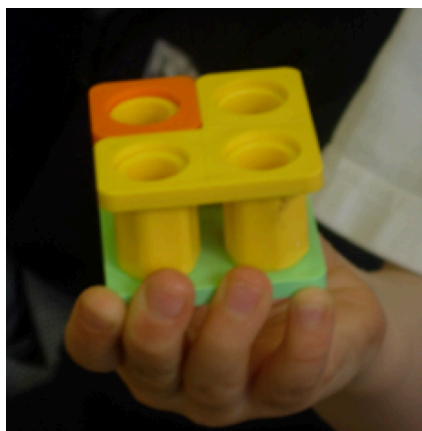


Numicon: Fun with fractions



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Hawkes Bay East Coast

Jan Ward Allen
jan@Edushop.nz

Change your
perception of
teaching maths
forever...



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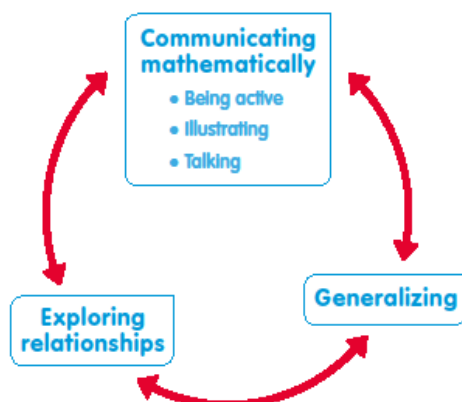
www.edushop.nz



Numicon is a distinctive approach to children's mathematical learning that emphasises three key aspects of doing mathematics:

This demands children to:

- Think and communicate with and about abstract objects
- Look for patterns in abstract objects
- Generalise and use these patterns

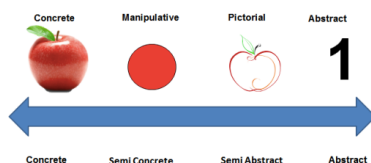


All aspects work together when 'doing' mathematics

*But it's hard to **generalise** in the abstract at first...*

Which is why Numicon is founded on the CPA approach:

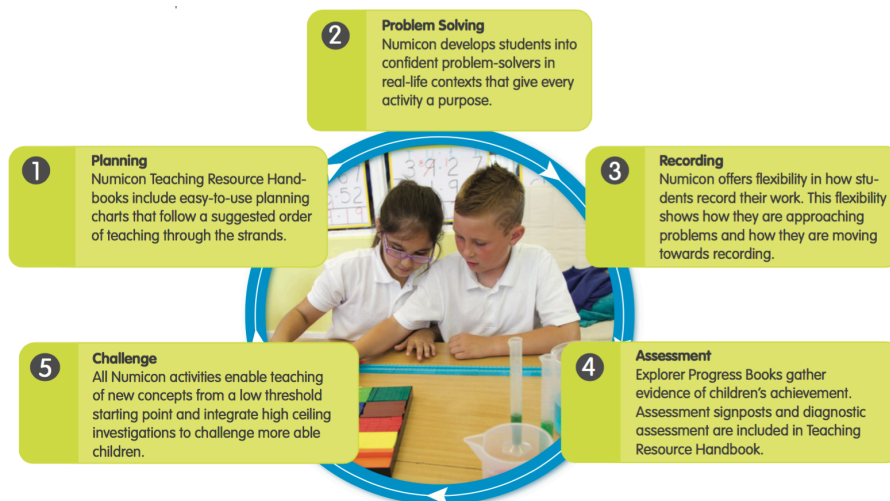
Concrete - Pictorial - Abstract



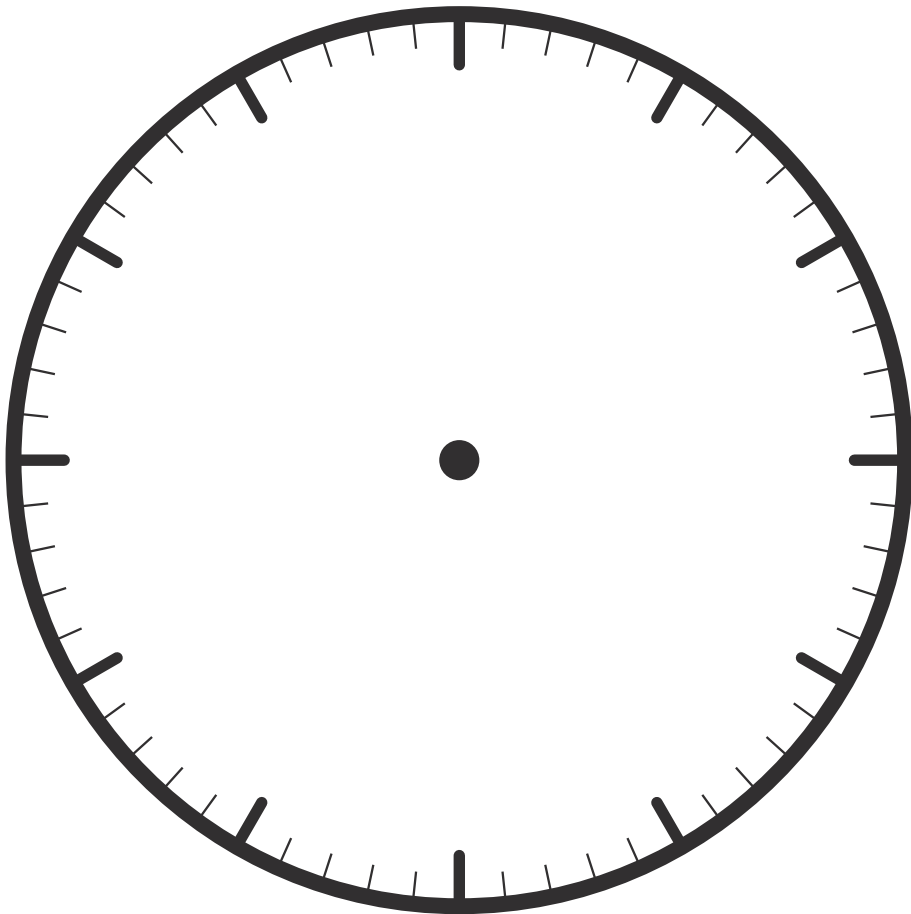
Numicon acknowledges the difficulties and teaches how to generalise to solve problems

