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Some Remediation Principles for Dyscalculia and Acquired Dyscalculia

Conditions of mathematics disabilities in one form or the other have been observed and described for about one hundred years (Gillespie, 1970; Wolman, 1977). It is almost hundred years since Josef Gerstmann, a German neurologist, defined a tetrad of symptoms for the condition that became to be known as Gerstmann syndrome that included finger agnosia. The syndrome later was implicated in the incidence of difficulty in quantitative calculations. However, it was Ladislau Kosc, a Czechoslovakian neuropsychologist's, work in the 1970, that formally introduced the term developmental dyscalculia.

Today, there is an awareness of the condition, but the parameters of the problem, its etiology, and its solutions are far from agreed upon.

A. Mathematics Learning Problems and Maths Anxiety

There are several reasons children, with and without learning disabilities, find learning mathematics difficult. Factors for the incidence of specific mathematics learning difficulties range from *language difficulties*, *neuropsychological* and *cognitive factors* to *environmental conditions. All of them,* to a varying degree, contribute to an individual's problems in learning and mastering numeracy and mathematics. An individual's consistent failures in numeracy create fear of learning mathematics and learned helplessness. This may convert into the condition popularly known as *mathematics anxiety*. Many people identify themselves experiencing 'maths anxiety'.

Mathematics anxiety is a person's emotional and psychological reaction to this consistent failure in mathematics. Generally, mathematics anxiety begins to be manifested when one has difficulty in learning one of the "*developmental milestones* in mathematics learning."[1] In early grades, a child's *math anxiety* may remain just a collection of symptoms about some mathematics difficulty and, in most cases, with "*effective teaching*" it can be remedied. However, after age 10, it begins to be internalized by the child and becomes a causative factor of mathematics learning difficulties and needs specialized remediation[2]. In many cases, these fears persist even in adulthood.

One of the causes of this cycle of failure, fear, and low achievement in mathematics, for many children, is known as ***dyscalculia.*** However, not all learning difficulties in mathematics are due to dyscalculia. Mathematics learning difficulties fall on a continuum of difficulties, problems, or disabilities. Students may have difficulty in learning Algebra with intact numeracy skills. That difficulty is not due to dyscalculia. Similarly, a student may have difficulty in learning geometry with intact numeracy and algebraic skills. However, some difficulties in algebra may be due to dyscalculia, particularly when numeracy is involved, and a student has poor numeracy skills. Dyscalculia, a developmental condition, *reflects difficulties in some of the associational skills basic to number conceptualization, number relationships (arithmetic facts), and understanding and executing numerical operations*. It affects early numeracy acquisition and then *wherever****[3]*** numerical manipulations are involved.

B. Dyscalculia, Acquired Dyscalculia and Acalculia

The term and condition of dyscalculia is better understood when we compare learning numeracy and literacy processes. ***Dyscalculia*** is to numeracy as dyslexia is to reading. Numeracy deals with ***number concept***, ***number sense*** and numerical procedures. ***Numeracy*** is a child's ability and facility to execute four whole number operations (e.g., addition, subtraction, multiplication, and division) correctly, consistently, efficiently, with conceptual understanding in multiple forms (including the standard algorithms) in diverse situations of their occurrences. Therefore, dyscalculia is a child's difficulty with numeracy and its elements.

Understanding the development of numbersense (number concept, number facts, and place value) in children provides a window into children's arithmetic difficulties, particularly dyscalculia. Most of the dyscalculia related difficulties in mathematics, particularly, in numeracy emanate from the difficulties experienced by a child in learning number concept and numbersense.

Dyscalculia is a quantity/number (with some overlap of spatial orientation/space organization) based disorder, so the remedial/intervention[4] should focus on the development of number concept, numbersense, and numeracy. The remedial interventions for children with math learning disabilities (including dyscalculics) initially should focus on the mastery of number concept—

1. *visual clustering* (generalized subitizing),
2. *decomposition/recomposition* of number,
3. and acquiring *sight facts*.

They form the triad of number attack skills.

Dyscalculia differs from ***acquired dyscalculia***—deficits in numeracy due to poor instruction, lower expectations, social emotional interferences, and poor reinforcements of learning. The number of individuals suffering from acquired dyscalculia is much larger than those with identification of dyscalculia. However, the manifested symptoms of acquired dyscalculia similar to dyscalculia. The causes of dyscalculia lie within the child, the causes of the incidence of acquired dyscalculia are outside the child.

Another learning problem that relates to numeracy is ***acalculia* —**a difficulty in dealing with number, numbersense, and numeracy, resulting from damage due to insult or injury to the brain. It is traumatic disruption of existing quantitative skills and memories related to mathematics skills. The symptoms may look the same as dyscalculia.

Dyscalculia or acquired dyscalculia, thus are the manifestation of difficulties in the integration of number concept, numbersense, and numeracy. However, just like effective teaching methods[5] for reading can mitigate the impact of dyslexia, similarly, one can have dyscalculia or acquired dyscalculia, but effective and efficient teaching methods can give students skills to minimize, counter, or mitigate the effects of dyscalculia and acquired dyscalculia (and other specific mathematics difficulties). Due to better understanding of the functioning of the brain, necessary cognitive functions, such as executive functions—working memory, focus/attention, organization, and flexibility of thought, numeracy can be improved. We can design effective, efficient, and elegant instruction materials and responsive teaching so a student can learn numeracy.