Action and imagery activities to give experience to understand symbols, necessary for communicating a generalisation.

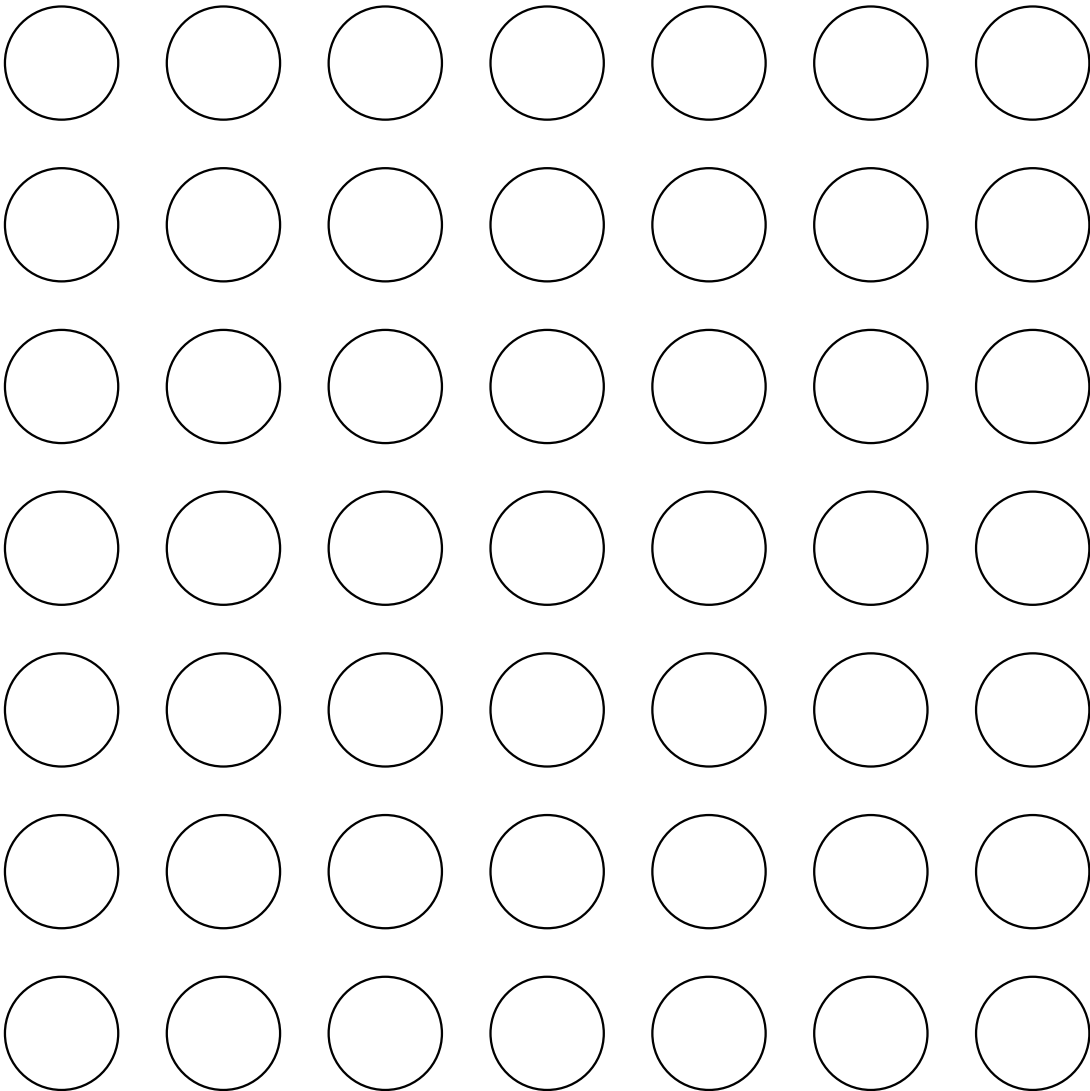
To aid this, a strong focus is placed upon the use of structured materials that will lead children into *doing* mathematics.



Name Date/...../.....



Name Date / /



Name Date/...../.....

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Birds in the Garden 2

A Numicon fractions activity for pairs.