Dyscalculia typically occurs in cases of normal intelligence, although scores on some intelligence subtests related to spatial orientation/space organization may be low. Some numeracy skills might be present, but the levels are significantly low, particularly mastery of arithmetic facts and the ability to estimate the outcome of numerical operations. Dyscalculics calculate haltingly and simple arithmetical errors are often present in their work. A fact might be known in one context and not present in another. Most of the calculations are based on counting (finger reckoning, number line, hash-marks, counting discrete objects, etc.) and often with the help of other devices, such as calculators, number tables, and charts. Frequently, the dyscalculic will guess answers with superficial number relationships. Their number attack strategies are hardly based on decomposition/recomposition of number. They lack effective, efficient, and generalizable strategies.

Dyscalculics may know some facts, but they lack the understanding of the fundamental concepts of subitizing, decomposition/recomposition, properties of the Base Ten-System (making tens, teens numbers, next tens, place value, meaningful estimation, reasons behind procedures, etc.), properties of operations and the concept of equality. Therefore, they have problems in extending their limited knowledge of facts, place value, and meaningful numbersense.
Some dyslexic's poor spatial sense mixed with poor numbersense compounds calculation problems. Associated with the poor spatial sense is confusion with left and right, therefore, a good chance of problems in place-value and numerical procedures. Some dyscalculics and dyslexics alike have confusion about months, seasons, and judgements of time, direction, distance, and size. On tests of motor development and coordination, they are likely to score low.

Dyscalculic cases often have poor memory (short-, working, and/or long-term), and indeed do for reading and numeracy material. Their learning strategies are not well-established. They use inefficient strategies that do not help them to receive, process, retain, recall, and produce information properly. Poor teaching exacerbates it. For example, learning arithmetic facts by rote or by counting places heavy load on working memory and since, by this process, the facts are placed in the long-term memory in isolation (because of memorizing isolated facts by flash cards without strategies), their retention (because few, poor, no connections are made between facts and concepts) is difficult, and the recall of unconnected information is difficult. Since they have fewer facts mastered, they have difficulty making connections, extensions, and applications.

Dyscalculia is comorbid in almost 40% of the dyslexia cases. Because of student's poor spatial sense and lack of organization skills, it is also comorbid with ***dysgraphia*** in a substantial number of cases. Dyscalculia thus represents an extreme and continuing lack of readiness for numbersense. In addition, domain-general cognitive risk factors, such as ***slow processing speed*** and ***working memory*** might be shared between the two disorders (dyslexia and dyscalculia) and could possibly explain why they may co-occur.

This comorbidity between dyslexia and dyscalculia indicates that there are fundamental components in both: *phonemic awareness* and *decomposition/recomposition*, respectively. On the other hand, high comorbidity rates between reading disorder (RD) and mathematics disorder (MD) indicate that, although the cognitive core deficits underlying these disorders are distinct, additional domain-general risk factors might be shared between the disorders. Three domain-general cognitive abilities *processing speed*, *temporal processing*, and *working memory* are studied in RD and MD literature. Since *attention problems* frequently co-occur with learning disorders, the three factors, which are known to be associated with attention problems, account for the comorbidity between these disorders. However, the attention problems observed in the case of MD, some of them are secondary, in the sense, that they might be the by-product of consistent failure in mathematics rather than the causative factors.

After controlling for attention, associations with RD and MD differed: Although *deficits in verbal memory were associated with both RD and MD*, *reduced processing speed was related to RD*, *but not with MD*; and the *association with RD was restricted to processing speed for familiar nameable symbols*. In contrast, *impairments in temporal processing and visuospatial memory were associated with MD, but not RD.* Visuospatial memory is essential for ***visual clustering***, ***decomposition/recomposition***, and therefore with ***development of sight facts***.

In the absence of effective interventions or with poor remediation, dyscalculic cases begin to show emotional distress and lack of confidence and math anxiety. They begin to manifest ***specific math anxiety***—anxiety activated by numerical and quantitative demands. Mathematics anxiety, in turn, exacerbates the difficulties in mathematics learning. The begin to avoid studies and professions related to STEM (Science, Technology, Engineering, and Mathematics) fields.

A dyscalculic may not show gross defects on a neurological level, however, in some cases, there may be some non-focal abnormalities in brain functioning. On neuropsychological examination, there may be some deficits in some aspects of ***executive functioning***. There may be some visuospatial-perceptual integration issues. Visual perception integration, working memory, and spatial orientation/space organization are highly correlated with mathematics learning and their poor development becomes a factor in the incidence of dyscalculia. As in all learning disability cases, children with mathematics difficulties, whether dyscalculia or other mathematics difficulties, feel inadequate, stupid, and guilty of their disability and their repeated failures.

The specific elements of disability associated with dyscalculia appear to be inefficiencies in the development of number concept: (a) associating and integrating (i.e., visual representation of a collection of objects, orthographic representation, and the oral representation of quantity); (b) decomposition/recomposition of number in developing arithmetic facts and operations; (c) developing and mastering arithmetic facts; and (d) estimating the outcomes of numerical operations. These deficiencies and inefficiencies grow into problems in mastering numeracy.

C. Catch Them Before They Fall

The neurobiological revelations in recent neurological research are inspiring new treatments to a variety of disorders. Given the importance of neuroplasticity in very young children, specialists now advise the opposite of a wait-and-see approach. The brain findings affirm the idea that we want to get involved in helping children as early as we can. Early identification and assessment of number concept are essential to prevent numeracy failure in young children and avoiding future mathematics difficulties. Students without adequate mastery of ***sight facts***[6] and ***decomposition/recomposition*** early, continue to demonstrate poor numbersense and numeracy skills, even into the middle grades and high school. There is a strong predictive validity of later mathematics achievement in mastering the components of early number concept: decomposition/recomposition and sight facts. Children continue to use the number concept and decomposition/recomposition whenever they encounter numeracy problems. For example, when students encounter work on fractions, integers and rational numbers, they continue to need and use them for new number relationships.