What you will need

- Multiple sets of 1–10 Numicon Shapes
- One copy of this sheet per pair (please note: this is an A3 document and needs to be printed at 100%)
- The Bird Cards cut from this sheet and placed face down in a pile
- 2 Garden Cards cut from this sheet

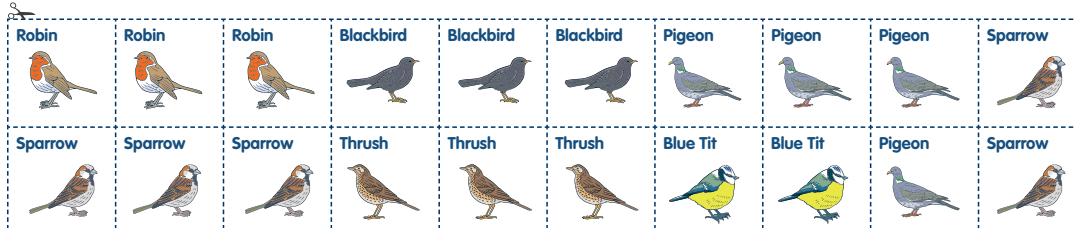
What to do

- Shuffle the pile of Bird Cards. The cards show some different types of birds that can be found in a garden.
- Up to 10 birds can visit your garden at a time. This is shown on your Garden Card.
- Take it in turns to select one card at a time from the pile and place it onto your 10-shape Garden Card until you collect 10 birds.
- Once you have collected all ten, rearrange them into groups of each type of bird on the Garden Card.
- Talk to your partner about how many of each bird you have out of 10, for example '3 out of 10 birds are robins'.
- Now show this as a fraction. For example, if 3 out of 10 birds are robins, that is $\frac{3}{10}$ (three tenths).
- Add up the fractions: what is the total?
- Repeat the activity, shuffling the cards first so you get different numbers of birds.

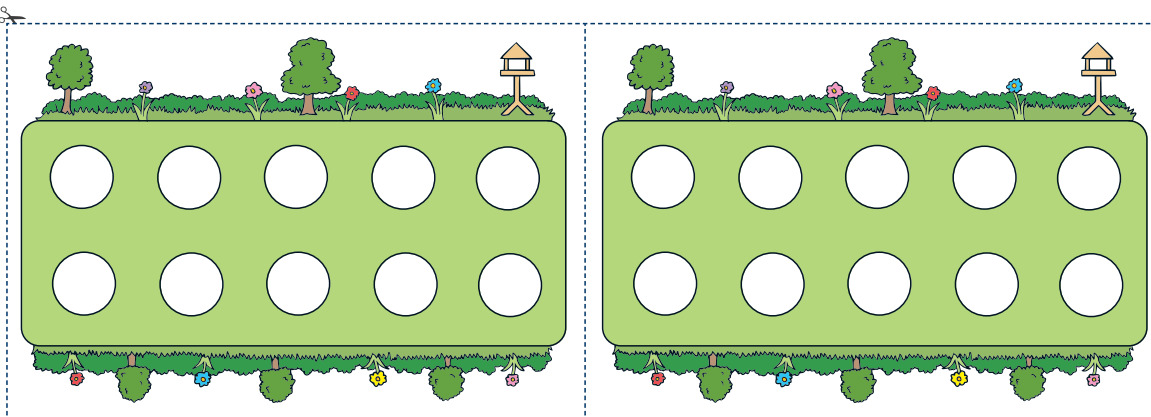
Extensions and questions

- Which birds make up the largest fraction of the total number of birds?
- Which birds make up the smallest fraction of the total number of birds?
- How many more... are in your garden than?
- Can you show your fractions in any other ways? For example, $\frac{2}{10}$ is the same as $\frac{1}{5}$.
- Compare the fractions with your partner to see who has more of each bird.
- Together, combine the two garden cards and add up the types of bird in both to find fractions out of 20.
- Repeat the activity with a garden that can have 8 or 12 birds visiting at once. (Use a Numicon Shape or Shapes as the garden.)