Research in reading processes indicates that domain-specific deficits such as measures of a child's phonemic awareness and phonological sensitivity are the best predictors of early reading performance (better than IQ tests, readiness scores, or socioeconomic level). It is widely accepted that deficits in phonological processing are the proximal cause of Reading Disability (Vellutino, Fletcher, Snowling, & Scanlon, 2004)).

Similarly, a domain-specific deficit in processing numerosities (***numberness***[7]***)*** has been implicated in mathematics disabilities (e.g., dyscalculia) (Butterworth, 2010; Wilson & Dehaene, 2007). Of the several sub-skills of numberness the most important skill involved as a deficit is decomposition/recomposition of number.

D. Interventions and Remedial Strategies for Dyscalculia

The following principles of remediation provide a framework to build effective interventions to use and practice. Several principles are restatements of principles common to all efficient and effective learning processes.

The Latin root for ***remediation*** means “to heal.” Dyscalculic students are behind in their level of performance, they need more than healing, they need acceleration. The Latin root for ***acceleration*** means “to hasten.” They need healing acceleration. They are not broken; they are just behind. The most important objective of *any intervention should be making students feel that they are still whole—not broken and they can progress faster than before.*

The important guiding principle in mathematics remediation is to help children acquire not only the necessary ***mathematics content***—language, concepts, procedures, skills, and their relationships, but also improve their ***learning ability***—cognitive, affective, perceptual, psycho-motoric, executive functions, and thinking skills. With this dual intertwined focus and corresponding strategies, student achievement and interest in learning mathematics can be improved. This creates conditions for maximizing their endowment and potential and thus accelerate their growth.

Another important principle is: *The student should get comprehensible, equitable input from instruction. It should have mathematical agency. Instruction is effective when it creates positive mathematics identity for the student.*

To achieve these dual goals, the theoretical foundations for intervention in mathematics education described, in following pages, are informed by:

- Piagetian and post-Piagetian research on how children learn informs us about what, when and how certain content is acquired by a student—the sequence of their acquisition and their relationship with cognitive development. It guides us in framing, sequencing, and presenting content. The child’s intellectual endowment guides the content—task formation, task analysis, identification of prerequisite skills, selection of activities, choice of instructional materials, and the emphasis points in the tasks to practice.
- The elements of pedagogy of the exercises/lessons draw heavily on research by Vygotsky, Bandura, and Skemp. Accordingly, various aspects of mediation are the cause of improved cognition for a particular content. That means the language, questioning process, and feedback loop must be appropriate and effective in its impact.
- The various mediating agents and their nature of implementation, their frames of reference, and methods of organizing the environment, and language derive from Feuerstein’s work on mediation learning and modifiability of children’s abilities.
- The expectations for the learning outcomes of mediation, nature and type of practice and mastery, focus, and feedback are informed by recent research about brain’s plasticity and potential, particularly the understanding the role of the memory systems: short-, working, and long-term memories.
- Presentation of skills and interactions with dyscalculic children are guided by science of reading research and nature of the problems exhibited by children and adults with dyscalculia and acquired dyscalculia.
- To acquire mastery in a concept or skill in arithmetic the determination of the nature and type of practice is important. It is informed by research in the area of optimal performance by novice and expert performers.
- Research in mathematics information shows the ideas are learned, retained, and recalled better when instruction emphasizes the integration of mathematics language, concepts, and procedures. The three components require different kind of teaching and amount of practice.
- Research in core brain physiology clearly shows that the ubiquitous behaviours related to learning are ***pattern generalizing*** for *understanding,* ***visualizations*** for *understanding* and *processing* of memory,

and ***making/using connections*** for *understanding* and *recall*. All brains use these functions at the core circuitry level not just for maths.

- The learning difficulties of dyscalculic children necessitate close adherence to effective general learning principles, effective use of time specific to task and focused concept and skill specific strategies.

These frameworks give rise to some working principles for remedial instruction. However, we should not forget that the incidence of acquired dyscalculia can be drastically reduced if regular classroom teaching also follows these principles.

Many teachers learn to violate general learning principles because normal children have such powers of independent learning that convenient violation of the principles do not handicap a child for long. And gifted and talented children may not even adhere to any general learning principles, because these children learn despite us. When we adhere to important general principles and learning strategies specific to dyscalculic children, then they sometimes make more progress in one session of well-planned, focused, direct remediation than they previously made in a year or more of haphazard remediation. This is because, in later case, violations of learning principles produced negative learning, confusion, and a sense of failure. *Remediation that does not help is likely to be harmful*.

In the child's development ***imitation*** and ***instruction*** play a major role. Imitation happens when the examples of models of behaviours and learning that we want children to imitate are present in abundance in their environment. Even in remediation, whenever possible work with a small group of children rather than one-to-one so that some level of imitation is possible. Vygotsky so rightly said:

"*In learning school subjects, imitation is indispensable. What a child can do in cooperation today he can do alone tomorrow. Therefore, the only good kind of instruction is that which marches ahead of development and leads it; it must be aimed not so much at the ripe as at the ripening functions. It remains necessary to determine lowest threshold at which instruction in , say, in arithmetic may begin since a certain minimal ripeness of functions is required. But we must consider the upper threshold as well; instruction must be oriented toward the future, not the past.* " (1962, p. 104)

This means instruction must be oriented to the child's strengths and in the process reducing the impact of the weaknesses and improving those functions that are contributing to the weakness. For example, suppose a child has strength in spatial orientation and weakness in executive function, such as working memory and does not have mastery of arithmetic facts. In such a case, using *Cuisenaire and Visual Cluster cards*[8], the child's arithmetic facts are strengthened, and, in the process, the working memory is also improved.