Bird Cards



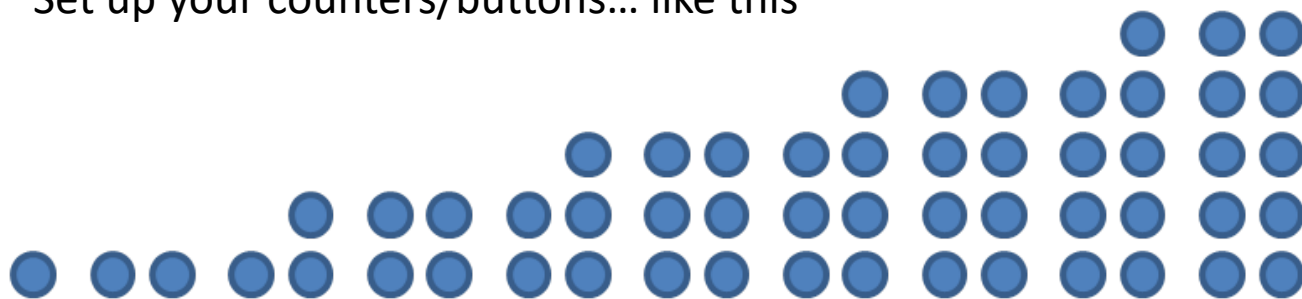
Garden Cards



Set up your shapes like this

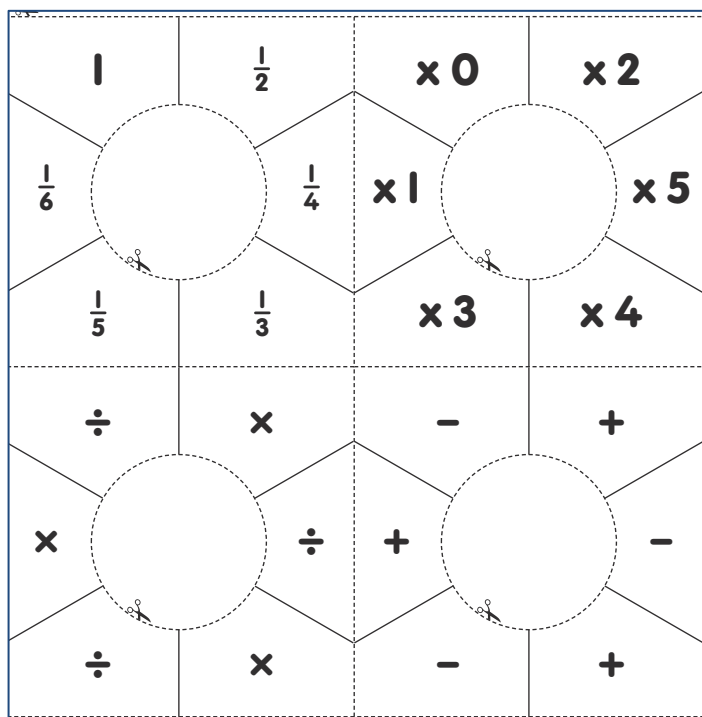
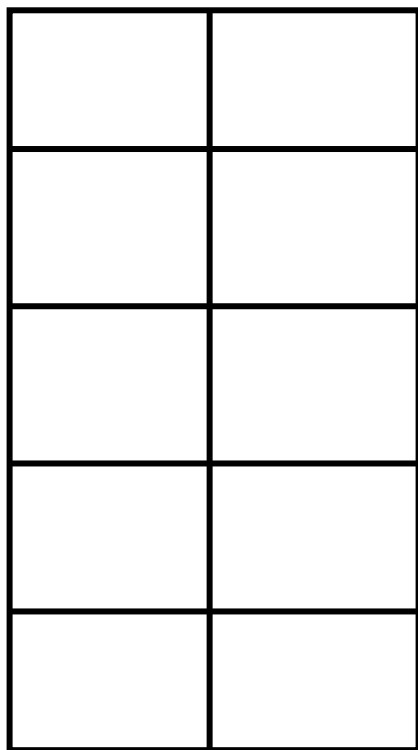


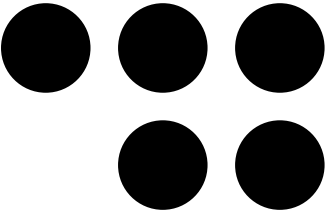
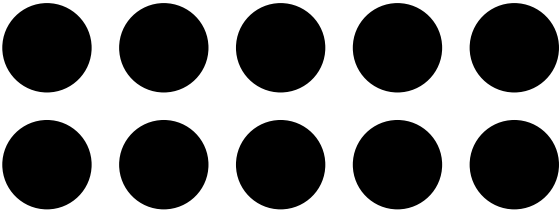
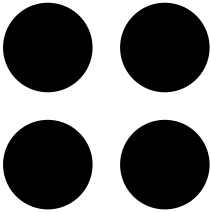
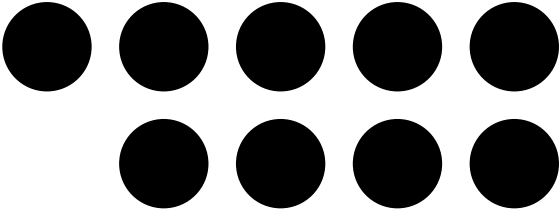
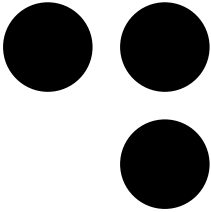
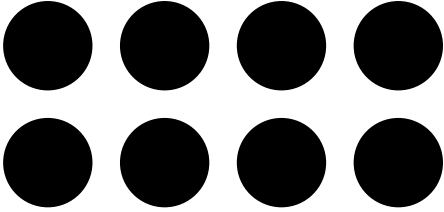
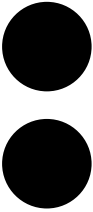
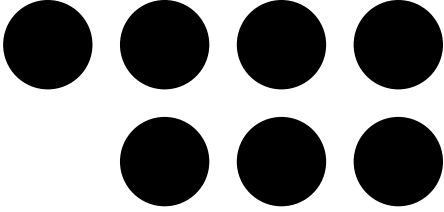

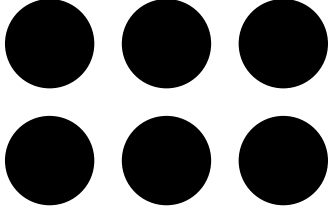
Set up your counters/buttons... like this



Set up your numeral cards like this

0 1 2 3 4 5 6 7 8 9



Number Sense to Place value

one	two	three	four
five	six	seven	eight
nine	ten	eleven	twelve

0	1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	---	----

What is Maths?

“Mathematics is the exploration and use of patterns and relationships in quantities, space, and time. Statistics is the exploration and use of patterns and relationships in data.

These two disciplines are related, but different ways of thinking and of solving problems. Both equip students with effective means for investigating, interpreting, explaining, and making sense of the world in which they live”. *NZ Curriculum*

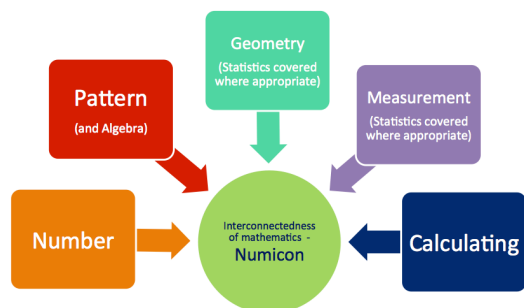
- An abstract concept, with numerals assigned to represent the concept
- Rules and methods to follow
- It is an axiom (a statement regarded as true)

It is made up of different components:

- Problem solving
- Patterns
- Relationships
- Calculations and counting
- Reasoning and generalizing & communicating

How does Numicon help?

- It makes numbers real
- It provides a structured learning process that follows the CPA sequence, using appropriate manipulatives.
- Children enjoy learning about maths which has a direct purpose and real-life context
- Reduces anxiety
- Supports all types of learners, AVK, as well as those with learning difficulties
- Reduces auditory processing and working memory load
- The teacher is supported with on-line and face to face support by a Numicon trained NZ teacher.
- Focuses on key aspects of mathematics: Communicating, Generalizing, Exploring Relationships



Mathematics and Statistics are interconnected. All strands should be taught together where relationships fall naturally

What does this mean for our students?

Visual prompts
Hands-on activities
One small step at a time, repeated
Working alongside others
Opportunity to say what they want to learn
Connections with real life experience

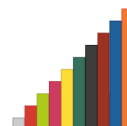
What does this mean for our teaching?

Open ended tasks and challenges
Effective questioning strategies
Making Skills and Capabilities explicit
Opportunities to work collaboratively
More self-directed learning
Making connections

Numicon Manipulatives

Numicon is essentially a whole school approach to mathematics that uses manipulatives to explore patterns and relationships in mathematics.

It uses number shapes and number rods,
Along with other manipulatives.



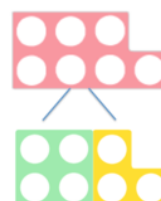
The Shapes

- Any group of objects can be arranged into a pattern that can be 'read'.
- A group then does not have to be counted
- The concept of 'number' or 'how many' is built
- Ready to explore relationships with groups
- Shapes and rods are tools for conversations about numbers
- and their relationships
- Generalisations about anything!
- Numeral symbols have meaning
- Mathematical thinking and communication grows
- Components or units in groups make up the whole – encourages part – whole thinking
- The 'whole idea' thinking is generalising
- Problem-solving, investigating, interpreting explaining and making sense of the world around them, follows readily

Numicon Pattern



Part - Whole

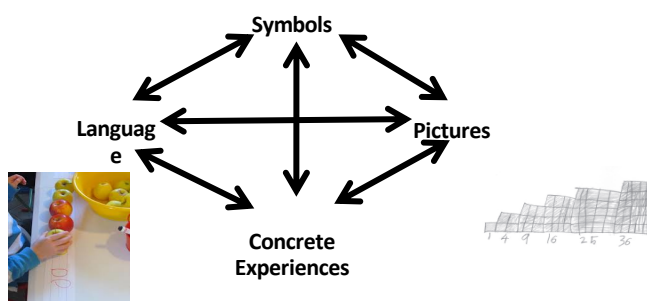


Bar Model



Foundations: Making Connections

Haylock and Cockburn (2008)



Abstraction of Manipulatives: No magic in the plastic

With any tool that you use in your teaching, you need to think about its purpose and if it is the best tool for the job. Each manipulative encourages certain thinking, for example, counters can be manipulated greatly as they can be counted separately and split into groups. Cuisenaire rods cannot be broken up but encourage thinking of numbers as a whole. Numicon shapes are extremely flexible in its abstraction – you can count the holes individually or think of the number as whole by looking at the shape. Once you remove the colour and physicality of the holes (perhaps by just using a cut out shape), you scaffold the thinking further.

Numicon and the New Zealand Curriculum

NZC NUMICON TABLE

NZC Level	1		2		3		4	
Year	0/1	2	3	4	5	6	7	8
NP Stages (Approx.)	0-3	4	Early 5	Late 5	6	7	8	
Numicon	FF	1	2	3	4	5	6	
Intervention	Numicon Intervention Programme							
Learning Needs	Breaking Barriers							
Acceleration	Big Ideas - Suitable for students in Years 5 – 9 as a catch-up							

Numicon covers all the strands of the New Zealand Curriculum in Mathematics including the Digital Technology Curriculum. It is based on years of sound and documented research.

Number and Algebra is taught in tandem with the strands demonstrating the interconnectedness of Mathematics, in a spiral approach giving students a long term confidence in knowledge and strategies.

The Concrete- Pictorial-Abstract Approach supports understanding, memory and application when solving problems. Structured concrete materials are incorporated into activities to develop fluency and aid children in problem-solving. Manipulatives work!