Explicit teaching of phonemic awareness skills and sound blending skills is important, but instruction that integrates 'in how to blend phonemes together' and how to 'pull apart' or 'segment words into phonemes' is more useful to students to acquire reading skills. Organized and intense supervised practice in building *sight vocabulary*, '*pulling apart,*' and '*blend it together*' converts novices into fluent readers.

The same principles work in teaching children numeracy and other related concepts. Learning sequential counting, one-to-one correspondence, and writing numbers in isolation are useful to an extent. But an instruction that focuses on integration of visual clustering (expanded subitizing), building sight facts, and decomposition/recomposition is even more important and productive. An organized, early, intensive supervised practice in the integration of these components develops effective number concept (numberness) and then aids in the optimal development of numbersense and numeracy.

In teaching number, most teachers help children acquire the number concept by connecting the *phoneme* (sound s-e-v-e-n, as they count) with *graphème* (symbol 7, looking at the number and writing it). But that is not enough because acquiring numberness is more than learning to read a number, write a number, or even count. It is integrating, the cluster (quantity), phoneme (the name), grapheme (the shape) associated with the number. When numerosity and oral representation are learnt before the writing of numbers, children develop number concept faster. The judicious integration of the two expedites the process.

Many planners of mathematics instruction for young children often do not fully consider that to increase proficiency, competence, and fluency with basic addition and subtraction facts, **children need to develop solid number concept (numberness) and flexible numbersense.** They stop short: as soon as a child can count one-to-one, *they assume the child has the concept of number.* Sequential counting and one-to-one correspondence, even when it is converted into conservation of number is not enough for competence in developing numbersense.

When one hears or sees a number, one does not see discrete objects; instead, one sees or hears a collection represented in its abstract symbolic form. Children can count objects one-by-one, but they have difficulty recognizing visual clusters of objects as represented by specific numbers. Counting is like recognizing and decoding individual letters and their sounds and recognizing visual clusters for individual numbers is like recognizing phonemes, syllables, and even words.

Able to decode individual letters does not make a child a fluent reader. For proficiency and fluency in reading with comprehension, one needs grapheme-phoneme connection and practice in blending sounds—the nature of sound and its relationship with orthographic form. Similarly, the ability to count does not make a child fluent in numberness or numbersense.[9] One must blend numberness of two numbers (i.e., producing sight facts and addition facts) to produce new numbers and break numbers into its components (i.e., producing subtraction facts). Arithmetic facts (blending of two numbers) and place value (blending of two or more numbers to represent large numbers) are like identifying the phonemes in a big word and then blending of those sounds in reading that word. The '***word-attack***' and '***number attack***' are, despite some specific differences, are parallel.

Weaknesses in *phoneme awareness* (PA), *rapid automatized naming* (RAN) and *working memory* (WM) are strong and persistent correlates of literacy problems, particularly spelling, even in adults. Similarly, decomposition/recomposition, working memory, and rapid automatized naming are strongly related to addition and subtraction and then with multiplication and division.

Strategies and instruction in arithmetic facts and procedures are much more productive when a child has acquired the number concept properly:

- Large Number Vocabulary (encourage the child to count every day for few minutes; while counting attention is brought to the patterns in numbers and structure of the number system),
- Visual Clustering[10] (extension of Subitizing),
- 45 Sight Facts[11] (i.e., with every number N, up to 10, N - 1 sight facts are associated: E.g., 7 = 1 + 6 = 2 + 5 = 3 + 4 = 4 + 3 = 5 + 2 = 6 + 1),
- Decomposition/recomposition (breaking a number according to context and combining two numbers by sight),

- Orthographic representation of number and recognition (writing a number properly with proper strokes and directionality), and
- Integrating these elements to acquire ***numberness***.

True number concept and iterative application of decomposition/recomposition is the basis of deriving addition and subtraction strategies:

[**Commutative Property of Addition;** N + 1 and 1 + N; *Making ten; N + 10 and N + 10; N + 9, 9 + 10 (add 10 – 1); N+N (doubles), N + (N+1) (doubles plus 1), N + (N - 1) (doubles minus 1), N + 2 (2 more), (N + 1) + (N - 1) (2-apart)*].

Mastery of arithmetic facts, thus, is dependent on strategies built on Number concept: ***Visual clustering, sight facts***, ***making ten***, ***Teen's numbers,*** and ***decomposition/recomposition***. For example, 8 + 6 = 8 + 2 + 4 (applying the knowledge that we need 2 to make 8 into 10 and 6 is decomposed into 2 + 4) = 10 + 4 (8 and 2 are recomposed into 10 and with the knowledge of teen's numbers—place value, we get 8 + 6 = 14).

For fluency of numbersense and flexibility of thought, a child should derive the same fact using other decompositions and decompositions and choose the most efficient or most contextual, for example: 8 + 6 = (7 + 1) + 6 = 7 + 7 = 14; 8 + 6 = 2 + 6 + 6 = 2 + 12 = 14; or, 8 + 6 = 8 + 8 - 2 = 16 - 2= 14.

Using several strategies based on sound principles help make connections and develop ***flexibility of thought***—an important component of executive function and an ingredient for the development of cognitive potential.

As sight vocabulary and phonemic awareness are to the reading process, sight facts and decomposition/recomposition are to the development of number concept, numbersense and numeracy, therefore, they act as antidotes to the incidence of acquired dyscalculia and development of remediation strategies for dyscalculia*.*

To design effective remedial methods, it is essential to understand the definitions and characteristics of dyscalculia and the developmental trajectories of number, number relationships, and numerical operations. Gifted children may compensate for even massive deficits using one or more of their equally massive strengths. A child with tremendous memory and fantastic oral comprehension might be able to get around abysmal arithmetic fact fluency for a while to produce adequate arithmetic results. But if they have deficits in the understanding, fluency, and applicability of number concept and numbersense and procedures, they will have difficulty in future mathematics. Similarly, a child using counting methods may be able to do well on tests and exams, in the early grades, will have difficulty later.