Numicon meets the needs of *all* learners therefore providing a truly inclusive programme.

Key features for teaching:

- Long-term, medium-term and short-term planning AND Assessment is provided. *It's all done for you!*
- Lesson structure is based on explorative activities (play), language, problem-solving, then recording.
- Real life contexts are the opportunity to apply the generalisations to solve problems.
- One lesson, planned well for the whole class, or groups provides more time for teacher participation and enjoyment.
- Focus activities during the week sequenced providing conceptual development. These include direct teaching, demonstration, problem-solving and investigations written by and trialled by teachers over many years and settings. This enables easy differentiation for mixed ability and targeted teaching.
- Formative Assessment is supported with 'look and listen for' prompts in the activities. Learning is recorded weekly along with Explorer Progress Booklets to show concrete evidence of learning. Summaries of learning is provided at each Milestone at 4 – 6 week intervals. This can be shared via Linc-ED, eTAP, or Kamar.

Key Mathematical ideas in Numicon 1-4

Teaching Progressions can be found under Supporting Resources on www.numicon.co.nz. These show when to connect NPC and GMS into your programme during the year.

	Number, Pattern and Calculating	Geometry, Measure and Statistics
Numicon 1 NZC Level 1	<ul style="list-style-type: none"> Counting objects to at least 30 Ordering numbers to 20 Counting in two's, five's, and ten's Understanding place value of 2-digit numbers Reading, writing and understanding $+$, $-$, $<$, $>$ Adding and subtracting facts to 10 Recognising halves and quarters of wholes 	<ul style="list-style-type: none"> Making tiling, repeating and growing patterns Making, naming and sorting 2D and 3D shapes Exploring properties of 2D and 3D shapes Giving directions, describing, turns and rotations Comparing and ordering mass, capacity and length Understanding time duration Telling the time to the hour and half hour Understanding money
Numicon 2 NZC Level 2	<ul style="list-style-type: none"> Patterns and sequences of 2s, 5s, and 10s Counting to 100 and beyond Comparing and ordering numbers to 100 Recognise the place value of 2-digit number When/how to add/subtract to solve problems Adding and subtracting facts to 20 Working with multiplying and dividing Recognising halves, quarters and thirds of wholes Understanding fractions as numbers 	<ul style="list-style-type: none"> Making and classifying polygons Identifying/describing faces, edges, vertices of 3D Symmetrical patterns, identifying lines of symmetry Identifying and naming prisms Exploring fractions of rotations Creating block graphs and bar graphs Telling the time to five minutes, including quarter past/to the hour
Numicon 3 NZC Level 3	<ul style="list-style-type: none"> Developing fluency $+$ $-$ in 2- and 3-digit numbers Exploring multiplying and dividing Partitioning 2- and 3-digit numbers Comparing and ordering numbers to 1000 Using apparatus and imagery in $+$ $-$ \times \div Understanding fractions of a wholes & numbers Using fraction notation 	<ul style="list-style-type: none"> Building skeleton 2D and 3D shapes Identifying regular and irregular polygons Making and identifying right angles and types of lines Sorting 2D and 3D shapes using sorting diagrams Describing position and movement on a grid Telling the time (analogue and digital) 12-hour clocks Measuring mass, capacity, length using standard units Understanding discrete and continuous scales
Numicon 4 NZC Level 3	<ul style="list-style-type: none"> Understanding place value in 4-digit numbers Ordering and comparing numbers to 1000+ Developing fluency with mental and written methods for adding and subtracting Developing fluency with multiplying and dividing facts to 12×12 Developing fluency with mental and written methods for multiplying and dividing Exploring negative numbers Exploring decimal fractions Exploring equivalent fractions 	<ul style="list-style-type: none"> Sorting/classifying triangles and quadrilaterals Making/identifying symmetrical figures Making/identifying types of angles in polygons Plotting /reading co-ordinates in the first quadrant Describing/drawing translations on a co-ordinate grid Measuring mass, capacity and length using decimals Calculating area and perimeter of rectilinear shapes Collating, comparing, presenting monetary data Reading/creating tables and graphs Telling the time (analogue/digital 24-hour clocks) Time duration

Breaking Barriers covers a summary of the concepts in Numicon 1, 2 and 3 at a pace to enable students with high Learning Needs to participate in the same class environment as their peers. Numicon supports inclusive education practice.

Numicon Intervention Programme covers the key mathematical ideas in Numicon 1, 2, and 3 in a 12-15 week intervention either as part of the classroom environment or in a separate environment. A Diagnostic Assessment in mathematics determines the starting point and teaching programme for each student to close the gap between the students who are struggling and their average-achieving peers.

'Investigations with Numicon' teaching book contains ten open-ended investigations with a low threshold and high ceiling, with the potential to stretch children to Level 3 and beyond of the NZ Curriculum. Suitable for bright children in maths, including bright children who are not succeeding in mathematics known as 'twice exceptional'.

Key Mathematical ideas in Numicon 5 and 6

Teaching Progressions can be found under Supporting Resources on www.numicon.co.nz.
These show when to connect NPC and GMS into your programme during the year.