1. Focus on Key Concepts

Due to the cumulative nature of numeracy and mathematics, few key concepts are important to focus to help dyscalculic students to become numerates and prepare them for future mathematics. They are:

(a) ***Number concept***—Mastering numberness, decomposition/recomposition, and 45 sight facts,
(b) ***Additive Reasoning***: Mastering addition and subtraction concepts and facts—45 sight facts, decomposition/recomposition, making ten, teens' numbers, next-tens, and adding multiples of ten to a number, and commutative property of addition, and mastery of sums and differences up to 20,
(c) ***Place value:*** Forming large numbers and decomposing large numbers in the base ten system,
(d) **Multiplicative Reasoning:** Mastering multiplication and division concepts and facts—four models of multiplication and division[12], decomposition/recomposition, 10 × 10 multiplication and division facts.

These are parallel to what we do with dyslexic students to become fluent readers:

(a) constantly increasing **sight vocabulary**,
(b) sustained, systematic work on phonics and phonological sensitivity to 'breaking the code' and build ***proficiency*** by focusing on intense practice in phonemic awareness, and
(c) repeated readings, interesting and motivating, using efficient strategies for blending sounds to build ***fluency***.

This process helps children to move from decoding of individual letters to chunking and blending sounds. This insight helps interrupt the cycle of failure for poor readers.

Phonemic awareness and decomposition/recomposition are parallel processes in literacy and numeracy. As focus on decoding individual letters for reading and counting to derive arithmetic facts are parallel misconceptions in literacy and numeracy.

The reading research demonstrates phonemic awareness (PA) is one of the biggest building blocks of the reading success, similarly, decomposition/recomposition process is the building block to mathematics success. PA should not be abandoned until the child has demonstrated advanced levels of PA skills. However, decomposition/recomposition, as a process, is present in mathematics at all levels; it is never abandoned. Its arena moves—from whole numbers to other numbers (i.e., fractions, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers). *Dyscalculics' biggest problem is decomposition of numbers contextually.* Practice in this skill is the key to solving their arithmetic facts mastery problem. **Mastering the decomposition/recomposition of numbers makes the difference in mathematics—it is the key.**

Working with hundreds and thousands of children, we have observed that fluent numberness is a better predictor of future proficiency and fluency in arithmetic and even higher mathematics. Remediation of numberness related key skills result in better understanding and mastery of numeracy. With proper intervention using appropriate instructional materials, it can be developed easily.

Proper Intervention begins with: Looking at a cluster of objects (up to ten) on a Visual Cluster Card[13], Domino, Dice, or another pattern-based arrangement of objects and recognizing it and giving it a numerical name, instantly, is the goal. However, to get there we need to focus on recognizing smaller clusters and as a starting point even some counting. As in reading a word, we do not focus on each letter, nor we begin with the whole word memorization, similarly, in ***numberness***, we neither focus on one object at a time nor on the whole cluster to start with. When it is absent or is not at the appropriate level, then remediation of number concept should be the focus before anything else.

The process is three-fold: (a) instantly associate a visual cluster (*quantity*) with its number representation (*orthographic image*), (b) associating a sound (*aural-auditory representation*) with number (*quantity or orthographic image*), and (c) decomposition and recomposition (*recognizing parts-to-whole* and *whole-to-parts*).

The recognition of a visual cluster and decomposing a cluster into its sub-clusters and combining sub-clusters into a larger cluster and instantly recognizing the larger cluster as the combination of the sub-clusters are the key processes to acquiring the number concept and numbersense (arithmetic facts and place value).

The efficient, elegant, and effective methods applied using proper instructional principles works for all children. Unfortunately, dyscalculic and learning-disabled children are taught using inefficient methods. School personnel often due to their own limited understanding of dyscalculia and mathematics disabilities may affect resources available for appropriate instruction and interventions in quantity and quality. The problem is compounded by the inadequate preparation of special education teachers in mathematics and mathematics learning disabilities. Many parents and school personnel mistakenly assume that dyscalculia equates to permanent condition of deficit or disability. Since, true number concept is at the basis of the development of fluent numbersense, the condition of dyscalculia and acquired dyscalculia can be positively affected proper teaching of number concept. However, it is difficult to master[14], arithmetic facts and procedures without proper number concept teaching.

2. Role of Practice and Integration of Old and New Learning

Because of dyscalculics' avoidance of mathematics tasks, practice of the new learning and review of the previously learned material must be planned and conducted in every session using efficient strategies that are based on decomposition/recomposition of number. The practice should be performed orally (first of the basic arithmetic facts and then of extended facts using scripts that are derived with efficient strategies)[15] and then writing and applying them in problem situations. Review and practice should be slanted towards rapid discrimination[16] and forming rapid associations—all being automatic skills.

Every remedial lesson should be divided into three parts:
(a) ***Tool Building component***—developing pre-requisite skills (content and learning skills related) (¼ of the session),
(b) ***Main Concept***—language, concept, procedure, and skills) (½ of the session), and
(c) ***Review and reinforcement:*** reteaching, practice, integration of skills and application (¼ of the session).
For example, let us assume that the focus of Main Session is on deriving and mastering facts of the type: N+ 9. Then Tool Building will focus on prerequisite skills will be on:
(i) 45 Sight Facts, (ii) Decomposition/recomposition of number, (iii) Commutative property of addition, (iv) N + 1, 1 + N, (v) N + 1 (N - 1), (vi) Making 10, and (vi) Teens' numbers, and (vii) Decomposition/recomposition Script[17].
Similarly, for developing other strategies for arithmetic facts, one will identify and work on the appropriate pre-requisite skills. However, the Tool Building component will also include newly learned strategies. The sequence of teaching/learning the strategies[18] for addition facts is: (i) *N + 9*, (ii) N + N, (iii) N + (N +1), (iv) N + (N + 2), (v) N + 2, (vi) Facts near 10, and (vii) Remaining facts 4 + 8, 8 + 4, 5 + 8, and 8 + 5.