<p>Numicon 5</p> <p>NZC Level 3 - 4</p>	<ul style="list-style-type: none"> • Reading/writing -digits & multiples to seven places • Interpreting negative numbers in context • Recognise/describe linear number sequences, rules • + And - numbers 4 plus digits, algorithms reasoning • Square numbers (2) and cubed (3) • Scaling by simple fractions and simple rates • Fractions –multiples, equivalent, tenths and hundredths, mixed, improper fractions • + And – fractions, x proper fractions/mixed numbers • Decimal -fractions, hundredths, tenths & decimal equivalents, rounding • Per cent %, fraction and as a decimal • Percentage & decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$ $\frac{1}{10}$, with a multiple of 10 or 25 	<ul style="list-style-type: none"> • Convert between different units of metric measure and solve problems involving converting between units of time • Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres • Calculate and compare the area of rectangles • Estimate volume • Use all four operations to solve problems involving measures using decimal notation, including scaling. • Angles -drawn, measured in degrees • Line graphs, complete, read and interpret information in tables, including timetables
<p>Numicon 6</p> <p>NZC Level 4</p>	<ul style="list-style-type: none"> • Read, write, order and compare numbers to 10 million • use negative numbers and calculate across 0 • long multiplication up to 4 digits • long division up to 4 digits, and interpret remainders as whole number remainders, fractions, or by rounding • common factors, common multiples and prime numbers • Addition and subtraction multi-step problems in contexts • common factors to simplify fractions • + - fractions with different denominators and mixed numbers, multiply simple pairs of proper fractions, divide proper fractions by whole numbers • calculate decimal fraction equivalents for a simple fraction • identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 • multiply one-digit numbers with up to 2 decimal places • written division methods in cases where the answer has up to 2 decimal places • equivalences between simple fractions, decimals and percentages, including in different contexts • use integer multiplication and division facts where missing values can be found • calculation and comparison of percentages • solve problems involving similar shapes where the scale factor is known or can be found • Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples • simple formulae and linear number sequences • Express missing number problems algebraically • Find pairs of numbers that satisfy an equation with 2 unknowns • enumerate possibilities of combinations of 2 variables 	<ul style="list-style-type: none"> • Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate • Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places • Convert between miles and kilometres • Recognize that shapes with the same areas can have different perimeters and vice versa • Recognize when it is possible to use formulae for area and volume of shapes • Calculate the area of parallelograms and triangles • Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3] • Draw 2-D shapes using given dimensions and angles • Recognize, describe and build simple 3-D shapes, including making nets • Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons • Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius • Recognize angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles • Describe positions on the full coordinate grid (all 4 quadrants) • Draw and translate simple shapes on the coordinate plane, and reflect them in the axes • Interpret and construct pie charts and line graphs and use these to solve problems • Calculate and interpret the mean as an average

Numicon overview chart

	NURSERY	FOUNDATION	AGE 5-6	AGE 6-7	AGE 7-8	AGE 8-9	AGE 9-10	AGE 10-11	
TEACHING SUPPORT	1ST STEPS WITH NUMICON IN THE NURSERY KIT	NUMICON FIRM FOUNDATIONS KIT	<div>Teaching Resource Handbook and Implementation Guide (Number, Pattern and Calculating, and Geometry, Measurement and Statistics teaching packs available separately)</div> <div><div>NEW</div><div>Handbook 1</div><div>Handbook 2</div><div>Handbook 3</div><div>Handbook 4</div><div>Handbook 5</div><div>Handbook 6</div></div>						NUMICON ONLINE ONLINE PLANNING AND ASSESSMENT SUPPORT
APPARATUS			STARTER APPARATUS PACK A	STARTER APPARATUS PACK B	STARTER APPARATUS PACK C				
ASSESSMENT			Three Explorer Progress Books for Number, Pattern and Calculating, and Geometry, Measurement and Statistics			<div>NEW</div> <div>Explorer 3b</div> <div>Explorer 4b</div> <div>Explorer 5b</div> <div>Explorer 6b</div>			
ACTIVITIES FOR HOME	1ST STEPS WITH NUMICON AT HOME KIT		Explore More Copymasters 1 2		Explore More Copymasters 3 4 5 6 <div>NEW</div>				
<div>ALSO AVAILABLE</div> <div>Investigations with Numicon: a supplementary teaching manual and apparatus pack to stretch able children in Key Stage 2.</div> <div>ALSO AVAILABLE</div> <div>Breaking Barriers designed specifically for pupils with SEND or those experiencing learning difficulties with maths.</div>									

- First Steps with Numicon in the Nursery
- First Steps with Numicon at Home
- Firm Foundations
- Numicon at the Seaside

Teaching Resources Numicon 1- 6:

1. Teaching Resource Handbook
2. Implementation Guide
3. Explore More – extra activities or for home work
4. Pupil Books – Problem-solving for deeper learning and investigations (Numicon 3 – 6)
5. Explorer Progress Books – formative assessment

- Investigations with Numicon
- Breaking Barriers
- Big Ideas
- Numicon Intervention Programme

Apparatus Packs

- Apparatus Packs – Starter Class and Starter 1:1
- Apparatus Pack A – Supports Numicon 1 and 2
- Apparatus Pack B – Supports Numicon 3 and 4
- Apparatus Pack C – Supports Numicon 5 and 6
- Breaking Barriers – Supports Breaking Barriers Teaching Pack
- Firm Foundations Apparatus Packs

Starter Apparatus Pack A



Starter Apparatus Pack B



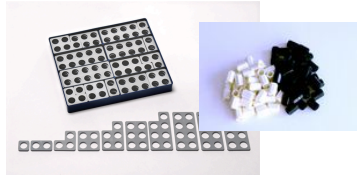
Starter Apparatus Pack C



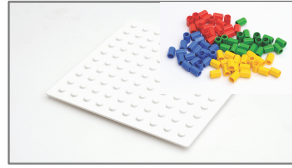
Numicon- Apparatus & Resources



Box of 80 Shapes



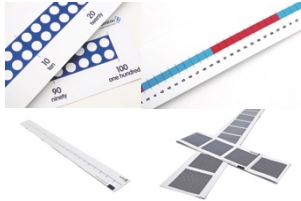
Grey Shapes and pegs



Pegs and baseboard



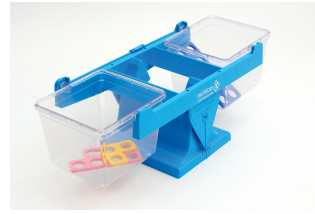
Little Box of shapes



Numerous number lines



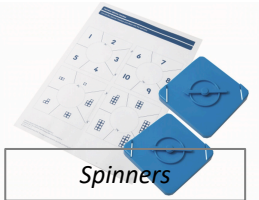
Intervention CD



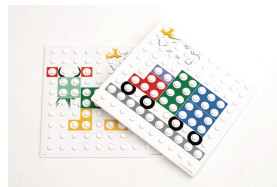
Pan Balance



Training DVD



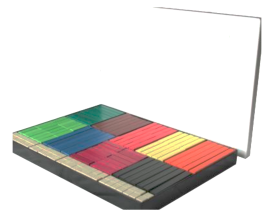
Spinners



Picture Overlays



Number overlays



Cuisenaire Rods



Large Foam Shapes



www.numicon.co.nz



the home of smart teaching



Numicon Teaching & Assessment Resources

Activity (0 of 24)

Activity group summaries (0 of 213)

Classroom display (0 of 2)

Photocopy master (1 of 923)

Assessment (61 of 293)

Curriculum coverage (115 of 115)

Planning (1 of 203)

Professional development (11 of 35)

Teacher support video (29 of 60)



Oxford Owl Website

OUP Numicon website

Activity Group Components

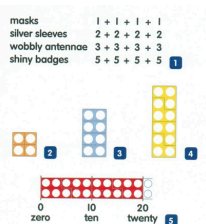
Focus Activity – Direct Teaching

To provide students with a model representing the relationship and action



Focus Activity – Problem-solving in real life context

To provide students with a setting or reason for the maths they are learning.



Activity 2: Connecting grouping in tens with place value

Have ready: Numicon 0–100 cm Number Line, Numicon Coloured Pegs or counters, Tens and Units Frame (photocopy master 47), sheets of wrapping paper with between 21 and 99 pictures, *Numicon Software for the Interactive Whiteboard* (optional)

Step 1

Show children the wrapping paper; discuss and agree which pictures to count. Look and listen for children who talk about tagging each picture with an object and then arranging the objects into groups of 10 (see Numbers and the Number System 1, Activities 4 and 5).

Step 2

When children have tagged the pictures with Pegs or counters and arranged them into Numicon 10-patterns, ask them to say how many they have in total, and how many tens and how many units they have. Look and listen for children who can say how many tens and units from the Numicon Shape patterns.

Activity 1: Introducing the word 'times' with repeated adding

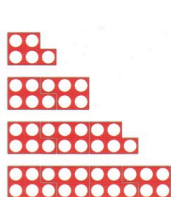
Have ready: Numicon Shapes, Numicon 10s Number Line, *Numicon Software for the Interactive Whiteboard* (optional)

Step 1

Set the scene by telling children about preparing for a school play about space travel. Four children will be dressed up as aliens, each wearing one mask, two silver sleeves, three wobbly antennae and five shiny badges. Ask children how many of each item are needed. Remind them about organizing work when working on problems and write a list of the different costume parts.

Whole Class, Independent, Explore More, and Explorer Progress

Well-structured activities to provide exploration, enrichment, reinforcement, review, discuss their learning, and to generalise their learning to other settings



$1 \times 5 =$	
$2 \times 5 =$	
$3 \times 5 =$	
$4 \times 5 =$	

Pairs Of Socks

Dad was sorting 17 socks into pairs.

Can you work out how many pairs Dad had and if there were any left over?



Whole-class

- Discuss with children how and when the mathematics they have been learning could help them in solving problems.
- Ask children to suggest things we do a number of times, such as the number of times they go out to play during the school day.

Paired work for Activities 5–8

Have ready: Numicon Shapes, number rods, Numicon Post Box, Multiplying Sentences for 2, 3, 5 and 10 Times Tables (cut from photocopy master 21)

Children take turns to post a multiplying card (photocopy master 21) through the Post Box. The second child completes the number sentence by filling in the product and posts it back. Children can check the product with Shapes or rods.

Learning Journal – Explorer Progress

To provide students the opportunities to share their thoughts and their learning

My Learning Log	
I can write a number story.	Date: / /
I know these facts.	Date: / /

My learning about numbers.	
Date: / /	
Reflections.	
Date: / /	
My favourite maths was...	
I would like more time to think about...	
I felt proud when...	