Again, before the child derives, learns, and practices the 10 × 10 multiplication facts (multiplication tables), one should have mastered: (1) 45 sight facts, (2) Making Ten, (3) Teens numbers, (4) Identifying next tens (e.g., next tens to 33 is 40 and for 48, the next tens is 50) (5) Making next tens, (6) Mentally adding multiples of 10 to any number (i.e., 46 + 20 = 66), (7) Commutative property of multiplication, (8) Decomposition/recomposition of number and/or Distributive property of multiplication.

The order for mastering multiplication facts (the tables) is: Tables of 1, 10, 2, 5, 9, 4, 3, 6, 7, and 8. For example, once the student knows the tables of 1, 2, 5, 10, and 9, she knows 75 facts out of 100 (10 × 10 facts) and number left to learn is 25 facts. But, by using commutative property of multiplication, one has only 13 new facts to learn. These 13 facts can be derived by decomposition/recomposition and with the knowledge of facts already mastered. For example, to derive 8 × 7 = 8 × 7

= (8 × 5) + (8 × 2) (by decomposition/recomposition; using sight facts) (always break one of the numbers into 5 + another number)
= 40 + 16 (by using the known tables of 5 and 2)
= 40 + (10 + 6) (Place-value)
= 50 + 6 = 56.
7 × 8 = 8 × 7 (by commutative property of multiplication)
= 56.

First, these facts are derived at *Concrete level of knowing (**C**)*[19] using concrete materials (e.g., Cuisenaire rods using area model of multiplication), then *Pictorial representation* (***P***) (area model), then *visualizing and manipulating the steps* (***V***) in the working memory (to transfer the information from working memory to long-term memory) using the ***script*** developed during the concrete and reinforced during pictorial representations, and finally the facts are derived at the *abstract level* (***A***) using the script at the abstract level. This is known as the ***CPVA*** level.

In ordinary situations, transfer of learning from concrete to abstract is difficult for many children including dyscalculics. Visualization of the process in the working memory is the link for the transfer of learning from concrete to abstract. The concept of decomposition/ recomposition, appropriate instructional materials, proper language, questions process, adequate challenge, and caring, immediate, and constant feedback make integration of executive functions and the content acquisition.

Many teachers and researchers use *addition is counting up*, *subtraction is counting backwards*, *multiplication is skip counting* and *division is skip counting backwards* the default approach to teaching and researching numeracy issues. This is particularly so in special education settings and in most regular classrooms. It works for many students, but it is not a useful approach to dyscalculic students due to their difficulty with acquiring frequency. Special educators need to understand and use those principles and strategies that meet the needs of all students, including dyscalculics. Some students may arrive with workable strategies despite counting as the method of teaching. However, the strategies derived and used should be *effective, efficient,* and *elegant*[20]. Since some students will find learning numeracy exceptionally difficult (because of dyscalculia), numeracy instruction should be oriented toward a structured strategy numeracy model (CPVA). Orton Gillingham Method for reading and CPVA for numeracy are parallel.

2. **What is Intervention and Remediation**

Intervention is well understood in the medical practice but less well understood in the context of education, except reading. The purpose of medical intervention is to assist the patient realize his/her genetic potential and optimize its functioning. The effect of an intervention is then assessed by measurements and clinical observations to see the extent to which the patient approaches the norms expected. The interventions also happen to prevent disease, to fix some deficits—a defective heart valve, for a person to function optimally. Some interventions correct the impact of environmental variables—urban Asthma, accident. The role of a patient in interventions varies—from minimal as in surgery, except pre- and post-surgical precautionary care to involved as in changing lifestyle.

In the context of reading the nature of intervention is also clear: When a child does not know how to read, is poor reader due to inferior instruction, or displays reading disabilities such as dyslexia, s/he is sent to a reading specialist. She does the diagnosis, task-analysis of needs, and introduces the skills systematically, carefully, and helps the child practice them judiciously and properly till they are automatized. Her methods, strategies, and approach define the scope and level of intervention. She helps the child to learn the sub-skills of reading till child integrates them and can read. She makes sure that the child learns to read as fluently as possible with comprehension. It is a systematic process. There is a beginning, the middle, and end to the process. Then the individual applies it to learn anything and everything. The individual is transformed. As Stanislaus Dehaene writes in his book: "*The reading brain is a different brain*." The reading process has transformed the child's brain.

Proper interventions work, in both professions—medicine and reading. The process is fascinating and a marvel to watch. It is truly more than the *"rocket science."*

What is then the equivalent process in the case of learning problems in mathematics—whether the mathematics difficulties are due to environmental reasons or specific mathematics disabilities such as dyscalculia? What is a suitable model for intervention, treatment, and successful outcomes in mathematics?

In the field of mathematics learning problems, the issues are not yet clear and well-defined. Researchers and practitioners use a variety of definitions for diagnosis and interventions. The processes are not well-developed. There are many reasons for this.

First, **mathematics learning is developmental and cumulative**. As the child with deficits is ***catching up*** the earlier mathematical developmental milestones[21], new and more complex topics and skills are being introduced. When the students have a modicum of understanding of whole number concepts and mastery of operations on them, they are introduced to new type of numbers in quick succession—fractions, decimals, integers, rational numbers, irrational numbers, real numbers, imaginary numbers, complex numbers, etc. The cognitive and emotional demands of mathematics learning keep on increasing. Therefore, dyscalculics need efficient, effective, and elegant strategies so they can be accelerated. The complex of cognitive, perceptual, linguistic, and neuropsychological traits and skills needed to learn a myriad of *quantitative* (i.e., arithmetic and algebra), *spatial* (i.e., geometry), and *integrated domains* (i.e., measurement, coordinate geometry, higher mathematics—calculus, etc.) is quite complex. Interventionists should be well versed in numeracy and early mathematics so that they can anticipate student problems.

Second, the nature of mathematics learning problems is multi-faceted, ***from*** difficulties due to dyscalculia, which relates to numeracy only ***to*** mathematics difficulties due to complexity of content, language and/or reading deficits. There is a range and diversity in the distribution of learner differences due to manifestation of children's learning difficulties in mathematics.

Third, as students move from grade to grade, some of the mathematics learning difficulties are contributed by the nature of mathematics itself. Different mathematics content areas—arithmetic, algebra, geometry, probability, and their integration (e.g., coordinate geometry, calculus, etc.) pose their own specific difficulties.

Dyscalculia and acquired dyscalculia deal with only numeracy—concepts, skills, and procedures. Dyscalculia may affect other areas of mathematics learning where numerical operations are involved. On the other hand, a student may have difficulties in learning mathematics without having any dyscalculia. Even after a student has acquired sound numeracy skills, because of language, spatial, and reasoning difficulties he may encounter mathematics learning difficulties. Therefore, not all mathematics difficulties can be categorized as dyscalculia. The following illustrates the continuum of numeracy and mathematics learning difficulties.

Therefore, developing and implementing an intervention program in mathematics education is not a one-shot deal—a quick-fix activity. It involves important social, technical, practical, and intellectual—content and pedagogy considerations and decisions.

Schools and parents need to consider:
Who should do the intervention?
What should be the focus of intervention at different grades?
What should be the nature of interventions?

3. A great deal of these decisions are hinged by what is affected by dyscalculia.
When we compare acquiring literacy and numeracy skills, we become aware that our focus in intervention should be on developing numeracy skills in children. Since teaching higher mathematics is the domain of those who are well versed in mathematics and only those who are interested in learning mathematics join them. This paper, therefore, does not deal with mathematics related difficulties. However, developing numeracy is fundamental for survival, being secure and, functioning as a productive citizen in a highly technological society. In a democratic society, a well-informed citizenry demonstrates facility in numeracy. The fundamental role of interventionists is to focus on developing numeracy.

Learning and teaching numeracy is informed by several fields. For example, *cognitive psychology* informs us how children conceptualize and learn mathematics; *neuropsychology* informs us about what role do neurological and psychological skills such as executive function play in receiving, comprehending, retaining and communicating mathematics learning; through neurology we learn about what and when it is possible for a child to learn particular mathematics concepts; *linguistics* is important for us to know about the role of language in conceptualizing and communicating mathematics ideas; whereas the keen knowledge of pedagogy helps us to make it all happen.

All of this calls for developing a cadre of people who are skilled in this information and can work with children to make it happen. This is not any different than the preparation of reading specialists in schools who are specially trained in working with children with reading related difficulties, disabilities, and specific problems such as dyslexia. These math learning specialists should be skilled in working with difficulties, disabilities, and specific problems with mathematics such as dyscalculia. A generalist educator or a generalist special educator is not equipped for remediation of learning problems in mathematics. As a professional role, it is an emerging one.

These specialists, in their work with children should focus on key developmental milestones of mathematics: Number concept, Additive reasoning, multiplicative reasoning, Place value, and proportional reasoning. Their work should be in addition to students' classroom work.

Therefore, it is important that the ***interventionists*** prepare and engage in

- Addressing essential aspect of **learning gaps** of students that may prevent them from reaching their potential.
- Developing a deep understanding of the ***conceptual connections*** in spatial, additive, multiplicative, and proportional reasoning to build student capacity to learn new concept.
- **Developing the confidence and knowledge** of the math content she teaches so that colleagues seek her advice about lessons, children, and strategies about teaching dyscalculic children, just as they seek the advice from reading specialists.
- Helping students ***think like mathematicians*** so they begin to think themselves capable of learning and then learn and make connections between different concepts.

4. What to focus on during remediation?

Intervention takes place at two levels: *Classroom level* and then at *individual level*. Most significant issues that we confront in implementing intervention strategies are not merely technical but social.

From student's perspective, the intervention takes place at three places:
(a) ***Preventive (****by special education teacher/tutor****)***: before the class pre-teaching or in the beginning of a lesson, when the teacher helps her students prepare for a topic, concept, skill to be covered during the lesson (i.e., *tool building segment*),
(b) ***During the class lesson (****by classroom teacher****)***: During the (i) lesson when the teacher specifically helps each student in a particular aspect of the concept or procedure being taught (*scaffolding*) by giving a specific example to a specific student, (ii) during the class when the teacher connects concepts (from earlier learning and to future concepts) and differentiates the content by using differentiated questioning and examples, (iii) during the class when the teacher does extensive task analysis of the content for the benefit of all students, including dyscalculics, (iii) provides timely feedback to all students, including dyscalculics, and (iv) provides help at the end of a session and,
(c***) Intentional specialized intervention*** *(outside the class by a specialist)*: focused, diagnostic, guided instruction practice and reinforcement more than the teacher/tutor provides, specific practice toward the end of the class or outside of the class on a particular concept, skill or procedure.

Focus here does not mean one small skill—unrelated to others, in isolation. Children do not develop at a single point on the continuum of learning mathematics, but across a wide range of ideas that are often related. While our focus may be the centre of the range of a concept, children are also just beginning to develop at the far reaches of their ability, at the same time they are finishing off mastery of skills which were the focus of teaching weeks or months earlier. Learning and teaching across a range is better, as opposed to focusing too narrowly on a single point in time and content. While the goal is certainly to develop not just a skill—like facts, or telling time, or a definition in geometry, but there should be always an underlying concept aimed at proficiency, for example, numbersense. We should not dwell too long (extended time on a session) on lower-level skills to the detriment of developing numbersense. That doesn't mean we neglect those skills entirely, or that some group of children will not benefit from a stepwise development of arithmetic facts alone that culminates with arithmetic mastery.

In educational intervention, the student and the interventionist have almost equal role. However, the task and the activity determine the focus of the role—orchestrated and directed by teacher, student, or both. For example, the process of "scaffolding,"[22] where it works is initiated by the interventionist but is followed through together by the teacher and student. Then the student practices in the presence and under teacher supervision and later alone.

If the intervention assists students in improving cognitive skills along with content, then this dual development will produce not only measurable effects on psychological tests and on content learning outcomes, but also accelerated learning. The latter happens because the psychological—both socio-emotional and cognitive development increase learning ability, potential and eagerness to learn.

About the Author:
Mahesh is the founder and President of the Centre for Teaching/Learning of Mathematics; former President & Professor of Mathematics Education at Cambridge College, and internationally known for his ground-breaking work in dyscalculia and other maths learning difficulties.

HYPERLINKS below:

[1] The milestones are: *Quantitative reasoning domain*: (a) Number concept, (b) Number relationships (arithmetic facts), (c) Place value, (d) Concept of fraction, (e) Operations on integers, and (f) Algebraic thinking.
Spatial Reasoning domain: (a) Spatial orientation/space organization from egocentric perspective, opposite perspective, and any perspective, (b) Classification of objects with multiple perspective, (c) Relationships between objects and their parts, (c) Compound objects.
Integration of quantitative and Spatial reasoning: (a) Measurement of different attributes of objects (idea of unit and its usage—length, temperature, area, volume, weight, compound measurements, derived units, etc.).
[2] See *Mathophobia* by Sharma (*Math Notebook*, 200_) and *Causes of Math Anxiety I and II* by Datta (Math Notebook, 200_).
[3] Any setting—academic or otherwise. For example, an individual (child/adult) with dyscalculia having problems with change, making tip or calculating interval of time.
[4] Intervention should be remedial acceleration. We call it vertical acceleration.
[5] For example, programs such as Orton Gillingham approach.
[6] See How To Teach Number Concept Easily and Effectively: Preventing Learning Difficulties in Mathematics by Sharma, 2019. ***Sight facts***, just like ***sight words*** are number relationships acquire by just visual observation. This is a perceptual activity.
[7]Numberness is the integration of the recognition of a cluster of objects, knowing its number name, its orthographical representation, and decomposition/recomposition of the cluster into sub-clusters.
[8] The information presented through instructional materials that have color, size, shape, and pattern enters short-term memory and can be held in the working memory longer and can be manipulated.
[9] See How To Teach Number Concept: Preventing Mathematics Failure (Sharma, 2019) an eBook.
[10] *Same as above*
[11] See Sharma How To Teach Number Concept Easily and Effectively: Preventing Mathematics Difficulties (eBook, 2019).
[12] The four models of multiplication and division: *Repeated addition* (*repeated subtraction*); *Groups of* (*partitioning*), *Array (Array)*; and *Area model/Area model*. The area is the most general model for both as it applies to fractions, decimals, and algebraic multiplication.
[13] We have achieved a great deal of success in mastering number concept, fact mastery using Visual Cluster Cards (VCC) and Sight Fact Cards (SFC) and Cuisenaire rods. VCC and SFC are specially designed for teaching number concept and facts. Originally, they were derived for dyscalculic children and adults, but now we use them with all children and the results are very impressive.
[14] Mastery means: presence of (a) appropriate language, (b) efficient, effective, and elegant strategies , (c) fluency, and (d) ability to apply.
[15] See Addition and Multiplication ladders for arithmetic facts and extended facts, CT/LM
[16] ***Addition Ladders, Multiplication ladders, Decomposition/recomposition exercises***, Visual Cluster cards War Games are very effective devices for practicing and reviewing arithmetic facts.
[17] The decomposition script for N + 9 or 9 + N will be: to find 9 + 7: Is the answer more than ten, less than ten or ten? Which is bigger 9 or 7? How do you make 9 into 10? Where is 1 coming from? After taking 1 from 7 what is left in 7? Now, what is 9 + 1? What is 10 + 6? So, what is 9 + 7? What is 7 + 9? Etc.
[18] Strategies for Teaching Addition Facts (Sharma, 2008)
[19] See The Mathematics Notebook on Levels of Knowing Mathematics by Sharma (20,,,,)
[20] Effective means that the strategy is not just problem specific and its application results in a student accessing the problem or getting the result; efficient means it takes less effort and supports and capitalizes on even the limited working memory space; and elegant means that it can be generalized to other situations, extrapolated to various settings, and abstracted to other number systems and procedures.

[21] The developmental milestones in the quantitative domain are: Number concept, number relationships (arithmetic facts), place-value, fractions, integers, and algebraic thinking; in the quantitative domain they are: Spatial orientation/space organization, geometrical objects and their elements (e.g., shapes, figures, diagrams, etc.), measurement, properties and relationships between geometrical objects, and geometrical thinking.

[22] In certain educational contexts scaffolding is a useful component of conceptual structure for delivering intervention. Studies have shown that scaffolding is successful in providing intervention in one-to-one or very small group setting but has limited impact in large group intervention. For example, scaffolding in mathematics tutoring in small groups is very effective. Pulling it off in a regular classroom setting requires the teacher to be especially adept in the mathematics content (the language, conceptual trajectory, and tools), strategies, and understanding how children learn.

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